



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

A

T R E A T I S E

ON

SURVEYING AND NAVIGATION:

UNITING

THE THEORETICAL, PRACTICAL, AND

EDUCATIONAL FEATURES OF THESE SUBJECTS

By HORATIO N. ROBINSON, A. M.

FORMERLY PROFESSOR OF MATHEMATICS IN THE UNITED STATES NAVY; AUTHOR OF
MATHEMATICAL, PHILOSOPHICAL, AND ASTRONOMICAL WORKS.

THIRD EDITION.

CINCINNATI:

JACOB ERNST, 112 MAIN STREET.

BOSTON:

B. B. MUSSEY & CO.; ROBERT S. DAVIS & CO.; W. J. REYNOLDS & CO.

NEW YORK:

MASON BROTHERS; D. BURGESS & CO.; A. S. BARNES & CO.; NEWMAN & IVISON;
PRATT, WOODFORD & CO.

PHILADELPHIA:

LIPPINCOTT, GRAMBO & CO.; THOMAS, COWPERTHWAIT & CO.;
E. H. BUTLER & CO.; URIAH HUNT & SON.

1853.

ENTERED, according to act of Congress, in the year 1852,
By H. N. ROBINSON,
in the Clerk's Office of the District Court for the Northern District of New York.

~~~~~  
A. C. JAMES, STEREOTYPER,  
167 WALNUT ST., CINCINNATI.

## PREFACE.

---

THIS book is more than its title page proclaims it to be: it is the practical application of the Mathematical Sciences to Mensuration, to Land Surveying, to Leveling, and to Navigation.

Nor is the work merely practical. Elementary principles are here and there brought before the mind in a new light; and original investigations will be found in many parts of the work. To show the reader *how* a thing is to be done, is but a small part of the object sought to be obtained: the great stress is put upon the *reasons* for so doing, which gives true discipline to the mind, and adds greatly to the educational value of any book.

We have illustrated the subject of logarithms, and their practical uses, the same in this book as is common to be found in other books, and this is sufficient for the common pupil, or the ordinary practical man, whether surveyor or navigator; but in addition to this, we have carried logarithms much further in this work than in any I have seen. I do not mean by this that we have more voluminous tables than others. Such is not the fact.

Voluminous tables are not necessary for those who really understand the nature of logarithms, and such are mainly intended for those who are not expected to understand principles. To give a more practical illustration of logarithms, and to suggest artifices in using logarithms generally, we have given Table III and its auxiliaries, on page 70 of tables, showing logarithms to *twelve places* of decimals, a degree of accuracy which *practice never* demands. By the help of this table combined with a true knowledge of the subject, the logarithm of *any number* may be readily found true to *ten places* of decimals, or, conversely, the number corresponding to any given logarithm may be found to almost any degree of accuracy.

Our Traverse Table is not so full as in some other books, but it



is full enough to answer every purpose ; and latitude and departure, corresponding to any course and distance, can be found by it, provided the operator's good judgment is awake. Indeed a contracted table, in an educational point of view, is better than a full one ; for the former calls forth and cultivates tact in the student, but the latter is best for the unanimated plodder.

In running lines, and computing the areas of surveys, we have endeavored to present the subject in such a manner that the reader must constantly keep Elementary Geometry in view, and the whole is so clear and simple, that many will think it unworthy of the rank that it seems to hold in the public estimation, but there are other reasons for this.

The chapter on surveys and surveyors will be found to be a little peculiar, but the information there given, will be highly useful to all those who are inclined to look upon a survey as a mathematical problem only.

On the compass, and the declination of the needle, we have been very full : the subject embraces meridians and astronomical lines drawn on the earth.

The manner in which we should proceed to make a survey, provided no such instrument as the compass existed, and there were no such thing as a magnetic needle, is taken up and illustrated in this work.

The subject of dividing lands is fully discussed and illustrated, and if any one has occasion to complain of mathematical abstrusity in this work, it will be found in this connection ; yet there is nothing here above elementary algebra and geometry.

The method of taking levels and making a profile of the vertical section of a line for rail roads, is set forth in this work. The profile shows the necessary excavation or embankment, which it is necessary to cut down or build up at any particular point, to conform to any proposed grade that may be contemplated.

To determine the elevation of any place above the level of the sea, by means of the barometer, has been, and now is, a very interesting problem to all philosophical students, yet very few of them have been able to comprehend it beyond its first great principle, the variation of atmospheric pressure. To trace, or rather to discover the mathematical law which connects the elevation of any locality with the mean height of the barometer at the same place, has been an obscure problem, and we have taken hold of it with a determination to break open some avenue of light (if such were possible) by which the simplicity of the problem might be brought to the comprehension of the every-day mathematical student, and we believe that we have succeeded in the undertaking.

The part on Navigation, might be regarded, at first view, an abridgment of that subject, and in one sense it is, for we have studied to be as brief as possible, but we would never let brevity stand in the place of perspicuity; and however it may appear, we have given all the mathematical essentials of the subject, and whoever acquires what is here given, will find very little necessity of looking elsewhere for the continuation of the study, unless it is for *sea terms* and seamanship; but these have nothing to do with Navigation as a science. Our method of working *lunars* is more brief than any other, where auxilliary tables and methods of approximation are not resorted to, but to attain this brevity, we have been compelled to use Natural Sines in part of the operation; but on the other hand, this should be no objection, for it gives us a clearer view of the unity and harmony of the mathematical sciences.

# CONTENTS.

---

## INTRODUCTION.

### CHAPTER I.

|                                                                             |           |
|-----------------------------------------------------------------------------|-----------|
| Introduction . . . . .                                                      | PAGE<br>9 |
| Construction of Geometrical Problems, with the use of instruments . . . . . | 12        |

### CHAPTER II.

|                                                                                              |       |
|----------------------------------------------------------------------------------------------|-------|
| Logarithms. . . . .                                                                          | 22    |
| Application of Logarithms to Multiplication, Division, and the extraction of Roots . . . . . | 29—32 |
| Artifices in the use of Logarithms (Art. 12). . . . .                                        | 32—34 |
| Logarithms extended to a greater number of Decimals (Art. 13)                                | 34—43 |

### CHAPTER III.

|                                             |    |
|---------------------------------------------|----|
| Plane Trigonometry. . . . .                 | 44 |
| Explanation of Tables . . . . .             | 58 |
| Oblique-angled Plane Trigonometry . . . . . | 64 |

## SURVEYING.

|                                    |       |
|------------------------------------|-------|
| Introductory Remarks . . . . .     | 70    |
| Finding Areas in general . . . . . | 70—79 |
| Mensuration of Solids . . . . .    | 79    |

### CHAPTER I.

|                                     |       |
|-------------------------------------|-------|
| Mensuration of Lands . . . . .      | 80    |
| To measure a Line. . . . .          | 81    |
| Surveyor's Compass . . . . .        | 82—85 |
| Vernier Scales in general . . . . . | 85—86 |

### CHAPTER II.

|                                        |    |
|----------------------------------------|----|
| Latitude and Departure . . . . .       | 87 |
| Taking angles by the Compass . . . . . | 89 |

## CONTENTS.

vii

|                                              | PAGE.  |
|----------------------------------------------|--------|
| To close a Survey. . . . .                   | 89     |
| To find the true from a random Line. . . . . | 90     |
| Computation of Areas by trapezoids. . . . .  | 91—103 |

### CHAPTER III.

|                                    |         |
|------------------------------------|---------|
| To find Meridian Lines. . . . .    | 103     |
| Variation of the Compass . . . . . | 104—112 |
| Practical Difficulties. . . . .    | 112     |

### CHAPTER IV.

|                                       |     |
|---------------------------------------|-----|
| To Survey without a Compass . . . . . | 113 |
| The Circumferentor. . . . .           | 114 |

### CHAPTER V.

|                                                |     |
|------------------------------------------------|-----|
| Original and subsequent Surveys . . . . .      | 116 |
| Difficulties and Duties of a Surveyor. . . . . | 118 |
| United States' Land. . . . .                   | 121 |

### CHAPTER VI.

|                                                   |         |
|---------------------------------------------------|---------|
| Very irregular Figures. . . . .                   | 123     |
| Division of Lands—a variety of Problems . . . . . | 124—145 |

### CHAPTER VII.

|                                      |         |
|--------------------------------------|---------|
| Triangular Surveying . . . . .       | 146     |
| The Plane Table—its uses &c. . . . . | 147—152 |
| Marine Surveying. . . . .            | 152     |
| Piloting Ships . . . . .             | 154     |

### CHAPTER VIII.

|                                                                                          |         |
|------------------------------------------------------------------------------------------|---------|
| Leveling. . . . .                                                                        | 155     |
| Description of the Level. . . . .                                                        | 158     |
| Adjusting the Level. . . . .                                                             | 159—161 |
| Keeping Book . . . . .                                                                   | 163     |
| Contour of Ground . . . . .                                                              | 165     |
| Elevation determined by atmospheric Pressure, as indicated by<br>the Barometer . . . . . | 166—174 |

## NAVIGATION.

### CHAPTER I.

|                                 |     |
|---------------------------------|-----|
| Introduction . . . . .          | 175 |
| The Log-line. . . . .           | 176 |
| The Mariner's Compass . . . . . | 177 |

## CONTENTS.

## CHAPTER II.

|                                   | Page.   |
|-----------------------------------|---------|
| Plane Sailing . . . . .           | 180—186 |
| Middle Latitude Sailing . . . . . | 186     |
| Traverse Sailing . . . . .        | 186     |
| Sailing in Currents . . . . .     | 191     |

## CHAPTER III.

|                                            |         |
|--------------------------------------------|---------|
| Mercator's Chart—its construction. . . . . | 194—196 |
| Mercator's Sailing . . . . .               | 196     |

## CELESTIAL OBSERVATIONS.

## CHAPTER I.

|                                                                     |         |
|---------------------------------------------------------------------|---------|
| Definition of Terms. . . . .                                        | 198—199 |
| Quadrant and Sextant . . . . .                                      | 202     |
| Construction of the Sextant . . . . .                               | 204     |
| The adjustments of the Sextant. . . . .                             | 205     |
| To take an Altitude of the Sun at sea . . . . .                     | 206     |
| To find the Latitude by the Sun or Stars . . . . .                  | 207     |
| To find the Latitude by the meridian Altitude of the Moon . . . . . | 209—211 |

## CHAPTER II.

|                                      |     |
|--------------------------------------|-----|
| A perfect Time-piece . . . . .       | 213 |
| Local Time—Rule to find it . . . . . | 216 |
| Longitude by Chronometer. . . . .    | 217 |

## CHAPTER III.

|                                                                                                      |         |
|------------------------------------------------------------------------------------------------------|---------|
| Lunar Observations . . . . .                                                                         | 220     |
| Formulæ, for clearing the observed Distance from the effects of<br>Parallax and Refraction . . . . . | 223     |
| Examples for working Lunars. . . . .                                                                 | 224—227 |

## APPENDIX.

|                                                                  |     |
|------------------------------------------------------------------|-----|
| Artifices to be resorted to in difficult circumstances . . . . . | 228 |
| TABLES.                                                          |     |

# INTRODUCTION.

---

## CHAPTER I.

MENSURATION, SURVEYING, and NAVIGATION, are but branches of the same science, and should be regarded as the application of geometry and trigonometry, and in this light we shall present them to our readers.

In this volume we shall not demonstrate geometrical truths unless we wish to present them in some new form, or unless the demonstration is not readily to be found in the proper places, in the elementary books.

It is expected that all readers of a work of this kind, have previously made themselves more or less acquainted with Algebra and Geometry, and where this is the case the reader will have no difficulty ; and readers who are not thus prepared should be careful not to charge *imaginary* defects to the book : in no work of this kind would it be proper to demonstrate every elementary principle. These remarks apply only to the *educational character* of the book.

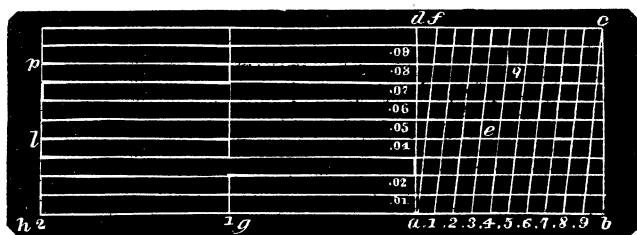
Preparatory to a course of practical mathematics, it is proper to give such descriptions of the instruments to be used as will enable the operator to understand their use. But some of these instruments can never be understood from a book, it must be from the instrument itself ; we might as well attempt to give a person an idea of color by the means of language, as to give a person a correct idea of the sextant and theodolite by a mere book description. It is true we can do something by drawings and descriptions, and that something we intend to do.

To represent plane surfaces and tracts of land on paper, no other instruments are necessary than the scale, and dividers, and a pro-

tractor to measure angles. In fact, every thing can be done with the scale and dividers only — other instruments, as the protractor, sector, and parallel rulers, only add to our convenience; at the same time they could be dispensed with.

#### THE PLANE SCALE.

The plane scale, or the plane diagonal scale of equal parts, as here represented, is the most common and useful of all the instruments used in drawing. It is also a ruler, and if wide and well made will serve as a square also, by which right angles may be drawn.



The very appearance of this scale will show its construction, the side of the square  $a b$  may be of any length whatever, it is generally taken an inch, but this is not imperative.

By means of the 10 parallel lines running along the length of the scale, and the 10 diagonal lines parallel to each other in the square  $a b c d$ , we have 100 *intersections* in the square, by which we are enabled to find any and every hundredth part of the division of  $a b$ .

For example, I wish to find 27 hundredths of the line  $a b$ . I go to the division 2 on  $a b$ , and then run up that diagonal line to the 7th parallel, and from that intersection to the line  $a d$  is 27 hundredths of  $a b$ .

The distances  $a b$ ,  $a g$ ,  $g h$ , &c., may be taken to represent 1, 10, 100, or in fact any number we please. Suppose we take any one of the equal divisions  $a b$ ,  $a g$ , &c., to represent 100, and then require 234. From  $l$  to  $e$  represents that distance.

If the base  $a b$  were 10, from  $l$  to  $e$  would be 23.4; if 1, then from  $l$  to  $e$  would be 2.34; and so on proportionally for any other change of base, or change of the *unit*.

To transfer distances from the scale (as  $l e$ ,  $p q$ , &c.) to paper, we require

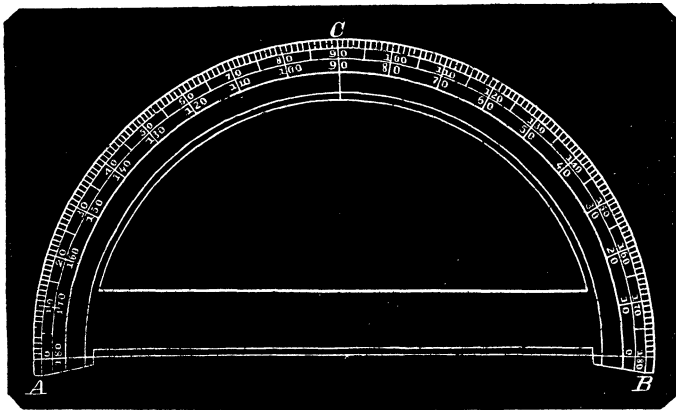
## DIVIDERS.

Dividers are nothing more than a delicate pair of compasses — two bars turning on a joint. They are too well known to require representation by a figure.

They are also used for describing circles and parts of circles.

## THE PROTRACTOR.

The following diagram accurately represents this instrument. It consists of a semicircle of brass  $ABC$ , divided into degrees.



The degrees are numbered both ways, from  $A$  to  $B$  and from  $B$  to  $A$ . There is a small notch in the middle of  $AB$ , to indicate the center.

*To lay off an angle.* Place the diameter  $AB$  on the line, so that the center shall fall on the angular point.

Then at the degree required, at the edge of the semicircle make a point with a pin. Then remove the protractor and draw a line through the point so marked and the angular point; this line, with the given line, will make the required angle.

The reader will observe a great similarity between this instrument and the circumferentor, which is described in a subsequent portion of this work.

This instrument is designed merely to draw angles on paper, that to draw lines marking given angles, with other lines, in the field.



In addition to this, both the protractor and circumferentor may be used in taking levels, and measuring angles of altitude, when *no better instruments* for such purposes are at hand.

For instance, if a delicate plumb should be suspended from the center of the protractor, and the thread rest at the point  $C$ , while the instrument is held in a frame, then  $A$  and  $B$  would be as a level, and as many degrees as the plumb line rested from  $C$  so many degrees would be the inclination of  $A$  and  $B$  from a horizontal level.

Levels and angles of altitudes were formerly taken in this way.

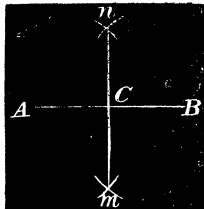
With the instruments previously described, solve the following problems. The references are to Robinson's Geometry. Thus, (th. 15, b. 1, cor. 1,) indicates theorem 15, book 1, corollary 1, where the demonstrations of the problem referred to will be found.

### PROBLEM 1.

*To bisect a given finite straight line.*

Let  $AB$  be the given line, and from its extremities,  $A$  and  $B$ , with any radius greater than the half of  $AB$  (Post. 3), describe arcs, cutting each other in  $n$  and  $m$ . Join  $n$  and  $m$ ; and  $C$ , where it cuts  $AB$ , will be the middle of the line required.

Proof, (th. 15, b. 1, cor. 1).

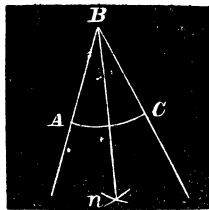


### PROBLEM 2.

*To bisect a given angle.*

Let  $ABC$  be the given angle. With any radius, from the center  $B$ , describe the arc  $AC$ . From  $A$  and  $C$ , as centers, with a radius greater than the half of  $AC$ , describe arcs, intersecting in  $n$ ; and join  $Bn$ , it will bisect the given angle.

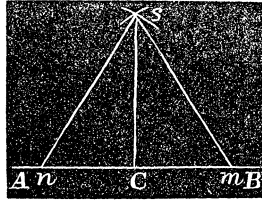
Proof, (th. 19, b. 1).



## PROBLEM 3.

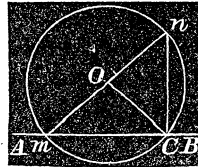
*From a given point, in a given line, to draw a perpendicular to that line.*

Let  $AB$  be the given line, and  $C$  the given point. Take  $n$  and  $m$  equal distances on opposite sides of  $C$ ; and from the points  $m$  and  $n$ , as centers, with any radius greater than  $nC$  or  $mC$ , describe arcs cutting each other in  $S$ . Join  $SC$ , and it will be the perpendicular required. Proof, (th. 15, b. 1, cor. ).



The following is another method, which is preferable, when the given point,  $C$ , is at or near the end of the line.

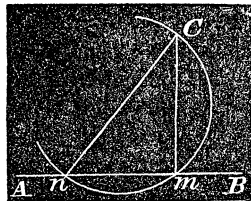
Take any point,  $O$ , which is manifestly one side of the perpendicular, and join  $OC$ ; and with  $OC$ , as a radius, describe an arc, cutting  $AB$  in  $m$  and  $C$ . Join  $mO$ , and produce it to meet the arc, again, in  $n$ ;  $mn$  is then a diameter to the circle. Join  $Cn$ , and it will be the perpendicular required. Proof, (th. 9, b. 3).



## PROBLEM 4.

*From a given point without a line, to draw a perpendicular to that line.*

Let  $AB$  be the given line, and  $C$  the given point. From  $C$ , draw any oblique line, as  $Cn$ . Find the middle point of  $Cn$  by (problem 1), and from that point, as a center, describe a semicircle, having  $Cn$  as a diameter. From the point  $m$ , where this semicircle cuts  $AB$ , draw  $Cm$ , and it will be the perpendicular required. Proof, (th. 9, b. 3).



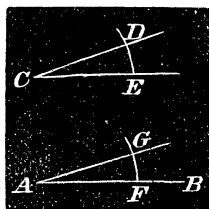
## PROBLEM 5.

*At a given point in a line, to make an angle equal to another given angle.*

Let  $A$  be the given point in the line  $AB$ , and  $DCE$  the given angle.

From  $C$  as a center, with any radius,  $CE$ , draw the arc  $ED$ .

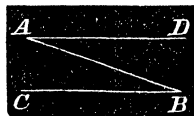
From  $A$ , as a center, with the radius  $AF=CE$ , describe an indefinite arc; and from  $F$ , as a center, with  $FG$  as a radius, equal to  $ED$ , describe an arc, cutting the other arc in  $G$ , and join  $AG$ ;  $GAF$  will be the angle required. Proof, (th. 5, b. 3).



## PROBLEM 6.

*From a given point, to draw a line parallel to a given line.*

Let  $A$  be the given point, and  $CB$  the given line. Draw  $AB$ , making an angle,  $ABC$ ; and from the given point,  $A$ , in the line  $AB$ , draw the angle  $BAD=ABC$ , by the last problem.

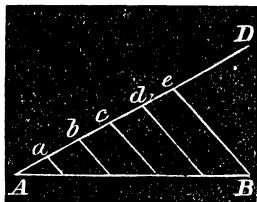


$AD$  and  $CB$  make the same angle with  $AB$ ; they are, therefore, parallel. (Definition of parallel lines).

## PROBLEM 7.

*To divide a given line into any number of equal parts.*

Let  $AB$  represent the given line, and let it be required to divide it into any number of equal parts, say five. From one end of the line  $A$ , draw  $AD$ , indefinite in both length and position. Take any convenient distance in the dividers, as  $Aa$ , and set it off on the line  $AD$ ;



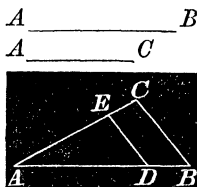
thus making the parts  $Aa$ ,  $ab$ ,  $bc$ , &c., equal. Through the last point,  $e$ , draw  $EB$ , and through the points  $a$ ,  $b$ ,  $c$ , and  $d$ , draw parallels to  $eB$  (problem 6.); these parallels will divide the line as required Proof (th. 17, b. 2).

## PROBLEM 8.

*To find a third proportional to two given lines.*

Let  $AB$  and  $AC$  be any two lines. Place them at any angle, and join  $CB$ . On the greater line,  $AB$ , take  $AD = AC$ , and through  $D$ , draw  $DE$  parallel to  $BC$ ;  $AE$  is the third proportional required.

Proof, (th. 17, b. 2).

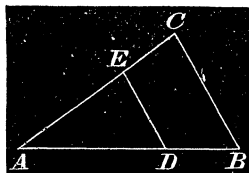


## PROBLEM 9.

*To find a fourth proportional to three given lines.*

Let  $AB$ ,  $AC$ ,  $AD$ , represent the three given lines. Place the first two together, at a point forming any angle, as  $BAC$ , and join  $BC$ . On  $AB$  place  $AD$ , and from the point  $D$ , draw (problem 6)  $DE$  parallel to  $BC$ ;  $AE$  will be the fourth proportional required.

Proof, (th. 17, b. 2).

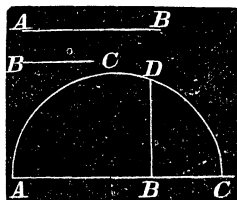


## PROBLEM 10.

*To find the middle, or mean proportional, between two given lines.*

Place  $AB$  and  $BC$  in one right line, and, on  $AC$ , as a diameter, describe a semicircle (postulate 3), and from the point  $B$ , draw  $BD$  at right angles to  $AC$  (problem 3);  $BD$  is the mean proportional required.

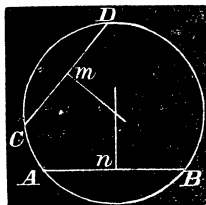
Proof, (scholium to th. 17, b. 3).



## PROBLEM 11.

*To find the center of a given circle.*

Draw any two chords in the given circle, as  $AB$  and  $CD$ ; and from the middle point,  $n$ , of  $AB$ , draw a perpendicular to  $AB$ ; and from the middle point,  $m$ , draw a perpendicular to  $CD$ ; and where these two perpendiculars intersect will be the center of the circle. Proof, (th. 1, b. 3).

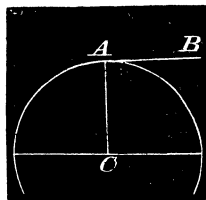


## PROBLEM 12.

*To draw a tangent to a given circle, from a given point, either in or without the circumference of the circle.*

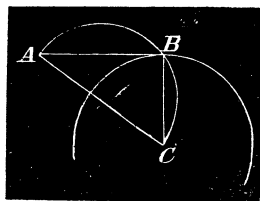
When the given point is in the circumference, as  $A$ , draw  $AC$  the radius, and from the point  $A$ , draw  $AB$  perpendicular to  $AC$ ;  $AB$  is the tangent required.

Proof, (th. 4, b. 3).



When  $A$  is without the circle, draw  $AC$  to the center of the circle; and on  $AC$ , as a diameter, describe a semicircle; and from the point  $B$ , where this semicircle intersects the given circle, draw  $AB$ , and it will be tangent to the circle.

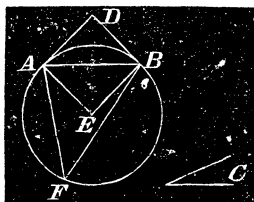
Proof, (th. 9, b. 3), and (th. 4, b. 3).



## PROBLEM 13.

*On a given line, to describe a segment of a circle, that shall contain an angle equal to a given angle.*

Let  $AB$  be the given line, and  $C$  the given angle. At the ends of the given line, make angles  $DAB$ ,  $DBA$ , each equal to the given angle,  $C$ . Then draw  $AE$ ,  $BE$ , perpendicular to  $AD$ ,  $BD$ ; and from the center,  $E$ , with radius,  $EA$  or  $EB$ , describe a circle; then  $AFB$  will be the segment required, as any angle  $F$ , made in it, will be equal to the given angle,  $C$ .

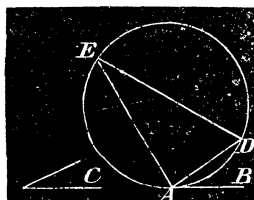


Proof, (th 11. b. 3), and (th. 8, b. 3).

### PROBLEM 14.

*To cut a segment from any given circle, that shall contain a given angle.*

Let  $C$  be the given angle. Take any point, as  $A$ , in the circumference, and from that point draw the tangent  $AB$ ; and from the point  $A$ , in the line  $AB$ , make the angle  $BAD = C$  (problem 5), and  $AED$  is the segment required.

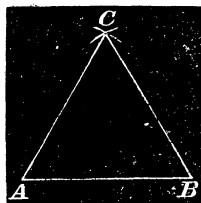


Proof, (th. 11, b. 3), and (th. 8, b. 3).

### PROBLEM 15.

*To construct an equilateral triangle on a given finite straight line.*

Let  $AB$  be the given line, and from one extremity,  $A$ , as a center, with a radius equal to  $AB$ , describe an arc. At the other extremity,  $B$ , with the same radius, describe another arc. From  $C$ , where these two arcs intersect, draw  $CA$  and  $CB$ ;  $ABC$  will be the triangle required.

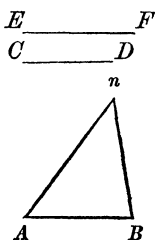


*The construction is a sufficient demonstration.* Or, (ax. 1).

## PROBLEM 16.

*To construct a triangle, having its three sides equal to three given lines, any two of which shall be greater than the third.*

Let  $AB$ ,  $CD$ , and  $EF$  represent the three lines. Take any one of them, as  $AB$ , to be one side of the triangle. From  $A$ , as a center, with a radius equal to  $CD$ , describe an arc; and from  $B$ , as a center, with a radius equal to  $EF$ , describe another arc, cutting the former in  $n$ . Join  $An$  and  $Bn$ , and  $AnB$  will be the  $\triangle$  required. Proof, (ax. 1).

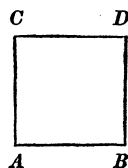


## PROBLEM 17.

*To describe a square on a given line.*

Let  $AB$  be the given line, and from the extremities,  $A$  and  $B$ , draw  $AC$  and  $BD$  perpendicular to  $AB$ . (Problem 3.)

From  $A$ , as a center, with  $AB$  as radius, strike an arc across the perpendicular at  $C$ ; and from  $C$ , draw  $CD$  parallel to  $AB$ ;  $ACDB$  is the square required. Proof, (th. 21, b. 1.)



## PROBLEM 18.

*To construct a rectangle, or a parallelogram, whose adjacent sides are equal to two given lines.*

Let  $AB$  and  $AC$  be the two given lines. From the extremities of one line, draw perpendiculars to that line, as in the last problem; and from these perpendiculars, cut off portions equal to the other line; and by a parallel, complete the figure.

When the figure is to be a parallelogram, with oblique angles, describe the angles by problem 5. Proof, (th. 21, b. 1).

PROBLEM 19.

*To describe a rectangle that shall be equal to a given square, and have a side equal to a given line.*

Let  $AB$  be a side of the given square, and  $CD$  one side of the required rectangle.

Find the third proportional,  $EF$ , to  $CD$  and  $AB$  (problem 8). Then we shall have,

$$CD : AB :: AB : EF$$

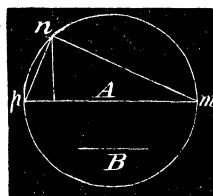
Construct a rectangle with the two given lines,  $CD$  and  $EF$  (problem 18), and it will be equal to the given square, (th. 13, b. 2).

PROBLEM 20.

*To construct a square that shall be equal to the difference of two given squares.*

Let  $A$  represent a side of the greater of two given squares, and  $B$  a side of the lesser square.

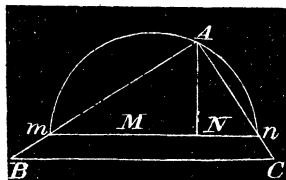
On  $A$ , as a diameter, describe a semicircle, and from one extremity,  $m$ , as a center, with a radius equal to  $B$ , describe an arc,  $n$ , and, from the point where it cuts the circumference, draw  $mn$  and  $np$ ;  $np$  is the side of a square, which, when constructed, (problem 17), will be equal to the difference of the two given squares. Proof, (th. 9, b. 3, and 36, b. 1.)



PROBLEM 21.

*To construct a square, that shall be to a given square, as a line,  $M$ , to a line,  $N$ .*

Place  $M$  and  $N$  in a line, and on the sum describe a semicircle. From the point where they join, draw a perpendicular to meet the circumference in  $A$ . Join  $Am$  and  $An$ , and produce them indefinitely. On  $Am$  or  $An$ , produced, take  $AB =$  to the side of the given square; and from  $B$ , draw  $BC$  parallel to  $mn$ ;  $AC$  is a side of the required square.





Besides the numerical scale of equal parts, we have scales of chords, sines, and tangents, which can be constructed corresponding to any radius.

Such scales of course *are not scales of equal parts*.

Such scales are constructed in the following manner.

Take  $CA$  any radius, and describe a semicircle. Draw  $CD$  at right angles to  $AB$ , and draw a tangent line from  $A$ . Divide the arc  $AD$  into equal parts 10, 20, 30, &c., beginning at  $D$ , and subdivide them as much as required.

Draw 10 10, — 20 20, — 30 30, &c., all parallel to  $CD$ .

From  $C$  to 10 on the line  $CA$ , is the sine of  $10^\circ$ . From  $C$  to 20 is the sine of  $20^\circ$  &c. &c.

The line 10 10 is the sine of  $80^\circ$ , and  $CD$  or  $CA$  is the sine of  $90^\circ$ .

The distance from  $A$  to  $D$  is the chord of  $90^\circ$ , from  $A$  to 10 is the chord of  $80^\circ$ , and from  $A$  to 20 is the chord of  $70^\circ$ , and so on down. Thus we perceive that we can take off any sine or chord and lay it down on a ruler; and chords and sines thus laid off constitute the scale of chords, sines, &c.

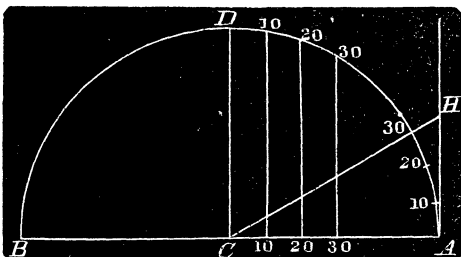
Lines drawn from  $C$  through any division of the arc, commencing at  $A$  to strike the tangent line, will mark off the tangent corresponding to that arc. Thus, if the angle  $ACH$  is  $30^\circ$ , then the line  $AH$  placed on a scale, will represent the tangent of  $30^\circ$  to the radius  $CA$ , and thus any other tangent can be laid down on the same scale.

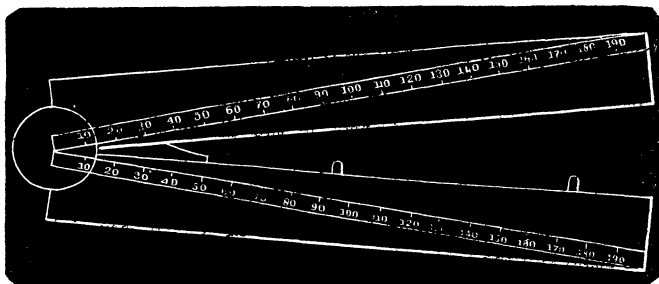
The scale of chords and sines, as well as the scale of equal parts, are to be found on the

#### SECTOR.

The sector is commonly made of ivory, and consists of two arms which open and turn round a joint at their common extremity.

For some operations, particularly the projection of solar eclipses, the sector is a very useful instrument.





The figure before us represents one side of a sector with the plane scale only upon it. More than one scale can be put on to a side, but we represent but one to avoid confusion.

The scale must be alike on both arms — and it must commence exactly at the joint — hence when near the center the different scales *crowd* each other.

The two arms of the sector always form two sides of a triangle, and by opening and closing them we vary the angle, yet the distance across from one arm to the other is always proportional to the sides of the triangle.

The advantage of the sector will appear from the following problem.

A map is before me, its scale is 20 miles to an inch; I wish to find the distance in a right line between two points laid down on it.

1st. I take one inch in the dividers and open the sector, so that the distance between 20 and 20 on the two arms, shall just correspond to the measure in the dividers, that is, shall be one inch. Let the sector lie on the table thus opened.

2nd. Now take the distance you wish to measure. in the dividers; place one foot on one arm of the sector, and the other foot on the other arm; so that the feet of the dividers shall fall on the same number on both arms of the sector.

The number thus marked by the dividers will be the distance required. The distance between any other two points may be measured on the same map, without any computation whatever.

For another illustration of the utility of the sector, let us suppose, that the sine of  $20^\circ$  is required corresponding to a radius of 6 inches.

Take 6 inches in the dividers, and open the sector so that the sine of  $90^\circ$  from arm to arm shall be 6 inches.

The sector being thus open, take the distance from 20 to 20, on the line of sines from arm to arm, in the dividers, and that is the distance required.

#### GUNTER'S SCALE.

Gunter's scale is commonly two feet in length, containing the plane scale and the scale of sines, chords, and tangents on one side of it, and the scale for the *logarithms* of numbers, sines, and tangents on the other.

This scale is very ingenious, but it is not so much used nor considered so important as formerly.

## CHAPTER II.

### LOGARITHMS.

ART 1. *Logarithms are exponents.*

Thus, if  $a^2=9$

and  $a^3=27$

Then  $a^5=243$ ; by multiplying the two equations together term by term.

The exponent 2 of the first equation may be considered the logarithm of 9; the exponent 3 the logarithm of 27, and the exponent 5 the logarithm of the number 243.

In these equations  $a=3$  the base of the system.

By the preceding operation it is obvious that adding the exponents 2 and 3, corresponds to multiplying the numbers 9 and 27.

If we take the equation  $a^5=243$ , and divide it by  $a^2=9$  member by member we shall have

$$a^{5-2}=a^3=27.$$

*Hence adding exponents (logarithms) corresponds to the multiplica-*

*tion of their corresponding numbers and subtracting the exponents (logarithms) corresponds to the division of their numbers.*

It is this property of logarithms that gives them their utility and importance.

ART. 2. The base of our common system of logarithms is 10, and in any equation in the form  $10^x = n$ ,  $x$  is the logarithm of the number  $n$  whatever number  $n$  may represent. If  $n=10$ , then the equation becomes  $10^x=10$ . Whence  $x=1$  because  $10^1=10$ . Therefore in our common system of logarithms the logarithm of 10 must be 1.

Now because

$$10^0=1$$

$$10^1=10$$

$$10^2=100$$

$$10^3=1000$$

$$10^4=10000$$

&c. &c.; it is plain that the logarithm of 1 is 0, of 10 is 1, of 100 is 2, &c., every power of 10 increasing the logarithm by 1.

It is also obvious, that every number between 0 and 10 must have a fractional or *decimal* number for its logarithm, and every number between 10 and 100 must have *one, and some decimal* for its logarithm.

In the equation  $10^x=3$ ,  $x$  is the logarithm of 3, and if we multiply this by  $10^1=10$ , member by member, we shall have

$$10^{1+x}=30$$

Multiply this by

$$10^1 = 10$$

and we have

$$10^{2+x}=300.$$

These results show that 3, 30, 300, have logarithms containing the same *decimal* number;  $x$  differs from each exponent only by whole numbers, and thus, generally; *any number multiplied or divided by 10, or any power of 10, will have logarithms containing the same decimal part.*

ART. 3. For the general computation of logarithms we refer to algebra, and in a work like this we shall only attend to such portions of theory as to enable the student to use them understandingly and with as much practical facility as possible.

Let it be observed that the logarithm

|    |                         |    |          |
|----|-------------------------|----|----------|
| of | 10000                   | is | 4,00000  |
| of | 1000                    | is | 3,00000  |
| of | 100                     | is | 2,00000  |
| of | 10                      | is | 1,00000  |
| of | 1                       | is | 0,00000  |
|    | $\frac{1}{10}$          | is | —1,00000 |
|    | $\frac{1}{100}=10^{-2}$ | is | —2,00000 |

For every division of the number by 10 we subtract 1 from its logarithm, and when the number comes down to 1, and its logarithm of course to 0, if we again divide by 10, making it  $\frac{1}{10}$  or  $10^{-1}$ , we must subtract *one* from the logarithm, making it —1.

The decimal portion of a logarithm is *always positive*, but the *index or whole number part of it, becomes minus when the value of the number is less than 1*.

ART. 4. The whole number belonging to any logarithm is called its *index*, a very appropriate term, because it *indicates*, it points out where to place the decimal point between whole numbers and decimals.

The *index*, or (as some call it) the *characteristic*, is never put in the tables (except from 1 to 100), because we always know what it is. It is always *one less* than the number of digits in the whole number. This is obvious from Art. 3.

Thus, the number 3754 has 3 for the index of its logarithm, because the number consists of 4 digits; that is, the logarithm is 3, *and some decimal*.

The number 34.785 has 1 for the index of its logarithm, because the number is between 34 and 35, and 1 is the index for all numbers between 1 and 100.

All numbers consisting of the same figures, whether integral, fractional, or mixed, have logarithms consisting of the same decimal part. (Art. 2.) The logarithms differ only in their *indices*.

|       |            |       |     |          |              |
|-------|------------|-------|-----|----------|--------------|
| Thus, | the number | 7956. | has | 3.900695 | for its log. |
|       | the number | 795.6 | has | 2.900695 | “            |
|       | the number | 79.56 | has | 1.900695 | “            |
|       | the number | 7.956 | has | 0.900695 | “            |

the number .7956 has —1.900695 for its log.  
 the number .07956 has —2.900695 “  
 &c., &c.

For every division by 10, we diminish the index by 1. When the index is *minus* it indicates a decimal number; but let the learner remember that the index only is minus; the decimal part is *always positive*.

ART. 5. To take out the logarithm of any number from the tables, we only consider the digits; for the logarithms of 7956, or of 7.956, or of .007956, have the same *decimal part*; and when that decimal part is found we then consider the value of the number to prefix the index.

To prefix the index to a decimal, count the decimal point 1, and each cipher 1, up to the first significant figure, and this is the negative index.

For example, find the logarithm of the decimal .00085. To accomplish this we must look for the logarithm of the whole number 85, and we find its decimal part to be .929419; and now, to determine the index, we count *one* for the decimal point and three ciphers, making 4; hence, we have Num. .00085 - log. —4.929419.

The smaller the decimal, the greater the negative index; and when the decimal becomes 0, the logarithm becomes *negatively infinite*.

ART. 6. The logarithm of any number consisting of *four digits or less*, can be taken out of the table directly and without the least difficulty.

Thus, to find the logarithm of the number 3725, we find the number 372 at the side, and over the top we find 5, and opposite the former and under the latter we find .571126 for the decimal part of the logarithm. The 57, the first two decimal, is under 0, which is the same for the whole horizontal column.

Hence, the logarithm of 3725 is 3.571126  
 of 37250 is 4.571126  
 of 3725 is 0.571126  
 &c., &c.

Find the logarithm of 1176. We find 117 at the side, and 6 at the top, and opposite the former and under the latter we find .407

The point here demands a cipher, and is put in to arrest attention to make the operator look to the next horizontal line below for the first two decimals. Thus, we find .070407 for the decimal part of the logarithm required.

Hence, the log. of 1176 is 3.070407

- |                                                 |                |
|-------------------------------------------------|----------------|
| 1. What is the log. of .001176 ?                | Ans. —3.070407 |
| 2. What is the log. of 13.81 ?                  | Ans. 1.140194  |
| 3. What is the log. of 72.55 ?                  | Ans. 1.860637  |
| 4. What is the log. of .6762 ?                  | Ans. —1.830075 |
| 5. What is the logarithm of the number 834785 ? |                |

This number is so large that we cannot find it in the table, but we can find the numbers 8347 and 8348. The logarithms of these numbers are the same as the logarithms of the numbers 834700 and 834800, except the indices.

|             |            |      |           |
|-------------|------------|------|-----------|
|             | 834700     | log. | 5.921530  |
|             | 834800     | log. | 5.921582  |
| Difference, | <u>100</u> |      | <u>52</u> |

Now, our proposed number, 834785, is between the two preceding numbers; and, of course, its logarithm lies between the two preceding logarithms; and, without further comment, we may proportion to it thus,

$$100 : 85 = 52 : 44.2$$

$$\text{Or,} \quad 1. : .85 = 52 : 44.2$$

|                  |           |
|------------------|-----------|
| To the logarithm | 5.921530  |
| Add              | <u>44</u> |

|                                   |                 |
|-----------------------------------|-----------------|
| Hence, the logarithm of 834785 is | 5.921574        |
| the logarithm of 83.4785 is       | <u>1.921574</u> |

From this we draw the following rule to find the logarithm of any number consisting of more than four places of figures.

*RULE.*—Take out the logarithm of the four superior places directly from the table, and take the difference between this logarithm and the next greater logarithm in the table. Multiply this difference by the inferior places in the number as a decimal, and add the result to the logarithm corresponding to the superior places, the sum will be the logarithm required.

Example. Find the log. of 357.32514.

The four superior digits are 3573; the logarithm of these

corresponds to the decimal, .553033, for its decimal part. The inferior digits, taken as a decimal, are

$$\begin{array}{r} .2514 \\ \underline{122} \\ 5028 \\ 5028 \\ \underline{2514} \\ 30.6708 \end{array}$$

This result shows that 30, or more nearly, 31, should be added to the logarithm already found, thus giving .553064 for the decimal part of the logarithm 357.32514.

Therefore, as three digits of the given number are whole numbers, the index must be 2, and the logarithm

$$\begin{array}{lll} \text{of } 357.32514 & \text{is} & 2.553064 \\ \text{of } 3573251.4 & \text{is} & 6.553064 \\ \text{of } .035732514 & \text{is} & -2.553064 \end{array}$$

The change between the place of the decimal point in a number, and the corresponding change of the index to its logarithm, should be strongly impressed on the mind of a learner.

Example 2. What is the log. of 366.25636 ? Ans. 2.563785

3. What is the log. of 39.37079 ? Ans. 1.595174

4. What is the log. of 2.37581 ? Ans. 0.375812

ART. 7. We now give the converse of the last article; that is, we give the decimal part of a logarithm to find its corresponding number.

Taking the decimal in Example 1, (Art. 6,) .553064, we demand its corresponding number.\*

The next less logarithm in the table, is .553033, corresponding to the figure 3573. The difference between this given logarithm and the one next less in the table, is 31; and the difference between two consecutive logarithms in this part of the table, is 122. Now divide 31 by 122, and write the quotient after the number 3573.

---

\* To take out a number from its logarithm, never enter the first part of the table between 1 and 100. Go to the main table, as it contains many more logarithms.



$$\begin{array}{r}
 \text{That is,} \qquad 122)31.(254 \\
 \qquad \qquad \qquad 244 \\
 \hline
 \qquad \qquad \qquad 660 \\
 \qquad \qquad \qquad 610 \\
 \hline
 \qquad \qquad \qquad 500 \\
 \qquad \qquad \qquad 488 \\
 \hline
 \end{array}$$

The figures, then, are 3573254, which corresponds to the decimal logarithm .553064; and the value of these figures will, of course, depend on the index to the logarithm.

If this given logarithm contained an index, such index would point out how many of these figures must be taken for whole numbers, the others will be decimals; thus, if the index had been 4, the number would be 35732.54

If the given decimal had been .553063.67, which is the exact converse of example 1, then we should have found that number, 35732514; but we did not give that decimal logarithm, because the table contains only six decimal places. From this obvious operation we derive the following rule to find the number corresponding to a given logarithm.

*RULE. — If the given logarithm is not in the table, find the one next less, and take out the four figures corresponding; and if more than four figures are required, take the difference between the given logarithm and the next less in the table, and divide that difference by the difference of the two consecutive logarithms in the table, the one less, the other greater than the given logarithm; and the figures arising in the quotient, as many as may be required, must be annexed to the former figures taken from the table.*

#### EXAMPLES.

1. Given, the logarithm 3.743210, to find its corresponding number true to three places of decimals. Ans. 5536.182
2. Given, the logarithm 2.633356, to find its corresponding number true to two places of decimals. Ans. 429.89
3. Given, the logarithm —3.291742, to find its corresponding number. Ans. .0019577

MULTIPLICATION BY LOGARITHMS.

ART. 8. If the principle first laid down in (ART. 1) is true, the sum of the exponents will be the exponent of the product of any number of factors. In other words,

*The sum of the logarithms of any number of factors will be the logarithm of the product of those factors.*

N. B. The logarithmic table corresponds to this principle, and we may see by the following

EXAMPLES.

The log. of 3 (taken from the table,) is 0.477121

The log. of 4 “ “ “ “ is 0.602060

Therefore the log. of 12 must be 1.079181

Given, the log. of 7 and the log. of 9, to find the logarithm of 63. Because  $7 \times 9 = 63$ , therefore,

To log. 7 = 0.845098

Add log. 9 = 0.954243

Sum 1.799341

By inspecting the table, we shall find this logarithm stands opposite 63, and by this process the logarithms of all the composite numbers have been found. In this we may consider that the logarithm pointed out the product 63.

Hence we have the following rule for obtaining the product of any number of factors.

RULE. — *Find the logarithm of each factor, add those logarithms together and the sum will be the logarithm of the product. The number corresponding to this last logarithm taken from the table, will be the product itself.*

EXAMPLES.

|                                |                 |                                      |                 |
|--------------------------------|-----------------|--------------------------------------|-----------------|
| 1. To multiply 23.14 by 5.062. |                 | 2. To multiply 2.581926 by 3.457291. |                 |
| Numbers.                       | Logs.           | Numbers.                             | Logs.           |
| 23.14                          | 1.364363        | 2.581926                             | 0.411944        |
| 5.062                          | 0.704322        | 3.457291                             | 0.538736        |
| Product 117.1347               | <u>2.068685</u> | Product.                             | <u>0.950680</u> |

3. To mult. 3.902 and 597.16  
and .0314728 all together.

| Numbers.      | Logs.           |
|---------------|-----------------|
| 3.902         | 0.591287        |
| 597.16        | 2.776091        |
| .0314728      | —2.497935       |
| Prod. 73.3333 | <u>1.865313</u> |

Here the—2 cancels the 2, and  
the one to carry from the decimals  
is set down.

4. To mult. 3.586, and 2.1046,  
and 0.8372, and 0.0294 all  
together.

| Numbers.        | Logs.            |
|-----------------|------------------|
| 3.586           | 0.554610         |
| 2.1046          | 0.323170         |
| 0.8372          | —1.922829        |
| 0.0294          | —2.468347        |
| Prod. 0.1057618 | <u>—1.268956</u> |

Here the 2 to carry cancels the  
—2, and there remains the —1 to  
set down.

#### DIVISION BY LOGARITHMS.

ART. 9. As division is the converse of multiplication we draw the following rule for division by use of logarithms.

N. B. Addition and subtraction is to be understood in the algebraic sense.

RULE. — *From the logarithm of the dividend subtract the logarithm of the divisor, and the number corresponding to the remainder is the quotient required.*

#### EXAMPLES.

1. Divide 327.5 by 2207

|          |        |                  |
|----------|--------|------------------|
| log.     | 327.5  | 2.515211         |
| log.     | 2207   | 3.342028         |
| Quotient | .14839 | <u>—1.173183</u> |

2. Divide .054 by 1.75

|          |         |                  |
|----------|---------|------------------|
| log.     | .054    | —2.732394        |
| log.     | 1.75    | 0.243038         |
| Quotient | .030857 | <u>—2.489356</u> |

ART. 10. The preceding examples in multiplication and division were adduced only to show the nature of logarithms: had our object been results, the common arithmetical operations would have been more convenient for some of them; but there are cases that demand the use of logarithms, and such cases mostly occur in Involution and Evolution.

**RULE FOR INVOLUTION.**—*Take out the logarithm of the given number, and multiply it by the index of the proposed power. Find the number corresponding to the product, and it will be the power required.*

EXAMPLES.

|                                  |                               |
|----------------------------------|-------------------------------|
| 1. What is the 2d power of 351 ? | 2. What is the cube of 1.72 ? |
| log. 351 2.545307                | log. 1.72 0.235528            |
| 2                                | 3                             |
| Ans. 123201 5.090614             | Ans. 5.0884 0.706584          |

|                                     |                                     |
|-------------------------------------|-------------------------------------|
| 3. What is the 4th power of .0916 ? | 4. What is the 17th power of 1.04 ? |
| log. .0916 —2.961895                | log. 1.04 0.017033                  |
| 4                                   | 17                                  |
| Ans. .000070401 —5.847580           | 0.119231                            |
|                                     | 0.17033                             |

Here 4 times the negative index is —8, adding the 3 to carry gives —5.

Ans. 1.9476 0.289561  
N. B. This last example begins to disclose the utility of logarithms.

|                                     |                                     |
|-------------------------------------|-------------------------------------|
| 5. What is the 6th power of 1.037 ? | 6. What is the 21st power of 2.02 ? |
| log. 1.037 0.015779                 | log. 2.02 0.305351                  |
| 6                                   | 21                                  |
| Ans. 1.243+ 0.094674                | .305351                             |
|                                     | 6.10702                             |
|                                     | Ans. 2584454.6 6.412371             |

EVOLUTION.

**ART. 11.** Evolution is the converse of Involution; hence we have the following rule for the extraction of roots :

*Take the logarithm of the given number out of the table. Divide the logarithm, thus found, by the index of the required root; then the number corresponding is the root sought.*

## EXAMPLES.

|                                                |                                      |
|------------------------------------------------|--------------------------------------|
| 1. What is the cube root of 125?               | 2. What is the cube root of 200?     |
| log. 125      3)2.096910                       | log. 200      3)2.301030             |
| Ans.          5          0.698970              | Ans.          5.848+      0.767010   |
| 3. What is the 4th root of 751?                | 4. What is the 20th root of 1.035?   |
| log. 751      4)2.875640                       | log. 1.035    20)0.014940            |
| Ans.          5.235+      0.718910             | Ans.          1.001718      0.000747 |
| 5. What is the cube root of the decimal .00048 |                                      |
| log. .00048      —4.681241                     |                                      |

To the inexperienced here would be a difficulty, as the index is *negative*, and the decimal part *positive*. How then shall we divide by 3? Add  $-2$  and  $+2$  to the index; and this is, in effect, adding *nothing*; it merely changes the form of the index, thus,  $-6 + 2.681241$ . Now, we can divide by 3, and the quotient is  $-2.893747$ . The corresponding number, or root, sought, is .07829+ Ans.

REMARK.—In the preceding articles we have taught all the preliminary rules for the use of logarithms; “*but there is a wisdom beyond rules*,” and he who does not arrive at it, attains only the burdens of knowledge without its benefits. Rules are necessary through the first rudiments of any science; but he who can instantly fall back on to first principles, and do the most advantageous thing at the most advantageous point of time, has a practical tact of the highest value.

To awaken this faculty in the mind of the learner, we give what follows on the subject of logarithms. It may not be necessary for some, but to many it will be interesting and new

ART. 12. Persons who possess both theoretical knowledge and practical skill, rarely, if ever, use the first part of the logarithmic table of numbers, *except for exact numbers*, especially if they pretend to any thing like accuracy. Such persons take some artifice to *throw* their logarithm into the last part of the table, *where the variation of the logarithm is slower* than in the first part of the table.

To illustrate these remarks let us take into consideration the resulting logarithm to Example, 4, Art. 11.

It would be troublesome, and, indeed, quite impossible, to take out the number corresponding to this logarithm 0.000747 if we simply apply the usual rules, and go directly to the table with the logarithm.

|                          |       |                 |
|--------------------------|-------|-----------------|
| From the given log.      |       | 0.000747        |
| Subtract the log. of     | 1.01  | <u>0.004321</u> |
| This log. corresponds to | .9918 | —1.996426       |

Subtracting the logarithm of 1.01 was equivalent to dividing the number by 1.01. We must, therefore, multiply .9918 by 1.01 to produce the number corresponding to the log. 0.000747.

|       |             |
|-------|-------------|
| Thus, | 0.9918      |
|       | <u>9918</u> |
|       | 1.001718    |

Subtracting the logarithm of 1.01 produced a logarithm having a large decimal part, and this was the object. We can, then, take it into the table, and find its number, to great accuracy, by mere inspection. We might have added the logarithm of 9, and thus produced a *large decimal*, and then have divided the corresponding number by 9, for the required result.

Again: Suppose the logarithm of the number 101248, was required to as great exactness as our tables will allow. The operator who exercises no original thought, and depends only upon rules, will go directly to the table for the logarithm required; but he cannot find it there without some trouble to proportion to it; *and even then his result will not be accurate*, because the logarithms vary by no exact numerical ratio.

Take the number and divide by some number that will give a *large integer* quotient. Thus,

$$\begin{array}{r}
 102)101248(992.6\frac{1}{4} \\
 \underline{918} \\
 944 \\
 \underline{918} \\
 268 \\
 \underline{204} \\
 640 \\
 \underline{612}
 \end{array}$$

Now, we can find the logarithm of  $992.6\frac{1}{4}$  very accurately, by

inspection. Not regarding the fraction  $\frac{1}{4}$  it would be very accurate ; but the practical man always makes a little correction for such fractions, *without taking any proportion to do so*, or using any formality about the matter. In this case, we perceive that 10 or 11, added to the last two decimal figures, will correct it ; hence,

|                           |                 |
|---------------------------|-----------------|
| log. 992.6 $\frac{1}{4}$  | 2.996785        |
| log. 102                  | <u>2.008600</u> |
| Therefore, log. 101248 is | 5.005385        |

We give one more example.

|                                     |                 |
|-------------------------------------|-----------------|
| What number corresponds to the log. | 2.111497        |
| Add log. of 7                       | <u>0.845098</u> |
| Num. 904.88                         | log. 2.956595   |

Dividing this by 7, gives 129.27 for the number required.

REMARK.—The foregoing comments and illustrations are sufficient for all practical problems, that can come before the surveyor, navigator, or engineer. The common table of logarithms, to six decimal places, extending from page two to twenty, of tables, is sufficiently accurate for all common problems ; but there are cases in Astronomy, and in very delicate scientific investigations, where it is important to have the logarithms extend to a greater number of decimal places. Accordingly I have computed a table to 12 decimal places, corresponding to the consecutive whole numbers up to 110, and the prime numbers from thence to 1129. These logarithms, together with the auxiliary logarithms on page 71, are sufficient to find the logarithm of any number that can be proposed, and to find the number corresponding to any given logarithm containing ten decimal places.

It is a general impression that a table of logarithms must be practically useless, unless it is voluminous and complete ; and for constant practical use it should be so ; but for occasional service, the tables here given are sufficient, and for *educational purposes* they are better than they would be if they were more full, for now they demand proper theoretical knowledge to use them with success. On the contrary, whoever cannot use them with success, must be deficient in theoretical knowledge, or wanting in practical tact to bring such knowledge into immediate use.

To show the importance and practical utility of these tables, is the object of the following illustrations and examples.

ART. 13. To make a table of logarithms anew, to contain any particular number of decimal places, the following formula taken from algebra, appears to be the most practical and convenient.

The investigation of the formula belongs to the science of algebra, and not to a work like this.

$$\log(z+1) - \log z = 0.8685889638 \left( \frac{1}{2z+1} + \frac{1}{3(2z+1)^3} + \frac{1}{5(z+1)^5} \text{ \&c.} \right)$$

By this formula we perceive that the log. of  $(z+1)$  becomes known when that of  $z$  is known; but the log. of  $z$  is known when  $z=1, 10, 100, 1000, \text{ \&c.}$  Then the formula will give the logarithms of 2, 11, 101, 1001, &c.

After a commencement has been made and the logarithms of a few numbers obtained, the logarithms of others can be deduced from them — hence the formula is used for the prime numbers only.

When  $z$  is large, over 100, the series converges very rapidly and only two terms need be used. When  $z$  is over 2000 only one term is required *even for twelve decimal places*.

The auxiliary logarithms marked *A, B, C* page 71, were computed by this formula. For example, the log. of 1000 is 3,000000: make  $z=1000$ , then  $(z+1)=1001$ , and  $(2z+1)=2001$ . The formula now readily gives the log. of 1001; and the log. of 1.001 is the same, if we suppose the index or rather make the index 0. Having the log. of 1001, we find that of 1002, and thus we run through *A*. In the same manner we run through *B* and *C*.

The greater the number the more readily can its log. be computed.

That the learner may fully comprehend the application of the auxiliary logarithms *A, B*, and *C*, he must call to mind the following principle.

**ART. 14.** *The product of any number of factors consisting of one and a small fraction, is very nearly equal to one and the sum of those fractions.*

Thus the product of (1.0001) (1.00002) (1.000003) is very nearly equal to 1.000123.

If this be true, we can immediately separate 1.000123 or any other similar quantity in the following factors.

$$(1.0001) (1.00002) (1.000003)$$

The number 1.00008 may be taken as the product of (1.00002) (1.00005) (1.00001) without any material error.

This principle may be proved algebraically, thus: Let  $a, b$ , and  $c$  represent very small fractions, then the product of

$$(1+a) (1+b) = 1 + a + b + ab.$$



But  $a$  and  $b$  being very small fractions their product  $ab$  is still much less, and the material part of the whole product is  $1+a+b$ . Multiply this by  $(1+c)$  making the same consideration, and we shall have  $1+a+b+c$  for the essential value of  $(1+a)(1+b)(1+c)$ .

Try it by numbers, thus : multiply 1.0001 by 1.00004.

$$\begin{array}{r} 1.00004 \\ 1.0001 \\ \hline 100004 \\ 1.00004 \\ \hline 1.000140004 \end{array}$$

But the value of this is extremely near 1.00014, *the sum of unity and the fractional parts of the factors.*

ART. 15. When the difference of two quantities of the same kind, is very small in relation to the quantities themselves, such a difference is called a differential.

Thus, the difference between 1.000140004 and 1.00014 is .000000004, and it may be called the differential of 1.00014, and in reference to it may be omitted.

The difference between 8 and 9 is 1, but in this case 1 cannot be considered the differential of 8, it is too large.

But the difference between 8000 and 8001 is 1, and here 1 is sufficiently small to be considered as the differential of 8000.

There is no exact line of demarkation where a difference may be taken for a differential, that depends on the nature of the case ; hence the prejudice in a certain class of minds against the calculus.

Now, if we take the logarithmic formula from Art. 13, and conceive  $z$  to be very large, then the difference between  $(z+1)$  and  $z$  which is 1, may be considered as the *differential* between the two numbers ; and in that case  $\log. (z+1) - \log. z$ , is the same as the differential of the logarithm of  $z$ .

Making this supposition, the formula in Art. 13 becomes

$$(\text{dif.}) \log. z = 0.8685889638 \times \frac{(\text{dif.}) z}{(2z+1)}$$

We take only one term of the series, because the other terms are of no essential value, compared with the first ; moreover, as  $z$  is very large,  $2z+1$  is comparatively so little greater than  $2z$ , that *for all*

*practical purposes* it may be taken as  $2z$ ; this being admitted, the preceding equation reduces to

$$(\text{dif.}) \log. z = \frac{0.4342944819 (\text{dif.}) z}{z}$$

The symbol (dif.) stands for the *differential* of the quantity. Observe that the decimal 0.4342944819 is the *modulus* of our system of logarithms.

Now this equation put in words, is the following :

*The differential of a logarithm is equal to the modulus, into the differential of the number divided by the number.*

This equation also gives

$$(\text{dif.}) z = \frac{z (\text{dif.}) \log. z}{0.4342944819}$$

Or in words,

*The differential of a number is equal to the number into the differential of the logarithm of the number divided by the modulus.*

The practical use of these principles will be shown in the following articles.

**ART. 16.** When the diameter of a circle is 1, the circumference is 3.14159265359. Find the log. of this number, true to at least ten decimal places.

When the logarithm is found its index will be 0. Now, consider the digits as composing one whole number, and then pay no attention to the index.

Take the three superior digits 314. Its factors are 2.157. (Table commencing on page 67.)

|          |               |      |                |
|----------|---------------|------|----------------|
|          | Log. 157      |      | 2.195899653409 |
|          | Log. 2        |      | 0.301029995644 |
|          | Log. 314      |      | 2.496929649053 |
| Table B. | 1.0005        | log. | 0.000217099966 |
| Prod.    | 3141570       | log. | .497146749019  |
| Table C. | 1.000007      | log. | 3040058 (76)   |
|          | 2199099       |      |                |
|          | 314157        |      |                |
| Prod.    | 314159199099— | log. | .497149789077  |

REMARK.—Let the learner take hold of the preceding problem with great deliberation, understand the reason of every part of the process — and make every necessary consideration, and then he will understand how to manage every problem of the like kind.

Here then, we have the exact decimal part of the logarithm to these digits. If the value of that superior digit 3 is simply three, then the index to the log. is 0 ; if it is 30, 1, if 300, 2, &c. Knowing the value to be 3, we shall in the end, put the index at 0.

We have obtained the exact logarithm of a certain number, but it is not the number required ; it is a number however, very near the number required. Take their difference

$$\begin{array}{r} \text{From } 314159265359 \\ \text{Take } 314159199099 \\ \hline 66260 \end{array}$$

This difference, great as it may appear by itself, is so small in relation to 314159265359 that it may be taken as its *differential*.

But corresponding to this differential of the number, there is a differential for the logarithm, which is given by the equation in Art. 15.

$$\text{That is, (dif.) log. } z = \frac{(0.4342944819)(66260)}{314159265359}$$

It would be a tedious operation to draw out the result of this expression arithmetically. We will, therefore, use the common table of logarithms, which will answer every purpose.

We have the logarithm of the *modulus* as a constant quantity on page 71 of tables ; and having the logarithm of the denominator, we have only to look for the logarithm of 66260, the operation stands thus,

$$\begin{array}{r} \log. 66260 \quad 4.821251 \\ \log. m. \quad \underline{-1.637784} \\ 4.459035 \\ \log. z. \quad 11.497150 \\ \text{Num. } 0.000000091596 \log. \quad \underline{-8.961885} \end{array}$$

$$\begin{array}{r} \text{To} \quad 0.497149789077 \\ \text{Add} \quad 0.000000091596 \\ \text{Log. of } 3.14159265359 = \quad 0.497149880673 \end{array}$$

The factor 1.0005 was obvious enough from inspection ; but the

other factor 1.000007 is not obvious. The question then arises, how did we find it?

Wanting a factor, which with the other factor 314157, would produce the given number, we represented it by  $x$ , then

$$314157x = 314159265359$$

By division  $x = 1000007 +$

The sidereal year consists of 365.2563744 mean solar days. What is the logarithm of this number?

We know that the index must be 2, therefore pay no attention to the index during the operation.

Take the three superior digits 365 : its factors are 73 and 5.

|          |              |      |                          |
|----------|--------------|------|--------------------------|
|          | 73           | log. | 1.863 322 860 120        |
|          | <u>5</u>     | log. | <u>698 970 004 336</u>   |
| Prod.    | 365          | log. | 2.562 292 864 456        |
| Table B, | <u>10007</u> | log. | <u>0.000 303 836 798</u> |
| Prod.    | 3652555      | log. | 2.562 596 701 254        |
| C        | { 1.000002   | log. | 868 588 (2b)             |
|          | { 1.0000003  | log. | 130 287 (3c)             |
|          | { 1.00000009 | log. | <u>39 087 (9d)</u>       |

Prod. = given number nearly, log. = 2.562 597 739 216

We found these factors by taking the value of  $x$  out of the following equation :

$$3652555x = 3652563744$$

Whence,  $x = 1.00000239$ .

And by Art. 14,  $x = (1.000002)(1.0000003)(1.00000009)$

In place of these last factors we might have taken the differential equation (Art. 15), and that would have been more direct and to the point. The factor method but approximates to the given number : the differential method comes directly to it, but it will not be so generally understood as the factor method.

The differential equation applied in place of the last three factors, gives 2.562 597 740 854 for the required logarithm. But, if *very great* accuracy was required, the factor (1.000002) should be employed and then the differential equation.

When the radius of a circle is 1, the natural sine of  $7^\circ 30'$  is expressed by the decimal 0.1305261921 what is the logarithm

of this number, true to nine places of decimals, consider the decimal a whole number?

Take its four superior digits 1305 : the factors of these are 15 and 87 ; therefore,

|       |              |             |                     |
|-------|--------------|-------------|---------------------|
|       | .87          | log.        | —1. 939 519 252 619 |
|       | <u>.15</u>   | log.        | —1. 176 091 259 059 |
| Prod. | .1305        | log.        | —1. 115 610 511 678 |
|       | <u>10002</u> | log.        | 0. 000 086 850 213  |
|       | 13052610     | log.        | —1. 115 697 361 891 |
|       |              | Correction, | <u>306 450</u>      |
| Num.  | .1305261921  | log.        | —1. 115 697 668 341 |

For the radius of our common tables add, 10

Hence, log. sine of  $7^{\circ} 30' = 9. 115 697 668 341$

In this manner the *logarithmic sine* of any other arc can be found when we have its *natural sine*.

The correction was found by the differential equation, Art. 15, thus,

$$(\text{dif.}) \log. z = \frac{(0.4342944819) (921)}{1305261000}$$

Having the logarithm of the modulus, and of the denominator, we can readily deduce the result by logarithms, using the common table to find the logarithm of 921.

|      |               |      |           |
|------|---------------|------|-----------|
|      | 921           | log. | 2.964260  |
|      | <i>m.</i>     | log. | —1.637784 |
|      | Numerator,    | log. | 2.602044  |
|      | Denom.        | log. | 9.115697  |
| Num. | .000000306450 | log. | —7.486347 |

We give one more problem of this kind.

The mean distance between the Sun and Earth is 1 ; the greatest distance in 1800 was 1.01685317 : what is the logarithm of this number ?

Take the whole as the whole number, 101685317, and pay no attention to the index during the process.

The four superior digits are 1016 ; the factors of this number are 8 and 127 : hence,

|                  |                  |      |                         |
|------------------|------------------|------|-------------------------|
|                  | 8                | log. | .903 089 986 992        |
|                  | <u>127</u>       | log. | <u>.103 803 720 956</u> |
| Prod.            | 1016             | log. | .006 893 707 948        |
| Table <i>B</i> . | <u>10008</u>     | log. | <u>.000 347 233 698</u> |
| Prod.            | 10168128         | log. | .007 240 941 646        |
| Table <i>C</i> . | <u>100004</u>    | log. | <u>17 371 430</u>       |
|                  | 10168128         |      |                         |
|                  | <u>40672512</u>  |      |                         |
| Prod.            | 1016853472512    | log. | .007 258313 076         |
| From             | 1016853472512    |      |                         |
| Take             | <u>101685317</u> |      |                         |
|                  | 30.25            |      |                         |

Taking the given quantity as a whole number, our last product exceeds it by  $30\frac{1}{4}$ , which is the differential of the number. The differential of the logarithm will therefore be  $\frac{m. 30.25}{z}$

|                   |                 |                         |
|-------------------|-----------------|-------------------------|
| By log.           | log. <i>m</i>   | —1. 637 784             |
|                   | log. 30.25      | <u>1. 480 725</u>       |
|                   |                 | 1. 118 509              |
|                   | log. <i>z</i> * | <u>8. 007 257</u>       |
| log. 0.0000001292 |                 | —7. 111 252             |
|                   | From            | .007 258 313 076        |
|                   | Take            | <u>.000 000 129 200</u> |
| Log. sought       |                 | 0. 007 258 183 876      |

From the foregoing problems, the reader will perceive that the logarithm of any number whatever, can be found by these tables — and to any degree of accuracy within ten decimal places. We are now prepared to take the converse problem ; that is,

*Given a logarithm, to find its corresponding number.*

What number corresponds to the log. 4. 636 747 519 487 ? Carrying this log. to the table, we find it corresponds to a number a little greater than 43, but the index being 4, the number must be a little over 43000. Let this be *one factor* of the number sought, and the reason for the following operation must be obvious.

---

\*  $z=101685317$ , a number now considered as a whole number, its log. is taken approximately, and its index is 8.

|                        |                      |                           |
|------------------------|----------------------|---------------------------|
|                        | Given log.           | 4.636747519487            |
| 1st Factor, 43000      | log.                 | <u>4.633468455579</u>     |
|                        |                      | 3279063908                |
| 2nd Factor, 1.007      | table <i>A.</i> log. | <u>3030465635</u>         |
|                        |                      | 248598273                 |
| 3rd Factor, 1.0005     | table <i>B.</i> log. | <u>217099966</u>          |
|                        |                      | 31498307                  |
| 4th Factor, 1.00007    | table <i>C.</i> log. | <u>30399546</u>           |
|                        |                      | 1098761                   |
| 5th Factor, 1.000002   | log.                 | <u>868588 (2<i>b</i>)</u> |
|                        |                      | 230173                    |
| 6th Factor, 1.0000005  | log.                 | <u>217145 (5<i>c</i>)</u> |
|                        |                      | 13028                     |
| 7th Factor, 1.00000003 | log.                 | <u>13029 (3<i>d</i>)</u>  |

The product of these factors is the number required, and that product can be obtained with great facility.

Prod. of 1st and 2nd factors=43301

This, by the 3rd 10005

43301

21.6505

43322.65050000

Prod. of 4th, 5th, 6th, 7th 1.00007253

43322.6505

3.032585535

866453010

2166132525

1299679515

Num. required 43325.792691840765

2. What number corresponds to log. 2. 563 785 181 020 ?

Taking this decimal log. to the table, we find that its place is between 36. and 37. and as the index is 2., it must be between 360. and 370., nearer the latter than the former. Suppose it near 366. It is not 367., for the table gives a log. of 367., greater than our given log.

|                   |                                    |
|-------------------|------------------------------------|
| 366=6×61          |                                    |
|                   | From given log. 2. 563 785 181 020 |
| Sub. log. 61      | 1. 785 329 835 011                 |
|                   | <hr/> 0. 778 455 346 009           |
| log. 6            | 0. 778 151 250 384                 |
|                   | <hr/> 304 095 625                  |
| 3rd Factor 1.0007 | 303 836 798                        |
|                   | <hr/> 258 827                      |

The product of these three factors is 366.2562, which is very near the number required.

*When a logarithm is reduced below its sixth decimal place, what is left may be taken as a differential of the given logarithm.*

This differential will give a corresponding differential, to be applied to the number by using the equation in Art 15.

$$\text{That is (dif.) } N = \frac{N \text{ (dif.) log.}}{\text{modulus}}$$

|                             |            |                |
|-----------------------------|------------|----------------|
| By the common table of log. | Given log. | 2. 563 785     |
| .0000002588                 | log.       | —7. 412 964    |
|                             |            | —5. 976 749    |
| log. <i>m</i>               |            | —1. 637 784    |
| 0.0002182                   | log.       | —4. 338 965    |
| Add                         |            | <hr/> 366.2562 |

Number sought 366.2564182

In this manner, the number to any logarithm may be found.

#### EXAMPLES.

1. What number corresponds to the log. 2.204923118054 ?  
Ans. 160.29616
2. What number corresponds to the log. 4.133409102 ?  
Ans. 13595.93
3. What number corresponds to the log. 3.278902074620 ?  
Ans. 1900.64967



## CHAPTER III.

ELEMENTARY PRINCIPLES OF PLANE  
TRIGONOMETRY.

TRIGONOMETRY in its literal and restricted sense, has for its object, the measure of triangles. When the triangles are on planes, it is plane trigonometry, and when the triangles are on, or conceived to be portions of a sphere, it is spherical trigonometry. In a more enlarged sense, however, this science is the application of the principles of geometry, and numerically connects one part of a magnitude with another, or numerically compares different magnitudes.

As the *sides* and *angles* of triangles are quantities of different kinds, they cannot be *compared* with each other; but the *relation* may be discovered by means of other complete triangles, to which the triangle under investigation can be compared.

Such other triangles are numerically expressed in Table II, and all of them are conceived to have one common point, the center of a circle, and as all possible angles can be formed by two straight lines drawn from the center of a circle, no angle of a triangle can exist whose measure cannot be found in the table of trigonometrical lines.

The measure of an angle is the arc of a circle, intercepted between the two lines which form the angle—the center of the arc always being at the point where the two lines meet.

The arc is measured by *degrees*, *minutes*, and *seconds*, there being 360 degrees to the whole circle, 60 minutes in one degree, and 60 seconds in one minute. Degrees, minutes, and seconds, are designated by °, ', ". Thus 27° 14' 21", is read 27 degrees, 14 minutes, and 21 seconds.

All circles contain the same number of degrees, but the greater the radii the greater is the absolute length of a degree; the circumference of a carriage wheel, the circumference of the earth, or the still greater and indefinite circumference of the heavens, have the same number of degrees; yet the same number of degrees in each and every circle is precisely the same angle in amount or measure.

As triangles do not contain circles, we can not measure triangles by circular arcs; we must measure them by *other triangles*, that is, by *straight lines*, drawn in and about a circle.

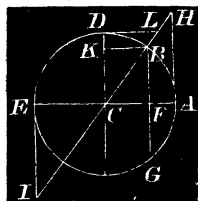
Such straight lines are called trigonometrical lines, and take particular names, as described by the following

## DEFINITIONS.

1. The *sine* of an angle, or an arc, is a line drawn from one end of an arc, perpendicular to a diameter drawn through the other end. Thus,  $BF$  is the sine of the arc  $AB$ , and also of the arc  $BDE$ .  $BK$  is the sine of the arc  $BD$ , it is also the cosine of the arc  $AB$ , and  $BF$  is the cosine of the arc  $BD$ .

N. B. The *complement* of an arc is what it wants of  $90^\circ$ ; the *supplement* of an arc is what it wants of  $180^\circ$ .

2. The *cosine* of an arc is the perpendicular distance from the center of the circle to the sine of the arc, or it is the same in magnitude as the sine of the complement of the arc. Thus,  $CF$  is the cosine of the arc  $AB$ ; but  $CF=KB$ , the sine of  $BD$ .



3. The *tangent* of an arc is a line touching the circle in one extremity of the arc, continued from thence, to meet a line drawn through the center and the other extremity.

Thus,  $AH$  is the tangent to the arc  $AB$ , and  $DL$  is the tangent of the arc  $DB$ , or the cotangent of the arc  $AB$ .

N. B. The *co*, is but a contraction of the word *complement*.

4. The *secant* of an arc, is a line drawn from the center of the circle to the extremity of its tangent. Thus,  $CH$  is the secant of the arc  $AB$ , or of its supplement  $BDE$ .

5. The *cosecant* of an arc, is the secant of the complement. Thus,  $CL$ , the secant of  $BD$ , is the cosecant of  $AB$ .

6. The *versed sine* of an arc is the difference between the cosine and the radius; that is,  $AF$  is the versed sine of the arc  $AB$ , and  $DK$  is the versed sine of the arc  $BD$ .

For the sake of brevity these technical terms are contracted thus: for sine  $AB$ , we write  $\sin.AB$ , for cosine  $AB$ , we write  $\cos.AB$ , for tangent  $AB$ , we write  $\tan.AB$ , &c.

From the preceding definitions we deduce the following obvious consequences :

1st, That when the arc  $AB$ , becomes so small as to call it nothing, its sine tangent and versed sine are also nothing, and its secant and cosine are each equal to radius.

2d, The sine and versed sine of a quadrant are each equal to the radius ; its cosine is zero, and its secant and tangent are infinite.

3d, The chord of an arc is twice the sine of half the arc. Thus, the chord  $BG$ , is double of the sine  $BF$ .

4th, The sine and cosine of any arc form the two sides of a right angled triangle, which has a radius for its hypotenuse. Thus,  $CF$ , and  $FB$ , are the two sides of the right angled triangle  $CFB$ .

Also, the radius and the tangent always form the two sides of a right angled triangle which has the secant of the arc for its hypotenuse. This we observe from the right angled triangle  $CAH$ .

To express these relations analytically, we write

$$\sin.^2 + \cos.^2 = R^2 \quad (1)$$

$$R^2 + \tan.^2 = \sec.^2 \quad (2)$$

From the two equiangular triangles  $CFB$ ,  $CAH$ , we have

$$CF : FB = CA : AH$$

$$\text{That is,} \quad \cos. : \sin. = R : \tan. \quad \tan. = \frac{R \sin.}{\cos.} \quad (3)$$

$$\text{Also,} \quad CF : CB = CA : CH$$

$$\text{That is,} \quad \cos : R = R : \sec. \quad \cos. \sec. = R^2 \quad (4)$$

The two equiangular triangles  $CAH$ ,  $CDL$ . give

$$CA : AH = DL : DC$$

$$\text{That is,} \quad R : \tan. = \cot : R \quad \tan. \cot. = R^2 \quad (5)$$

$$\text{Also,} \quad CF : FB = DL : DC$$

$$\text{That is,} \quad \cos. : \sin. = \cot : R \quad \cos. R = \sin. \cot. \quad (6)$$

By observing (4) and (5), we find that

$$\cos. \sec. = \tan. \cot. \quad (7)$$

$$\text{Or,} \quad \cos. : \tan. = \cot. : \sec.$$

The ratios between the various trigonometrical lines are always the same for the same arc, whatever be the length of the radius ; and therefore, we may assume radius of any length to suit our convenience ; and the preceding equations will be more concise, and more

readily applied, by making radius equal unity. This supposition being made, the preceding become

$$\sin.^2 + \cos.^2 = 1 \quad (1)$$

$$1 + \tan.^2 = \sec.^2 \quad (2)$$

$$\tan. = \frac{\sin.}{\cos.} \quad (3) \qquad \cos. = \frac{1}{\sec.} \quad (4)$$

$$\tan. = \frac{1}{\cot.} \quad (5) \qquad \cos. = \sin. \cot. \quad (6)$$

The center of the circle is considered the absolute *zero* point, and the different directions from this point are designated by the different signs + and —. On the right of *C*, toward *A*, is commonly marked plus (+), then the other direction, toward *E*, is necessarily minus (—). Above *AE* is called (+), below that line (—).

If we conceive an arc to commence at *A*, and increase continuously around the whole circle in the direction of *ABD*, then the following table will show the mutations of the signs.

|               | sin. | cos. | tan. | cot. | sec. | cosec. | vers. |
|---------------|------|------|------|------|------|--------|-------|
| 1st quadrant. | +    | +    | +    | +    | +    | +      | +     |
| 2d “          | +    | —    | —    | —    | —    | +      | +     |
| 3d “          | —    | —    | +    | +    | —    | —      | +     |
| 4th “         | —    | +    | —    | —    | +    | —      | +     |

### PROPOSITION I.

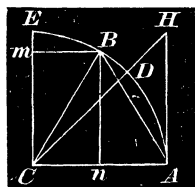
*The chord of 60° and the tangent 45° are each equal to radius; the sine of 30° the versed sine of 60° and the cosine of 60° are each equal to half the radius.*

(The first truth is proved in problem 15, book 1).

On *C* =, as radius, describe a quadrant; take *AD* = 45°, *AB* = 60°, and *AE* = 90°, then *BE* = 30°.

Join *AB*, *CB*, and draw *Bn*, perpendicular to *CA*. Draw *Bm*, parallel to *AC*. Make the angle *CAH* = 90°, and draw *CDH*.

In the  $\triangle ABC$ , the angle *ACB* = 60° by hypothesis; therefore, the sum of the other two angles is  $(180 - 60) = 120^\circ$ . But *CB* = *CA*, hence the angle *CBA* = the angle *CAB*, (th. 15 b. 1), and as the sum of the two is 120°, each one must be 60°; therefore, each of the angles of triangle *ABC*, is 60°





That is,  $R : \cos.a = \sin.b : FH$ , or  $FH = \frac{\cos.a \sin.b}{R}$

Also,  $GD : GO = GI : GN$

That is,  $R : \cos.a = \cos.b : GN$ , or  $GN = \frac{\cos.a \cos.b}{R}$

Also,  $GD : DO = FI : IH$

That is,  $R : \sin.a = \sin.b : IH$ , or  $IH = \frac{\sin.a \sin.b}{R}$

By adding the first and second of these equations, we have

$$IN + FH = FM = \sin.(a+b)$$

That is,  $\sin.(a+b) = \frac{\sin.a \cos.b + \cos.a \sin.b}{R}$

By subtracting the second from the first, we have

$$\sin.(a-b) = \frac{\sin.a \cos.b - \cos.a \sin.b}{R}$$

By subtracting the fourth from the third, we have

$$GN - IH = GM = \cos.(a+b) \text{ for the first member.}$$

Hence,  $\cos.(a+b) = \frac{\cos.a \cos.b - \sin.a \sin.b}{R}$

By adding the third and fourth, we have

$$GN + IH = GP = \cos.(a-b)$$

Hence,  $\cos.(a-b) = \frac{\cos.a \cos.b + \sin.a \sin.b}{R}$

Collecting these four expressions, and considering the radius unity, we have

$$(A) \quad \begin{cases} \sin.(a+b) = \sin.a \cos.b + \cos.a \sin.b & (7) \\ \sin.(a-b) = \sin.a \cos.b - \cos.a \sin.b & (8) \\ \cos.(a+b) = \cos.a \cos.b - \sin.a \sin.b & (9) \\ \cos.(a-b) = \cos.a \cos.b + \sin.a \sin.b & (10) \end{cases}$$

Formula (A), accomplishes the objects of the proposition, and from these equations many useful and important deductions can be made. The following, are the most essential:

By adding (7) to (8), we have (11); subtracting (8) from (7), gives (12). Also, (9)+(10) gives (13); (9) taken from (10) gives (14).

$$(B) \quad \begin{cases} \sin.(a+b) + \sin.(a-b) = 2\sin.a \cos.b & (11) \\ \sin.(a+b) - \sin.(a-b) = 2\cos.a \sin.b & (12) \\ \cos.(a+b) + \cos.(a-b) = 2\cos.a \cos.b & (13) \\ \cos.(a-b) - \cos.(a+b) = 2\sin.a \sin.b & (14) \end{cases}$$

If we put  $a+b=A$ , and  $a-b=B$ , then (11) becomes (15), (12) becomes (16), 13 becomes (17), and (14) becomes (18).

$$(C) \quad \begin{cases} \sin.A + \sin.B = 2\sin.\left(\frac{A+B}{2}\right) \cos.\left(\frac{A-B}{2}\right) & (15) \\ \sin.A - \sin.B = 2\cos.\left(\frac{A+B}{2}\right) \sin.\left(\frac{A-B}{2}\right) & (16) \\ \cos.A + \cos.B = 2\cos.\left(\frac{A+B}{2}\right) \cos.\left(\frac{A-B}{2}\right) & (17) \\ \cos.B - \cos.A = 2\sin.\left(\frac{A+B}{2}\right) \sin.\left(\frac{A-B}{2}\right) & (18) \end{cases}$$

If we divide (15) by (16), (observing that  $\frac{\sin.}{\cos.} = \tan.$  and  $\frac{\cos.}{\sin.} = \cot. = \frac{1}{\tan.}$  as we learn by equations (6) and (5) trigonometry), we shall have

$$\frac{\sin.A + \sin.B}{\sin.A - \sin.B} = \frac{\sin.\left(\frac{A+B}{2}\right)}{\cos.\left(\frac{A+B}{2}\right)} \times \frac{\cos.\left(\frac{A-B}{2}\right)}{\sin.\left(\frac{A-B}{2}\right)} = \frac{\tan.\left(\frac{A+B}{2}\right)}{\tan.\left(\frac{A-B}{2}\right)} \quad (19)$$

Whence,

$$\sin.A + \sin.B : \sin.A - \sin.B = \tan.\left(\frac{A+B}{2}\right) : \tan.\left(\frac{A-B}{2}\right)$$

or in words. *The sum of the sines of any two arcs is to the difference of the same sines, as the tangent of the half sum of the same arcs is to the tangent of half their difference.*

By operating in the same way with the different equations in formula (C), we find,

$$(D) \quad \begin{cases} \frac{\sin.A + \sin.B}{\cos.A + \cos.B} = \tan.\left(\frac{A+B}{2}\right) & (20) \\ \frac{\sin.A + \sin.B}{\cos.B - \cos.A} = \cot.\left(\frac{A-B}{2}\right) & (21) \\ \frac{\sin.A - \sin.B}{\cos.A + \cos.B} = \tan.\left(\frac{A-B}{2}\right) & (22) \\ \frac{\sin.A - \sin.B}{\cos.B - \cos.A} = \cot.\left(\frac{A+B}{2}\right) & (23) \\ \frac{\cos.A + \cos.B}{\cos.B - \cos.A} = \frac{\cot.\left(\frac{A+B}{2}\right)}{\tan.\left(\frac{A-B}{2}\right)} & (24) \end{cases}$$

These equations are all true, whatever be the value of the arcs designated by  $A$  and  $B$ ; we may therefore, assign any possible value to either of them, and if in equations (20), (21) and (24), we make  $B=O$ , we shall have,

$$\frac{\sin A}{1+\cos A} = \tan \frac{A}{2} = \frac{1}{\cot \frac{1}{2}A} \quad (25)$$

$$\frac{\sin A}{1-\cos A} = \cot \frac{A}{2} = \frac{1}{\tan \frac{1}{2}A} \quad (26)$$

$$\frac{1+\cos A}{1-\cos A} = \frac{\cot \frac{1}{2}A}{\tan \frac{1}{2}A} = \frac{1}{\tan^2 \frac{1}{2}A} \quad (27)$$

If we now turn back to formula (A), and divide equation (7) by (9), and (8) by (10), observing at the same time, that  $\frac{\sin}{\cos} = \tan$ , we shall have,

$$\tan(a+b) = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b}$$

$$\tan(a-b) = \frac{\sin a \cos b - \cos a \sin b}{\cos a \cos b + \sin a \sin b}$$

By dividing the numerators and denominators of the second members of these equations by  $(\cos a \cos b)$ , we find,

$$\tan(a+b) = \frac{\frac{\sin a \cos b}{\cos a \cos b} + \frac{\cos a \sin b}{\cos a \cos b}}{\frac{\cos a \cos b}{\cos a \cos b} - \frac{\sin a \sin b}{\cos a \cos b}} = \frac{\tan a + \tan b}{1 - \tan a \tan b} \quad (28)$$

$$\tan(a-b) = \frac{\frac{\sin a \cos b}{\cos a \cos b} - \frac{\cos a \sin b}{\cos a \cos b}}{\frac{\cos a \cos b}{\cos a \cos b} + \frac{\sin a \sin b}{\cos a \cos b}} = \frac{\tan a - \tan b}{1 + \tan a \tan b} \quad (29)$$

If in equation (11), formula (B), we make  $a=b$ , we shall have,

$$\sin 2a = 2 \sin a \cos a \quad (30)$$

Making the same hypothesis in equation (13), gives,

$$\cos 2a + 1 = 2 \cos^2 a \quad (31)$$

The same hypothesis reduces equation (14), to

$$1 - \cos 2a = 2 \sin^2 a \quad (32)$$

The same hypothesis reduces equation (28), to

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a} \quad (33)$$



The secants and cosecants of arcs are not given in our table, because they are very little used in practice; and if any particular secant is required, it can be determined by subtracting the cosine from 20; and the cosecant can be found by subtracting the sine from 20.

### PROPOSITION 3.

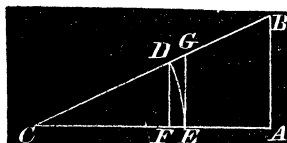
*In any right angled plane triangle, we may have the following proportions :*

1st. *As the hypotenuse is to either side, so is the radius to the sine of the angle opposite to that side.*

2d. *As one side is to the other side, so is the radius to the tangent of the angle adjacent to the first-mentioned side.*

3d. *As one side is to the hypotenuse, so is radius to the secant of the angle adjacent to that side.*

Let  $CAB$  represent any right angled triangle, right angled at  $A$ .  $AB$  and  $AC$  are called the sides of the  $\triangle$ , and  $CB$  is called the hypotenuse.



(Here, and in all cases hereafter, we shall represent the angles of a triangle by the large letters  $A, B, C$ , and the sides opposite to them, by the small letters  $a, b, c$ .)

From either acute angle, as  $C$ , take any distance, as  $CD$ , greater or less than  $CB$ , and describe the arc  $DE$ . This arc measures the angle  $C$ . From  $D$ , draw  $DF$  parallel to  $BA$ ; and from  $E$ , draw  $EG$ , also parallel to  $BA$  or  $DF$ .

By the definitions of sines, tangents, and secants,  $DF$  is the sine of the angle  $C$ ;  $EG$  is the tangent,  $CG$  the secant, and  $CF$  the cosine.

Now, by proportional triangles we have,

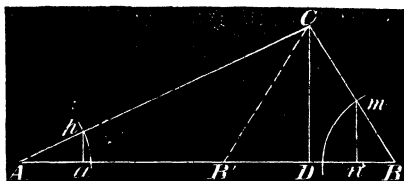
$$\left. \begin{array}{l} CB : BA = CD : DF \quad \text{or, } a : c = R : \sin. C \\ CA : AB = CE : EG \quad \text{or, } b : c = R : \tan. C \\ CA : CB = CE : CG \quad \text{or, } b : a = R : \sec. C \end{array} \right\} Q. E. D.$$

*Scholium.* If the hypotenuse of a triangle is made radius, one side is the sine of the angle opposite to it, and the other side is the cosine of the same angle. This is obvious from the triangle  $CDF$ .

PROPOSITION 4.

*In any triangle, the sines of the angles are to one another as the sides opposite to them.*

Let  $ABC$  be any triangle. From the points  $A$  and  $B$ , as centers, with any radius, describe the arcs measuring these angles, and draw  $pa$ ,  $CD$ , and  $mn$ , perpendicular to  $AB$ .



Then,  $pa = \sin A$ ,  $mn = \sin B$

By the similar  $\triangle$ s,  $Apa$  and  $ACD$ , we have,

$$R : \sin A = b : CD; \text{ or, } R(CD) = b \sin A \quad (1)$$

By the similar  $\triangle$ s  $Bmn$  and  $BCD$ , we have,

$$R : \sin B = a : CD; \text{ or, } R(CD) = a \sin B \quad (2)$$

By equating the second members of equations (1) and (2).

$$b \sin A = a \sin B.$$

Hence,  $\sin A : \sin B = a : b$   
Or,  $a : b = \sin A : \sin B$  } Q. E. D.

*Scholium 1.* When either angle is  $90^\circ$ , its sine is radius.

*Scholium 2.* When  $CB$  is less than  $AC$ , and the angle  $B$ , acute, the triangle is represented by  $ACB$ . When the angle  $B$  becomes  $B'$ , it is obtuse, and the triangle is  $ACB'$ ; but the proportion is equally true with either triangle; for the angle  $CB'D = CBA$ , and the sine of  $CB'D$  is the same as the sine of  $AB'C$ . In practice we can determine which of these triangles is proposed by the side  $AB$ , being greater or less than  $AC$ ; or, by the angle at the vertex  $C$ , being large as  $ACB$ , or small as  $ACB'$ .

In the solitary case in which  $AC$ ,  $CB$ , and the angle  $A$ , are given, and  $CB$  less than  $AC$ , we can determine both of the  $\triangle$ s  $ACB$  and  $ACB'$ ; and then we surely have the right one.

PROPOSITION 5.

*If from any angle of a triangle, a perpendicular be let fall on the opposite side, or base, the tangents of the segments of the angle are to one another as the segments of the base.*

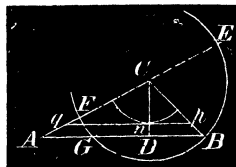
Let  $ABC$  be the triangle. Let fall the perpendicular  $CD$ , on the side  $AB$ .

Take any radius, as  $Cn$ , and describe the arc which measures the angle  $C$ . From  $n$ , draw  $qnp$  parallel to  $AB$ . Then it is obvious that  $np$  is the tangent of the angle  $DCB$ , and  $nq$  is the tangent of the angle  $ACD$ .

Now, by reason of the parallels  $AB$  and  $qp$ , we have,

$$qn : np = AD : DB$$

That is,  $\tan. ACD : \tan. DCB = AD : DB$  *Q. E. D.*



### PROPOSITION 6.

*If a perpendicular be let fall from any angle of a triangle to its opposite side or base, this base is to the sum of the other two sides, as the difference of the sides is to the difference of the segments of the base.*

(See figure to proposition 5.)

Let  $AB$  be the base, and from  $C$ , as a center, with the shorter side as radius, describe the circle, cutting  $AB$  in  $G$ ,  $AC$  in  $F$ , and produce  $AC$  to  $E$ .

It is obvious that  $AE$  is the sum of the sides  $AC$  and  $CB$ , and  $AF$  is their difference.

Also,  $AD$  is one segment of the base made by the perpendicular, and  $BD=DG$  is the other; therefore, the difference of the segments is  $AG$ .

As  $A$  is a point without a circle, by theorem 18, book 3, we have,

$$AE \times AF = AB \times AG$$

Hence,  $AB : AE = AF : AG$  *Q. E. D.*

### PROPOSITION 7.

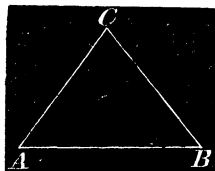
*The sum of any two sides of a triangle, is to their difference, as the tangent of the half sum of the angles opposite to these sides, to the tangent of half their difference.*

Let  $ABC$  be any plane triangle. Then, by proposition 4, trigonometry, we have,

$$CB : AC = \sin. A : \sin. B$$

Hence,

$$CB + AC : CB - AC = \sin. A + \sin. B : \sin. A - \sin. B \quad (\text{th. 9 b. 2})$$



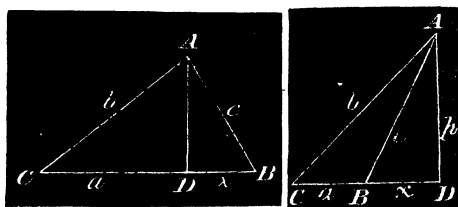
But,  $\tan. \left( \frac{A+B}{2} \right) : \tan. \left( \frac{A-B}{2} \right) = \sin. A + \sin. B : \sin. A - \sin. B$   
(eq. (1), trig.)

Comparing the two latter proportions (th. 6, b. 2), we have,  
 $CB+AC : CB-AC = \tan. \left( \frac{A+B}{2} \right) : \tan. \left( \frac{A-B}{2} \right)$  Q. E. D.

## PROPOSITION 8.

*Given the three sides of any plane triangle, to find some relation which they must bear to the sines and cosines of the respective angles.*

Let  $ABC$  be the triangle, and let the perpendicular fall either upon, or without the base, as shown in the figures; and by



recurring to theorem 38, book 1, we shall find

$$CD = \frac{a^2 + b^2 - c^2}{2a} \quad (1)$$

Now, by proposition 3, trigonometry, we have,

$$R : \cos. C = b : CD$$

Therefore,  $CD = \frac{b \cos. C}{R} \quad (2)$

Equating these two values of  $CD$ , and reducing, we have,

$$\cos. C = \frac{R(a^2 + b^2 - c^2)}{2ab} \quad (m)$$

In this expression we observe that the part of the numerator which has the minus sign, is the side opposite to the angle; and that the denominator is twice the rectangle of the sides adjacent to the angle. From these observations we at once draw the following expressions for the cosine  $A$ , and cosine  $B$ .

Thus,  $\cos. A = \frac{R(b^2 + c^2 - a^2)}{2bc} \quad (n)$

$$\cos. B = \frac{R(a^2 + c^2 - b^2)}{2ac} \quad (p)$$

As these expressions are not convenient for logarithmic computation, we modify them as follows :

If we put  $2a=A$ , in equation (31), we have,

$$\cos A + 1 = 2 \cos^2 \frac{1}{2} A$$

In the preceding expression ( $n$ ), if we consider radius, unity, and add 1 to both members, we shall have,

$$\cos A + 1 = 1 + \frac{b^2 + c^2 - a^2}{2bc}$$

$$\begin{aligned} \text{Therefore, } 2 \cos^2 \frac{1}{2} A &= \frac{2bc + b^2 + c^2 - a^2}{2bc} \\ &= \frac{(b+c)^2 - a^2}{2bc} \end{aligned}$$

Considering  $(b+c)$  as one quantity, and observing that we have the difference of *two squares*, therefore

$$(b+c)^2 - a^2 = (b+c+a)(b+c-a); \text{ but } (b+c-a) = b+c+a-2a$$

$$\text{Hence, } 2 \cos^2 \frac{1}{2} A = \frac{(b+c+a)(b+c+a-2a)}{2bc}$$

$$\text{Or, } \cos^2 \frac{1}{2} A = \frac{\left( \frac{b+c+a}{2} \right) \left( \frac{b+c+a}{2} - a \right)}{bc}$$

By putting  $\frac{a+b+c}{2} = s$ , and extracting square root, the final result for radius unity, is

$$\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$$

For any other radius we must write,

$$\cos \frac{1}{2} A = \sqrt{\frac{R^2 s(s-a)}{bc}}$$

$$\text{By inference, } \cos \frac{1}{2} B = \sqrt{\frac{R^2 s(s-b)}{ac}}$$

$$\text{Also, } \cos \frac{1}{2} C = \sqrt{\frac{R^2 s(s-c)}{ab}}$$

In every triangle, the sum of the three angles must equal  $180^\circ$ ; and if one of the angles is small, the other two must be comparatively large; if two of them are small, the third one must be large. The greater angle is always opposite the greater side; hence, by merely inspecting the given sides, any person can decide at once which is the greater angle; and of the three preceding equations, *that one* should be taken which applies to the greater angle, whether that be the particular angle required or not; because the equations bring out the

*cosines* to the angles; and the cosines, to very small arcs vary so slowly, that it may be impossible to decide, with sufficient numerical accuracy to what particular arc the cosine belongs. For instance, the cosine 9.999999, carried to the table, applies to several arcs; and, of course, we should not know which one to take; but this difficulty does not exist when the angle is large; therefore, compute the largest angle first, and then compute the other angles by proposition 4.

But we can deduce an expression for the sine of any of the angles, as well as the cosine. It is done as follows:

## EQUATIONS FOR THE SINES OF THE ANGLES.

Resuming equation (*m*), and considering radius, unity, we have,

$$\cos. C = \frac{a^2 + b^2 - c^2}{2ab}$$

Subtracting each member of this equation from 1, gives

$$1 - \cos. C = 1 - \left( \frac{a^2 + b^2 - c^2}{2ab} \right) \quad (1)$$

Making  $2a = C$ , in equation (32), then  $a = \frac{1}{2}C$ ,

$$\text{And} \quad . \quad . \quad 1 - \cos. C = 2 \sin.^2 \frac{1}{2} C \quad (2)$$

Equating the right hand members of (1) and (2),

$$\begin{aligned} 2 \sin.^2 \frac{1}{2} C &= \frac{2ab - a^2 - b^2 + c^2}{2ab} \\ &= \frac{c^2 - (a - b)^2}{2ab} \\ &= \frac{(c + b - a)(c + a - b)}{2ab} \\ &= \left( \frac{c + b - a}{2} \right) \left( \frac{c + a - b}{2} \right) \end{aligned}$$

$$\text{Or,} \quad . \quad . \quad \sin.^2 \frac{1}{2} C = \frac{ab}{ab}$$

$$\text{But,} \quad . \quad \frac{c + b - a}{2} = \frac{c + b + a}{2} - a \quad \text{and} \quad \frac{c + a - b}{2} = \frac{c + a + b}{2} - b$$

$$\text{Put} \quad . \quad \frac{a + b + c}{2} = s, \quad \text{as before; then,}$$

$$\sin. \frac{1}{2} C = \sqrt{\frac{(s - a)(s - b)}{ab}}$$

By taking equation (*p*), and operating in the same manner, we

$$\text{have} \quad . \quad . \quad \sin. \frac{1}{2} B = \sqrt{\frac{(s - a)(s - c)}{ac}}$$

$$\text{From (n)} \quad . \quad \sin. \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{cb}}$$

The preceding results are for radius unity; for any other radius, we must multiply by the number of units in such radius. For the radius of the tables, we write  $R$ ; and if we put it under the radical sign, we must write  $R^2$ ; hence, for the sines corresponding with our logarithmic table, we must write the equations

$$\text{thus,} \quad \sin. \frac{1}{2}A = \sqrt{\frac{R^2(s-b)(s-c)}{bc}}$$

$$\sin. \frac{1}{2}B = \sqrt{\frac{R^2(s-a)(s-c)}{ac}}$$

$$\sin. \frac{1}{2}C = \sqrt{\frac{R^2(s-a)(s-b)}{ab}}$$

A large angle should not be determined by these equations, for the same reason that a small angle should not be determined from an equation expressing the cosine.

In practice, the equations for cosine are more generally used, because more easily applied.

In the preceding pages we have gone over the whole ground of theoretical plane trigonometry, although several particulars might have been enlarged upon, and more equations in relation to the combinations of the trigonometrical lines, might have been given; but enough has been given to solve every possible case that can arise in the practical application of the science.

By the application of equations (1), (31), and (32), the table of natural sines and cosines has been computed.

The operation is as follows. *The sine of 30° is half radius*; making the radius unity, equation (1) gives

$$\frac{1}{2} + \cos.^2 30^\circ = 1 : \text{whence } \cos.^2 30^\circ = \frac{3}{4} \text{ or } \cos. 30^\circ = \frac{1}{2}\sqrt{3}.$$

$$\text{From (32) we have, } \sin.a = \sqrt{\frac{1 - \cos. 2a}{2}}$$

$$\text{Making } 2a = 30^\circ, \text{ then } \sin. 15^\circ = \left(\frac{1}{2} - \frac{1}{4}\sqrt{3}\right)^{\frac{1}{2}} = 0.25881904$$

$$\text{From (31) we have, } \cos.a = \sqrt{\frac{1 + \cos. 2a}{2}}$$

$$\text{Making } 2a = 30^\circ \text{ as before, } \cos.a = \left(\frac{1}{2} + \frac{1}{4}\sqrt{3}\right)^{\frac{1}{2}} = 0.96592582$$

Having sine and cosine of 15° the second application of these equations will give the sine and cosine of the half of 15°, and so on through as many bisections as we please.

Being desirous of giving a full exposition of the formation of table II, we give the following geometrical demonstration of equation 30, by the help of the figure in the margin.

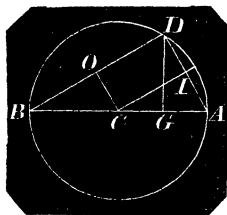
Let the arc  $AD=2a$

Then  $DG=\sin. 2a$ ,  $CG=\cos. 2a$ ,

$DI=\sin. a$ ,  $AD=2 \sin. a$ ,

$CI=\cos. a$ ,  $DB=2DO=2 \cos. a$ .

The angle  $DBA$  being at the circumference, is measured by half the arc  $AD$ , or by  $a$ .



Now, by applying proposition 4 to the triangle  $DBG$ , we have  
 $\sin. DBG : DG = \sin. 90^\circ : BD$ .

The  $\sin. DBG = \sin. a$ , and  $\sin. 90^\circ = 1$ , the radius being unity ; therefore, the preceding proportion becomes,

$$\sin. a : \sin. 2a = 1 : 2 \cos. a.$$

Whence  $2 \cos. a \sin. a = \sin. 2a$ . (Same as eq. 30.)

### PROBLEM.

*Given the sine and cosine of an arc, to find the sine and cosine of one half that arc.*

Designate the given arc by  $2a$ , the radius by unity, and whatever be the value of  $a$ , equation (1) gives

$$\cos.^2 a + \sin.^2 a = 1 \quad (m)$$

It is proved in proposition 1, that the sine of  $30^\circ$  is half the radius: therefore, let  $2a=30^\circ$ , then  $\sin. 2a=0.5$ : and equation 30, just demonstrated, gives

$$2 \cos. a \sin. a = 0.5. \quad (n)$$

Add (m) and (n), and extract the square root of both members.

$$\text{Then} \quad \cos. a + \sin. a = 1.22474486 \quad (o)$$

Subtracting (n) from (m), and extracting square root, gives

$$\cos. a - \sin. a = 0.70710678 \quad (p)$$

By subtracting, and adding (p) and (o), and dividing by 2, we find

$$\sin. a = \sin. 15^\circ = 0.25881904$$

$$\cos. a = \cos. 15^\circ = 0.96592582$$



Now let  $2a=15^\circ$ . Then

$$\cos.^2 a + \sin.^2 a = 1.$$

and

$$2 \cos. a \sin. a = 0.25881904$$

Operating as before, we find

$$\sin. a = \sin. 7^\circ 30' = 0.1305261921$$

$$\cos. a = \cos. 7^\circ 30' = 0.9914447879$$

Again, put

$$2a = 7^\circ 30'$$

then as before,

$$\cos.^2 a + \sin.^2 a = 1$$

$$2 \cos. a \sin. a = 0.1305261921$$

These equations give

$$\sin. a = \sin. 3^\circ 45' = 0.0654031291$$

$$\cos. a = \cos. 3^\circ 45' = 0.9978589222$$

Thus we can bisect the arc as many times as we please. After *five* more bisections, we have

$$\sin. a = \sin. 7' 1'' 52\frac{1}{2}''' = 0.0020453077$$

$$\cos. a = \cos. 7' 1'' 52\frac{1}{2}''' = 0.99999799$$

As the sines of all arcs *under*  $10'$ , may be considered as coinciding with the arc, and varying with it, we can now find the sine of  $1'$  by proportion.

$$\text{Thus, } 7' 1'' 52\frac{1}{2}''' : 1' :: 0.0020453077 : \sin. 1$$

$$\text{Or, } 25312.5'' : 3600 ::$$

$$\text{Or, } 10125 : 1440 :: 0.0020453077 : \sin. 1'$$

Whence

$$\sin. 1' = 0.0002908882$$

$$\sin. 2' = 0.0005817764$$

$$\sin. 3' = 0.0008726646$$

In formula (B) equation (11), we find

$$\sin. (a+b) + \sin. (a-b) = 2 \sin. a \cos. b$$

Now, if  $a=3'$  and  $b=1'$

$$\sin. 4' + \sin. 2' = 2 \sin. 3' \cos. 1'$$

We have already  $\sin. 2'$  and  $\sin. 3'$ , and  $\cos. 1'$  does not sensibly differ from *unity*, therefore

$$\sin. 4' = 2 \sin. 3' - \sin. 2' = 0.0011635528$$

$$\sin. 5' = 2 \sin. 4' \cos. 1' - \sin. 3' \text{ \&c. \&c. to } 15'$$

When the sine of any arc is known, its cosine can be found by the following formula, which is, in substance, equation (1) trigonometry

$$\cos. a = \sqrt{(1 + \sin. a)(1 - \sin. a)}$$

In formula (A) equation (7) we find that

$$\sin. (a+b) = \sin. a \cos. b + \cos. a \sin. b$$

Now, if we make  $a=30^\circ$  and  $b=4'$  Then

$$\sin. a = 0.5 \quad \cos. a = \frac{1}{2}\sqrt{3} = 0.8660254$$

$$\sin. b = 0.00116355 \quad \cos. b = 0.999999323$$

Whence

$$\begin{aligned} \sin. (30^\circ 4') &= (0.5) (0.999999323) + (0.8660254) (0.00116355) \\ &= 0.499999661 \quad + \quad 0.0010007620 \\ &= 0.501007281 \end{aligned}$$

Equation (8) gives

$$\sin. (29^\circ 56') = 0.498992041$$

When the sine and cosine of any arc are both known, the sine and cosine of the half or double of the arc can be determined by equation 30;— and thus, from equations (30), (7), (8), (11), and (1), the sines and cosines of all arcs can be determined.

But these sines and cosines are expressed in natural numbers, to radius unity, hence they are called *natural sines* and *natural cosines*, and they are all decimals, except the sine of  $90^\circ$  and the cosine of  $0^\circ$ , each of which is unity.

To form table II, we require logarithmic sines, and cosines, which are found by taking the logarithms of the natural sines and cosines, and increasing the indices by 10; to correspond to the radius of 10000000000. The radius of this table might have been greater or less, but custom has settled on this value.

To find the logarithmic sine of  $1'$ , we proceed thus,

$$\text{Nat. sin. } 1' = 0.0002908882 \quad \log. \quad -4.463726$$

$$\text{To which add} \quad 10.$$

$$\text{The log. sine of } 1', \text{ therefore is} \quad 6.463726$$

$$\text{Nat. sin. } 3' = 0.0008726646 \quad \log. \quad -4.950847$$

$$\text{Add} \quad 10.$$

$$\text{Log. sin. } 3' \text{ therefore is} \quad 6.940847$$

Thus the logarithmic sine and cosine of all arcs are found. After the logarithmic sine and cosine of any arc have been found, the tangent and cotangent of the same arc can be found by equations (3) and (5), and the secants by (4); that is,

$$\tan. a = \frac{R \sin. a}{\cos a} \quad \cot. a = \frac{R \cos. a}{\sin. a} \quad \sec. a = \frac{R^2}{\cos. a}$$

For example, the logarithmic sine of  $6^\circ$  is 9.019235, and its cosine 9.997614. From these, find tan., cot., and secant.

|           |   |   |   |   |          |                 |
|-----------|---|---|---|---|----------|-----------------|
| $R \sin.$ | - | - | - | - | -        | 19.019235       |
| Cos.      | - | - | - | - | subtract | <u>9.997614</u> |
| Tan. is   | - | - | - | - | -        | 9.021621        |
| $R \cos.$ | - | - | - | - | -        | 19.997614       |
| Sin.      | - | - | - | - | subtract | <u>9.019235</u> |
| Cotan. is | - | - | - | - | -        | 10.978379       |
| $R^2$ is  | - | - | - | - | -        | 20.000000       |
| Cos.      | - | - | - | - | subtract | <u>9.997674</u> |
| Secant is | - | - | - | - | -        | 10.002326       |

Thus we find all the materials for

#### TABLE II.

This table contains logarithmic sines and tangents, and natural sines and cosines. We shall confine our explanations to the logarithmic sines and cosines.

The sine of every degree and minute of the quadrant is given, directly, in the table, commencing at  $0^\circ$  and extending to  $45^\circ$ , at the head of the table; and from  $45^\circ$  to  $90^\circ$ , at the foot of the table, increasing backward.

The same column that is marked sine at the top, is marked cosine at the bottom; and the reason for this is apparent to any one who has examined the definitions of sines.

The difference of two consecutive logarithms is given, corresponding to *ten* seconds. Removing the decimal point one figure will give the difference for *one* second; and if we multiply this difference by any proposed number of seconds, we shall have a difference corresponding to that number of seconds, above the logarithm, corresponding to the preceding degree and minute.

For example, find the sine of  $19^\circ 17' 22''$ .

The sine of  $19^\circ 17'$ , taken directly from the table, is 9.518829

The difference for  $10''$  is 60.2; for  $1''$ , is  $6.02 \times 22$  133

Hence,  $19^\circ 17' 22''$  sine is 9.518952

From this it will be perceived that there is no difficulty in obtaining the sin. or tan., cos. or cot., of any angle greater than  $30'$ .

Conversely. Given the logarithmic sine 9.982412, to find its corresponding arc. The sine next less in the table, is 9.982404, and gives the arc  $73^{\circ} 48'$ . The difference between this and the given sine, is 8, and the difference for  $1''$ , is .61; therefore, the number of seconds corresponding to 8, must be discovered by dividing 8 by the decimal .61, which gives 13. Hence, the arc sought is  $73^{\circ} 48' 13''$ .

These operations are too obvious to require a rule. When the arc is very small, such arcs as are sometimes required in astronomy, it is necessary to be very accurate; and for that reason we omitted the difference for seconds for all arcs under  $30'$ . Assuming that the sines and tangents of arcs under  $30'$  vary in the same proportion as the arcs themselves, we can find the sine or tangent of any very small arc to great accuracy, as follows:

|                                                       |           |          |
|-------------------------------------------------------|-----------|----------|
| The sine of $1'$ , as expressed in the table, is      | . . . . . | 6.463726 |
| Divide this by 60; that is, subtract logarithm        | . . . . . | 1.778151 |
| The logarithmic sine of $1''$ , therefore, is         | . . . . . | 4.685575 |
| Now, for the sine of $17''$ , add the logarithm of 17 | . . . . . | 1.230449 |
| Logarithmic sine of $17''$ , is                       | . . . . . | 5.916024 |

In the same manner we may find the sine of any other small arc.

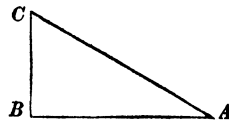
For example, find the sine of  $14' 21\frac{1}{2}''$ ; that is,  $861''$

|                                           |           |          |
|-------------------------------------------|-----------|----------|
| To logarithmic sine of $1''$ , is,        | . . . . . | 4.685575 |
| Add logarithm of 861.5                    | . . . . . | 2.935255 |
| Logarithmic sine of $14' 21\frac{1}{2}''$ | . . . . . | 7.620830 |

Without further preliminaries, we may now proceed to practical

EXAMPLES.

2. In a right angled triangle,  $ABC$ , given the base,  $AB$ , 1214, and the angle  $A$ ,  $51^{\circ} 40' 30''$ , to find the other parts.



To find  $BC$ .

|                                  |           |           |
|----------------------------------|-----------|-----------|
| As radius                        | . . . . . | 10.000000 |
| : $\tan A$ $51^{\circ} 40' 30''$ | . . . . . | 10.102119 |
| $\therefore AB$ 1214             | . . . . . | 3.084219  |
| : $BC$ 1535.8                    | . . . . . | 3.186338  |

N. B. When the first term of a logarithmic proportion is radius, the resulting logarithm is found by adding the second and third logarithms, rejecting 10 in the index, which is dividing by the first term.

In all cases we add the second and third logarithms together; which, in logarithms, is multiplying these terms together; and from that sum

we subtract the first logarithm, whatever it may be, which is dividing by the first term.

To find  $AC$ .

|                                                 |   |   |   |                 |
|-------------------------------------------------|---|---|---|-----------------|
| As sin. $C$ , or cos. $A$ $51^{\circ} 30' 40''$ | - | - | - | 9.792477        |
| : $AB$ 1214                                     | - | - | - | 3.084219        |
| :: Radius                                       | - | - | - | 10.000000       |
| : $AC$ 1957.7                                   | - | - | - | <u>3.291742</u> |

To find this resulting logarithm, we subtracted the first logarithm from the second, conceiving its index to be 13.

Let  $ABC$  represent any plane triangle, right angled at  $B$ .

1. Given  $AC$  73.26, and the angle  $A$   $49^{\circ} 12' 20''$ ; required the other parts.

Ans. The angle  $C$   $40^{\circ} 47' 40''$ ,  $BC$  54.46, and  $AB$  47.87.

2. Given  $AB$  469.34, and the angle  $A$   $51^{\circ} 26' 17''$ , to find the other parts.

Ans. The angle  $C$   $38^{\circ} 33' 43''$ ,  $BC$  588.5, and  $AC$  752.9.

3. Given  $AB$  493, and the angle  $C$   $20^{\circ} 14'$ ; required the remaining parts. Ans. The angle  $A$   $69^{\circ} 46'$ ,  $BC$  1338, and  $AC$  1425.

It is not necessary to give any more examples in right angled plane trigonometry, for *every distance* in the *traverse table* is but the hypotenuse of a right angled triangle, and its corresponding latitude and departure form the sides of the triangle.

*If any one should suspect an error in the traverse table, let him test it by computing the triangle anew.*

### OBLIQUE ANGLED TRIGONOMETRY.

Of the six parts of a triangle, three sides and three angles, three of them must be given and one of the given parts must be a side.

The subject presents four cases.

1. When two sides are given, and an angle opposite one of them.
2. When two sides are given, and the included angle.
3. When one side and two angles are given.
4. When the three sides are given.

The principles previously demonstrated are sufficient, and indeed ample, to give all solutions that can come under any one of these

cases. The operator must use his own judgment in applying these principles.

We give an example in each case, which, with the incidental examples, will be sufficient to fix the principles in the mind of the operator.

### EXAMPLE 1.

*In any plane triangle, given one side and the two adjacent angles, to find the other sides and angle.*

In the triangle  $ABC$ , given  $AB=376$ , the angle  $A=48^\circ 3'$ , and the angle  $B=40^\circ 14'$ , to find the other parts.

As the sum of the three angles of every triangle is equal to  $180^\circ$ , the third angle  $C$  must be  $180^\circ - 88^\circ 17' = 91^\circ 43'$ .

### INSTRUMENTALLY.

Take 376 from the scale, by means of the dividers, and place it on paper; making one extremity of the line  $A$ , and the other extremity  $B$ . From  $A$ , by means of the protractor (or otherwise), make the angle  $A=48^\circ 3'$ , and from  $B$ , make the angle  $B=40^\circ 14'$ . The intersections of the lines  $AC$ ,  $BC$ , will give the angle  $C$ , which being measured will be found to be a little more than a right angle.

Take  $AC$  in the dividers, and apply it to the scale, and it will be found to be 243; and  $BC$  will be found to be 279.8, if the projection is accurately made; but no one should expect numerical accuracy from this mechanical method.

N. B. Our figures in the book do not pretend to accuracy, they should be drawn on paper on a larger scale.

### BY LOGARITHMS.

To find  $AC$ .

|                             |   |   |   |           |
|-----------------------------|---|---|---|-----------|
| As sin. $91^\circ 43'$      | - | - | - | 9.999805  |
| : $AB$ 376                  | - | - | - | 2.575188  |
| :: sin. $AB$ $40^\circ 14'$ | - | - | - | 9.810167  |
|                             |   |   |   | <hr/>     |
|                             |   |   |   | 12.385355 |
| : $AC$ 243                  | - | - | - | 2.385550  |

Observe, that the sine of  $91^\circ 43'$  is the same as the cosine of  $1^\circ 43'$ .

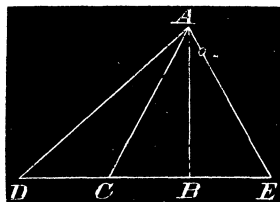
To find  $BC$ .

|                            |   |   |   |   |                  |
|----------------------------|---|---|---|---|------------------|
| As $\sin. 91^\circ 43'$    | - | - | - | - | 9.999805         |
| : $AB\ 376$                | - | - | - | - | 2.575188         |
| : : $\sin. A\ 48^\circ 3'$ | - | - | - | - | 9.871414         |
|                            |   |   |   |   | <u>12.446602</u> |
| : $BC\ 279.8$              | - | - | - | - | 2.446797         |

## EXAMPLE 2.

*In a plane triangle, given two sides, and an angle opposite one of them, to determine the other parts.*

Let  $AD=1751$ . feet, one of the given sides. The angle  $D=31^\circ 17' 19''$ , and the side opposite, 1257.5. From these data, we are required to find the other side, and the other two angles.



In this case we do not know whether  $AC$  or  $AE$  represents 1257.5, because  $AC=AE$ . If we take  $AC$  for the other given side, then  $DC$  is the other required side, and  $DAC$  is the vertical angle. If we take  $AE$  for the other given side, then  $DE$  is the required side, and  $DAE$  is the vertical angle; but in such cases we determine both triangles.

## INSTRUMENTALLY.

Draw  $DE$  indefinitely—from the point  $D$  make the angle  $D=31^\circ 17'$ .  $AD=1751$ ., but call it 175.1, which take from the scale. Place one foot of the dividers at  $D$ , the other foot will extend to  $A$ , thus finding the point  $A$ .

Take 125.75 in the dividers, place one foot at  $A$  as a center, and with the other strike an arc, cutting  $DE$  in  $C$  and  $E$ . Join  $AC$ ,  $AE$ , and one or the other of the triangles  $ADC$  or  $ADE$ , will be the triangle required.  $DC$  and  $DE$  applied to the scale, will give *one-tenth* of the required side, and the angle  $E$  or  $DCA$ , measured, will be one of the required angles.

We can also take one hundredth part of the sides, as well as the tenth; this will make no difference with the angles, the triangles thus formed will be similar.

In that case  $AD=17.51$ , and the side sought will be 23.64, which can be changed to 2364.

BY LOGARITHMS.

*To find the angle  $E=C$ .*

|                              |      |                 |
|------------------------------|------|-----------------|
| (Prop. 4.) As $AC=AE=1257.5$ | log. | 3.099508        |
| : $D 31^\circ 17' 19''$      | sin. | 9.715460        |
| :: $AD 1751$                 | log. | 3.243286        |
|                              |      | <hr/> 12.958746 |
| $E=C : 46^\circ 18'$         | sin. | 9.859238        |

From  $180^\circ$  take  $46^\circ 18'$ , and the remainder is the angle  $DCA = 133^\circ 42'$ .

The angle  $DAC = ACE - D$  (th. 11, b. 1) ; that is,  
 $DAC = 46^\circ 18' - 31^\circ 17' 19'' = 15^\circ 0' 41''$ .

The angles  $D$  and  $E$ , taken from  $180^\circ$ , give  $DAE = 102^\circ 24' 41''$ .

*To find  $DC$ .*

|                                |      |                 |
|--------------------------------|------|-----------------|
| As sin. $D 31^\circ 17' 19''$  | log. | 9.715460        |
| : $AC 1257.5$                  | log. | 3.099508        |
| :: sin. $DAC 15^\circ 0' 41''$ | log. | 9.413317        |
|                                |      | <hr/> 12.512825 |
| : $DC 626.86$                  |      | 2.797165        |

*To find  $DE$ .*

|                               |                 |
|-------------------------------|-----------------|
| As sin. $D 31^\circ 17' 19''$ | 9.715460        |
| : $AC 1257.5$                 | 3.099508        |
| :: sin. $102^\circ 24' 41''$  | 9.989730        |
|                               | <hr/> 13.089238 |
| : $DE 2364.5$                 | 3.373778        |

N. B. To make the triangle possible,  $AC$  must not be less than  $AB$ , the sine of the angle  $D$ , when  $DA$  is made radius.

## EXAMPLE 3.

*In any plane triangle, given two sides and the included angle, to find the other parts.*

Let  $AD=1751$  (see last figure),  $DE=2364.5$ , and the included angle  $D=41^\circ 17' 19''$ . We are required to find  $DE$ , the angle  $DAE$ , and angle  $E$ . Observe that the angle  $E$  must be less than the angle  $DAE$ , because it is opposite a less side.



## INSTRUMENTALLY.

Take  $DE=236.45$  from the scale (as near as possible), and from  $D$  draw  $DA$ , making the given angle  $41^\circ 17' 19''$ .

Take 175.1 from the scale, in the dividers, and with it mark off  $DA$ . Join  $AE$ ; and  $ADE$  will be the triangle in question, and  $AE$  applied to the scale will give the *tenth* part of the side sought; and measuring the angle  $E$  with the protractor (or otherwise), will determine its value.

## BY LOGARITHMS.

$$\begin{array}{rcl}
 \text{From} & - & - & - & - & 180^\circ \\
 \text{Take } D & - & - & - & - & 31^\circ 17' 19'' \\
 \hline
 \text{Sum of the other two angles} & = & 148^\circ 42' 41'' & (\text{th. 11, b. 1}) \\
 \frac{1}{2} \text{ sum} & - & - & - & - & = 74^\circ 21' 20''
 \end{array}$$

By proposition 7,

$$DE+DA : DE-DA = \tan. 74^\circ 21' 20'' : \tan. \frac{1}{2} (DAE-E)$$

That is,

$$\begin{array}{rcl}
 4115.5 : 613.5 = \tan. 74^\circ 21' 20'' : \frac{1}{2} (DAE-E) \\
 \text{Tan. } 74^\circ 21' 20'' & - & - & - & - & 10.552778 \\
 613.5 & - & - & - & - & 2.787815 \\
 \hline
 & & & & & 13.340593 \\
 4115.5 \log. (\text{sub.}) & & & & & 3.614423 \\
 \hline
 \frac{1}{2} (DAE-E) \tan. 28^\circ 1' 36'' & & & & & 9.726170
 \end{array}$$

But the half sum and half difference of any two quantities are equal to the greater of the two; and the half sum, less the half difference, is equal the less.

Therefore, to  $74^\circ 21' 20''$

Add  $28 \quad 1 \quad 36$

---

$DAE=102^\circ 22' 56''$

$E= 46 \quad 19 \quad 44$

To find  $AE$ .

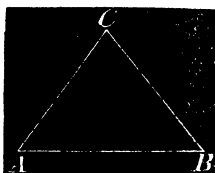
$$\begin{array}{rcl}
 \text{As sin. } E 46^\circ 19' 44'' & - & - & 9.859323 \\
 : DA 1751 & - & - & 3.243286 \\
 : : \sin. D 31^\circ 17' 19'' & - & - & 9.715460 \\
 \hline
 & & & 12.958746 \\
 : AE 1257.2 & - & - & 3.099423
 \end{array}$$

## EXAMPLE 4.

Given the three sides of a plane triangle to find the angles.

Given  $AC=1751$ ,  $CB=1257.5$ ,  $AB=2364.5$

If we take the formula for cosines, we will compute the greatest angle, which is  $C$ .



## INSTRUMENTALLY.

Construct a triangle with the three given sides 236.45, 125.75, and 175.1, according to problem 16, chapter 1. The angles then measured will show their value.

## BY LOGARITHMS.

|                                        |          |             |
|----------------------------------------|----------|-------------|
| $R^2$                                  |          | 20.000000   |
| $s=2686.5$                             |          | 3.429187    |
| $s-c=322$                              |          | 2.507856    |
| Numerator,                             | log.     | 25.937043   |
| $a$ 1257.5                             | 3.099508 |             |
| $b$ 1751.                              | 3.243286 |             |
| Denominator, log.                      | 6.342794 | 6.342694    |
|                                        |          | 2)19.594249 |
| $\frac{1}{2}C=51^\circ 11' 10'' \cos.$ |          | 9.797124    |
| $C=102 \quad 22 \quad 20$              |          |             |

The remaining angles may now be found by problem 4.

We give the following examples for practical exercises :

Let  $ABC$  represent any oblique angled triangle.

1. Given  $AB$  697, the angle  $A$   $81^\circ 30' 10''$ , and the angle  $B$   $40^\circ 30' 44''$ , to find the other parts.

Ans.  $AC$  534,  $BC$  813, and the angle  $C$   $57^\circ 59' 4''$ .

2. If  $AC=720.8$ , the angle  $A=70^\circ 5' 22''$ , and the angle  $B=59^\circ 35' 36''$ , required the other parts.

Ans.  $AB$  643.2,  $BC$  785.8, and the angle  $C$   $50^\circ 19' 6''$ .

3. Given  $BC$  980.1, the angle  $A$   $7^\circ 26' 26''$ , and the angle  $B$   $106^\circ 2' 23''$ , to find the other parts.

Ans.  $AB$  7284,  $AC$  7613.3, and the angle  $C$   $66^\circ 51' 11''$ .

## SURVEYING.

**SURVEYING** is the art of running definite lines on the surface of the earth, *measuring* them, and finding the contents of lands ; and the subject *necessarily* includes the measure of surfaces generally. We shall therefore commence with mensuration.

Mensuration is the application of the principles of Geometry, to the measure of surfaces and solids, and when lands are measured it is a part of surveying. We shall be very brief on mensuration proper, because the rules are so simple and obvious. For the demonstration of the rules, we refer to (Legendre and Robinson's Geometry.)

All surfaces are measured by the number of square units which they contain. The unit may be taken at pleasure ; it may be an inch, foot, yard, rod, mile, &c., as convenience and propriety may dictate.

The *square unit* is always the square or the *linear* unit.

### PROBLEM I.

To find the area of a square, or a parallelogram.

**RULE.**— *Multiply the length by the perpendicular breadth, and the product will be the area.*

(Leg. b. IV, prop. V. Rob. book I, th. 29).

1. What is the area of a square whose sides are 6 feet 3 inches ?

Ans.  $39\frac{1}{4}$  square feet. \*

2. How many square feet are in a board that is  $13\frac{1}{2}$  feet long and 10 inches wide ?

Ans.  $11\frac{1}{4}$  square feet.

3. A lot of land is 80 rods long, and 45 rods wide, how many square rods does it contain, and how many acres ?

Ans. 3600 rods,  $22\frac{1}{2}$  acres.

---

\* **NOTE.**— Reductions from one measure to another have no reference to the rules here given.

4. A man bought a farm 198 rods long, and 150 rods wide, at \$32 per acre; what did it come to?      Ans. \$5940.

PROBLEM II.

To find the area of a triangle, when the base and altitude are given.

RULE.— *Multiply one of these dimensions by half the other, and the product will be the area required.*

(Leg. book IV, p. VI. Rob. book I, th. 30).

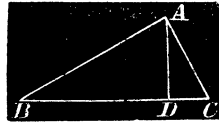
1. How many yards in a triangle whose base is 148 feet, and perpendicular 45 feet?      Ans. 370 yards.
2. What is the area of a triangle whose base is  $18\frac{1}{3}$  feet and altitude  $25\frac{1}{4}$  feet?      Ans.  $231\frac{1}{4}$  feet.

PROBLEM III.

*Investigate and give a rule for finding the area of a triangle when two sides and their included angle are given.*

Let  $ABC$  be the triangles,  $AB, BC$  the given sides, and  $B$  the given angle.

Represent the side opposite to the angle  $A$ , by  $a$ , opposite  $C$ , by  $c$ , and opposite  $B$ , by  $b$ .



Now  $a$  and  $c$  are the given sides, and by problem II, the area is

$$\frac{1}{2}a(AD) \quad (1)$$

The trigometrical value of  $AD$  can be found from the right angled triangle  $ABD$ .

Thus,  $\sin. ADB : c :: \sin. B : AD$ .

That is,  $1 : c :: \sin. B : AD$ .

Whence  $AD = c \sin. B$ .

This value of  $AD$  substituted in (1) gives

$$\frac{1}{2}ac \sin. B = \text{area } \triangle \quad (2).$$

This expression is the area of the triangle, and from it we draw the following rule.

RULE.— *Take half the product of the two given sides and multiply it by the natural sine of the included angle, and the last product will be the area required.*

1. One side of a triangle is 82 feet, another 90 feet, and their included angle is  $27^\circ 31'$ . What is the area?

Ans. 1749.4 square feet.

|                           |   |   |   |                  |
|---------------------------|---|---|---|------------------|
| $27^\circ 31'$ Nat. sine. | - | - | - | 46201            |
| $\frac{1}{2} ac$          | - | - | - | 3780             |
|                           |   |   |   | <hr/> 3996080    |
|                           |   |   |   | 323407           |
|                           |   |   |   | <hr/> 138603     |
|                           |   |   |   | <hr/> 1749.39780 |

Ans.

When we use logarithms we have the following rule :

RULE.— *To the logarithms of the two sides, add the log. sine of the included angle, and the sum rejecting 10, in the index, is the logarithm of twice the area of the triangle.*

2. A certain triangle has one side 125.81, another equal 57.65, and their included angle  $57^\circ 25'$ , what is its area? Ans. 3055.7.

|                |      |                |
|----------------|------|----------------|
| 125.81         | log. | 2.099715       |
| 57.65          | log. | 1.760799       |
| $57^\circ 25'$ | sine | <hr/> 9.925626 |

2 Area, 6111.4 log. 

---

3.786140

3. How many square yards in a triangle, two sides of which are 25 and  $21\frac{1}{2}$  feet, and their included angle  $45^\circ$ ? Ans. 20.8695.

#### PROBLEM IV.

*Investigate and give a rule for finding the area of a triangle when the three sides are given.*

(See figure to problem III). Let  $A$  represent the area of any plane triangle, then by problem III

$$A = \frac{1}{2}ac \sin. B. \quad (1)$$

But  $\sin. B = 2 \sin. \frac{1}{2}B \cos. \frac{1}{2}B$ . (Eq. 30, trigonometry).

Therefore,  $A = ac, \sin. \frac{1}{2}B \cos. \frac{1}{2}B$ . (2)

Now in proposition 8, trigonometry, we find

$$\sin. \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}}, \quad (3)$$

$$\text{and } \cos. \frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}} \quad (4)$$

The product of (3) into (4) is

$$\sin. \frac{1}{2}B \cos. \frac{1}{2}B = \sqrt{\frac{s(s-a)(s-b)(s-c)}{a^2 c^2}}, \quad (5)$$

$$\text{or } ac \sin. \frac{1}{2}B \cos. \frac{1}{2}B = \sqrt{s(s-a)(s-b)(s-c)}. \quad (6)$$

By comparing (2) and (6) we perceive that

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

Here  $s$  represents the half sum of  $a$ ,  $b$ , and  $c$ , therefore, we have the following rule to find the area when the three sides are given.

**RULE.**—Add the three sides together and take half the sum. From the half sum take each side separately, thus obtaining three remainders. Multiply the said half sum and the three remainders together; the square root of this product is the area required.

1. Find the area of a triangle whose sides are 20, 30, and 40.

Ans. 290.47.

$$\frac{1}{2} \text{ sum} = 45, \text{ 1st Rem.} = 25, \text{ 2d} = 15, \text{ 3d} = 5.$$

$$\sqrt{45 \cdot 25 \cdot 15 \cdot 5} = \sqrt{225 \cdot 25 \cdot 15} = 15 \cdot 5 \sqrt{15} = 75(3.873) = 290.474.$$

2. How many acres in a triangle whose sides are severally 60, 50, and 40 rods?

Ans.  $6\frac{1}{3}$  nearly.

3. How many square yards are there in a triangle whose sides are 30, 40 and 50 feet?

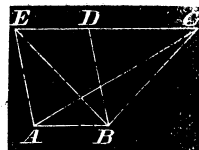
Ans.  $66\frac{2}{3}$ .

4. There is a triangular lot of land containing 8 acres, two of its sides are 64, and 46 rods respectively; what is the angle between these sides, and what is the length of the remaining side?

Ans. The angle is  $60^\circ 27'$ , or is supplement  $119^\circ 33'$ .

The side is 57.37 rods, or 95.535 rods; the less angle corresponding to the lesser side.

In short, there are two triangles answering to the conditions, the one is  $ABE$ , the other  $ABC$ . They are equal because they are on the same base and between the same parallels.  $AE = 57.37$ ,  $AC = 95.535$ .



## PROBLEM V.

To find the area of a trapezoid.

**RULE.**—Add the two parallel sides together, and take half the sum. Multiply this half sum by the perpendicular distance between the sides.

Or, The sum of the parallel sides multiplied by their distance asunder will give twice the area.

(Leg. book IV, prop. VII. Rob. b. I, th. 31).

**REMARK.**—The application of this problem is the most important of any in general surveying, as will appear in the sequel, and if the geometrical theorem is not familiar to the student he should again review it.

**Ex. 1.** In a trapezoid, the parallel sides are 750 and 1225, and the perpendicular distance between them 1540 links: to find the area.

1225

750

$1975 \times 770 = 152075$  square links = 15 acr. 33 perches.

**Ex. 2.** How many square feet are contained in a plank, whose length is 12 feet six inches, the breadth at the greater end 15 inches, and at the less end 11 inches? Ans.  $13\frac{1}{2}$  feet.

**Ex. 3.** In measuring along one side  $AB$  of a quadrangular field, that side, and the two perpendiculars let fall on it from the two opposite corners, measured as below, required the content.

$AP = 110$  links

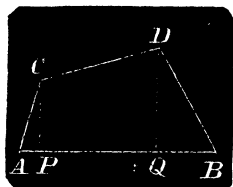
$AQ = 745$

$AB = 1110$

$CP = 352$

$DQ = 595$

Ans. 4 acres, 1 rood, 5.792 perches.



Here we perceive a trapezoid and two right angled triangles.

**N. B.** A chain is 4 rods, and contains 100 links; 10 square chains make an acre.

## PROBLEM VI.

To find the Area of any Trapezium.

**DIVIDE** the trapezium into two triangles by a diagonal; then find the areas of these triangles, and add them together.

Or thus, let fall two perpendiculars on the diagonal from the other two opposite angles ; then add these two perpendiculars together, and multiply that sum by the diagonal, taking half the product for the area of the trapezium.

Ex. 1. To find the area of the trapezium, whose diagonal is 42, and the two perpendiculars on it 16 and 18.

Here  $16+18=34$ , its half is 17.

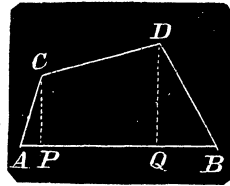
Then  $42 \times 17 = 714$  the area.

Ex. 2. How many square yards of paving are in the trapezium, whose diagonal is 65 feet ; and the two perpendiculars let fall on it 28 and  $33\frac{1}{2}$  feet ?

Ans.  $222\frac{1}{2}$  yards.

When the sides of a trapezium, and two of its opposite angles are given, the most convenient rule for finding its area is found in problem III.

Conceive  $CB$  joined, then the whole figure consists of two triangles and the whole area is found in the following expression  
 $(AB \times AC \times \sin.A) + (CD \times DB \times \sin.CDB).$



EXAMPLE.

In the quadrilateral  $ACDB$  we have  $AC$  15.7,  $CD$  20.4,  $DB$  14.24, and  $BA$  27.7 rods. The angle  $A$   $78^\circ 15'$  and the opposite angle  $CDB$   $97^\circ 30'$ . What is the area enclosed ?

Ans. 356.65 square rods.

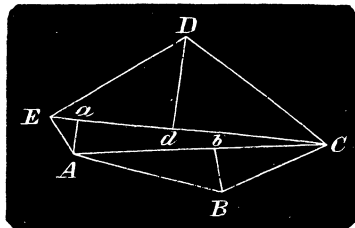
### PROBLEM VII.

To find the area of an irregular figure bounded by any number of right lines.

RULE. — Draw diagonals dividing the figure into triangles. Find the areas of the triangles so formed and add them together for the area of the whole.

Let it be required to find the area of the adjoining figure of five sides. On the supposition that  $AC=36.21$   $EC=39.11$   $Aa=4.18$   $Bb=4$  and  $Dd=7.26$

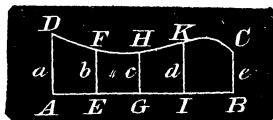
Ans. 296.129.





## PROBLEM VIII.

To find the area of a long irregular figure like the one represented in the margin, it is necessary to divide it into *trapezoids*. Then find the area of each one of the trapezoids (by problem V.) and add them together for the whole area.



If however the trapezoids have equal distances between their parallel sides we can take a more summary process, which we discover by the following investigation.

$$\text{The trapezoid } AEF D = \frac{1}{2} (a+b) \times AE.$$

$$\text{“ “ } EGH F = \frac{1}{2} (b+c) \times EG.$$

$$\text{“ “ } GIK H = \frac{1}{2} (c+d) \times GI.$$

$$\text{“ “ } IBCK = \frac{1}{2} (d+e) \times IB.$$

On the supposition that  $AE, EG, GI, \&c.$  are all equal to each other the sum of these is  $\left( \frac{a}{2} + b + c + d + \frac{e}{2} \right) \times AE$ ,

which represents the area of the whole figure.

From this we draw the following rule to find the area of a long and narrow figure bounded by a right line on one side, and a broken or curve line on the other, to which *off sets* are made at equidistant points along the right line.

**RULE.**—*Add the intermediate breadths or off sets together, and the half sum of the extreme one : then multiply this sum by one of the equal parts of the right line, the product will be the area required, very nearly.\**

1. The breadths of an irregular figure at five equidistant places, being 6.2, 5.4, 9.2, 3.1, 4.2, and the length of the base 60, what is the area ?

|                        |                  |
|------------------------|------------------|
| Mean of the Extremes   | 5.2              |
| Sum of 5.4, 9.2, 3.1   | 17.7             |
| Sum                    | 22.9             |
| One of the equal parts | 1 5              |
|                        | 114 5            |
|                        | 229              |
|                        | <hr/> Area=343.5 |

\* In case  $DF, FH, \&c.$  are right lines we shall have the area exactly, if they are other than right lines the area will be nearly.

2. The length of an irregular figure being 84, and the breadths at six equidistant places 17.4 20.6 14.2 16.5 20.1 24.4; what is the area?  
 Ans. 1550.64.

PROBLEM IX.

To find the area of a circle, also any sector or segment of a circle.

RULE 1. — *The area of a circle is found by multiplying the radius by half the circumference.*

(Leg. book V, prop. 12. Rob. book V, th. 1.)

RULE 2. *Multiply the square of the diameter by the decimal .7854.*

When the radius of a circle is 1, the length of one degree on the circumference is 0.01745 and the whole circumference is 3.1416.

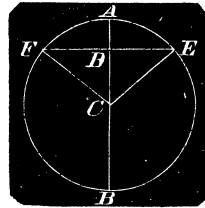
The radius and the circumference increase and decrease by the same ratio, therefore the length of any arc corresponding to any radius is easily computed.

A sector of a circle is to the whole circle as the number of degrees it contains is to 360.

The area of a segment of a circle as *FAE*, may be found by first finding the sector *FCE*, and from it taking the area of the triangle *FCE*.

This same triangle added to the greater sector will give the greater segment.

These principles and rules are sufficient to solve the following examples which are given merely as educational Exercises.



1. What is the area of a circle whose diameter is 10?

Ans. 78.54.

2. What is the area of a circle whose diameter is 20?

Ans. 4 times 78.54.

3. What is the area of a circle whose circumference is 12?

Ans. 11.4595.

4. How many square yards are in a circle whose diameter is  $3\frac{1}{2}$  feet?

Ans. 1.069.

5. Find the length of an arc of  $20^\circ$ , the radius being 9 feet.

Ans. 3.141.

6. Find the length of an arc of  $60^\circ$ , the radius being 18 feet.  
Ans. 18.846.
7. To find the length of an arc of 30 degrees, the radius being 9 feet.  
Ans. 4.7115.
8. To find the length of an arc of  $12^\circ 10'$ , or  $12^\circ \frac{1}{3}$ , the radius being 10 feet.  
Ans. 2.1231.
9. What is the area of a circular sector whose arc is  $18^\circ$  and the diameter 3 feet?  
Ans. 0.35343.
10. To find the area of a sector, whose radius is 10, and arc  $20^\circ$   
Ans. 100.
11. Required the area of a sector, whose radius is 25, and its arc containing  $147^\circ 29'$ .  
Ans. 804.3986.
12. What is the area of the segment, whose height is 18, and diameter of the circle 50?  
Ans. 636.375.
13. Required the area of the segment whose chord is 16, the diameter being 20?  
Ans. 44.728.
14. What is the length of a chord which cuts off one-third of the area from a circle whose diameter is 289?  
Ans. 278.6716.
15. The radius of a certain circle is 10; what is the area of a segment whose chord is 12?  
Ans. 16.35.
16. What is the area of a segment whose height is 2 and chord 20?  
Ans. 26.88.
17. What is the area of a segment whose height is 5, the diameter of the circle being 8?  
Ans. 33.0486.

### PROBLEM X.

*To find the Area of an Ellipse.*

RULE.—*Multiply the two semi-axes together and their product by 3.1416.*  
(See conic sections).

1. Required the area of an Ellipse whose two semi-axes are 25 and 20.  
Ans. 1570.8.
2. The two semi-axes of an Ellipse are 12 and 9, what is its area?  
Ans. 339.29.

To find the area of any portion of a parabola we *multiply the base by the perpendicular height, and take two-thirds of the product for the area required.*  
(See conic sections).

Required the area of a parabola, the base being 20, and the altitude 30. Ans. 400.

The surfaces of prisms, cylinders, pyramids, cones, &c., are found by the application of the preceding rules.

From theorem 16, book VII, Geometry, we learn that

*The convex surface of a sphere is equal to the product of its diameter into its circumference.*

*The surface of a segment is equal to the circumference of the sphere, multiplied into the thickness of the segment.*

*In the same sphere, or in equal spheres, the surfaces of different segments are to each other as their altitudes.*

## MENSURATION OF SOLIDS.

By the Mensuration of Solids are determined the spaces included by contiguous surfaces; and the sum of the measures of these including surfaces, is the whole surface or superficies of the body.

The measures of a solid, is called its solidity, capacity, or content.

Solids are measured by cubes, whose sides are inches, or feet, or yards, &c. And hence the solidity of a body is said to be so many cubic inches, feet, yards, &c., as will fill its capacity or space, or another of an equal magnitude.

The least solid measure is the cubic inch, other cubes being taken from it according to the proportion in the following table, which is formed by cubing the linear proportions.

*Table of Cubic or Solid Measures.*

|                   |                   |                 |
|-------------------|-------------------|-----------------|
| 1728              | cubic inches make | 1 cubic foot    |
| 27                | cubic feet        | 1 cubic yard    |
| 166 $\frac{2}{3}$ | cubic yards       | 1 cubic pole    |
| 64000             | cubic poles       | 1 cubic furlong |
| 512               | cubic furlongs    | 1 cubic mile.   |

As the mensuration of solids has little to do with surveying or navigation, we shall leave this subject after simply stating the following truths, which are demonstrated in solid geometry.

In fact, these truths may be called *rules* for practical operations.

1. *The solidity of a cube, parallelopiped, prism, or cylinder, is found by multiplying the area of its base by the altitude.*

2. *The solidity of a pyramid or cone is found by multiplying the base by the altitude, and taking one-third of the product.*

3. *The solidity of the frustum of a pyramid or cone is found by calculating the solidity of the pyramid when complete, and subtracting from it the solidity of the part removed; or by multiplying the top and bottom diameters together and that product by the altitude, and then to this last product adding a sum produced by squaring the difference between the top and bottom diameters and multiplying it by one-third of the height.*

4. *Gauging is performed by considering a cask to be made up of two frustums of cones placed base to base, and applying the rules for the measurement of such solids.*

5. *The solidity of a sphere is two-thirds of the solidity of its circumscribing cylinder.*

---

## CHAPTER I.

### MENSURATION OF LANDS.

LANDS are not only measured to find their areas, but their exact positions must be ascertained, the direction which each line makes with the meridians, or *with the north and south lines on the earth.*

The boundaries of each tract of land are referred to that meridian which runs through or by the side of it.

All meridian lines meet at the poles, therefore they are not parallel, (except at the equator,) but the poles are so far distant that no sensible error can arise from supposing them parallel, and all surveys are made on the supposition that the surface of the earth is a plane and the meridians parallel. When large surveys are made, like a county or a state, the spherical form of the earth should be taken into consideration.

Meridian lines in surveys are usually determined by the *magnetic needle*, but the needle does not settle exactly north and south, gen-

erally speaking, and the direction which it does settle is called the *magnetic meridian*.

Surveys are often made by the *magnetic meridian* as the true one, and this would answer every purpose, provided the difference between the magnetic and true meridians were *every where* and at *all times* the same, but this is not so.

The magnetic meridian is variable, and for this reason it is very difficult to trace old lines, unless visible monuments are left, or unless the record refers to the true meridian.

Lines are generally measured by a *chain* of 66 feet or 4 rods in length, containing 100 *links*, each link is therefore 7.92 inches.

The area of land is estimated in acres and hundredths, formerly in acres, roods, and perches, but the modern method is more simple and convenient; we have a clearer conception of 35 hundredths of an acre than we have of 1 rood and 16 perches.

*An acre is equal to 10 square chains or 100,000 square links.*

We may note down the length of a line in chains and hundredths, or in links only, for it is nearly one and the same thing: thus, 12 chains and 38 links may be written 12.38, or 1238 links.

The area of a field may be found by measuring with the chain only, and dividing it into rectangles and triangles, and computing each of them separately, according to the rules laid down in mensuration.

The most common method for measuring a field for calculation, is, to take the length of all the sides of the field with the chain, and their bearings with the surveyor's compass. With these notes an accurate plan or plot of the field may be made on paper, and then its contents ascertained by cutting it into triangles and measuring their bases and perpendiculars with a scale and dividers. A very little instruction from a teacher will enable the student to practice this method with success; *yet no instrumental measures pretend to be numerically accurate, they are but approximately so.*

#### TO MEASURE A LINE.

Provide a chain and 10 small arrows or marking pins to fix one into the ground, as a mark, at the end of every chain; two persons take hold of the chain, one at each end of it; and all the 10 arrows

are taken by one of them who goes foremost, and is called the leader ; the other being called the follower, for distinction's sake.

A picket, or station-staff being set up in the direction of the line to be measured, if there do not appear some marks naturally in that direction, they measure straight towards it, the leader fixing down an arrow at the end of every chain, which the follower always takes up, as he comes at it, till all the ten arrows are used. They are then all returned to the leader, to use over again. And thus the arrows are changed from the one to the other at every 10 chains' length, till the whole line is finished ; then the number of changes of the arrows shows the number of tens, to which the follower adds the arrows he holds in his hand, and the number of links of another chain over to the mark or end of the line. So, if there have been 3 changes of the arrows, and the follower hold 6 arrows, and the end of the line cut off 45 links more, the whole length of the line is set down in links thus, 3645.

In all these measures horizontal distances are required, and they are obtained, at least very nearly, by holding the chain in a *horizontal position*, both on ascending and descending ground. If the declivity is too great to admit of measuring a whole chain at a time, take a part of it, and in all cases the proper position of the elevated extremity should be determined by a plumb line. The reason of these operations is obvious by the adjoining figure ; we require the line  $AB$ , and not the line along the ground as  $AC$ .

$$AB = ab + cd + fC.$$

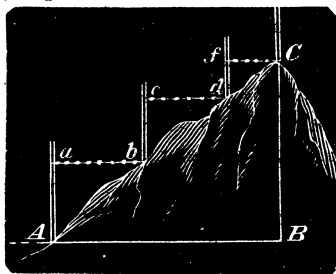
It is not only necessary to measure lines but we must also know their direction or the angles which they make with the meridian.

This is commonly determined by means of the

#### SURVEYOR'S COMPASS.

The surveyor's compass consists of a horizontal circle to which are attached sight-vanes and a magnetic needle delicately balanced on its centre.

When the compass is *set*, that is, standing in a free horizontal



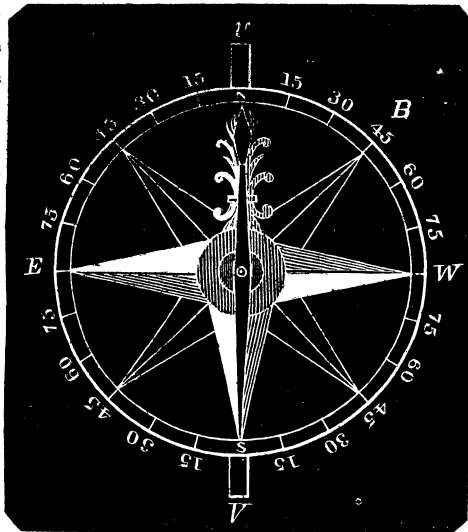
position and the needle free to move on the center, the needle will keep the magnetic meridian, and the circular plate *may be turned under it* to bring the sight-vanes to any line; — the needle will then point out the degree of inclination which the line makes with the meridian.

It is important that this part of the subject be most clearly understood by the learner, we therefore give the following minute illustration of it.

*Let the reader now face the north with the book open before him, his right hand is then toward the east, and his left hand toward the west.*

The following figure represents the compass set to the magnetic meridian, that is the sight-vanes *Vv* and the needle lie in the same direction. The degrees on the plate are numbered both ways from *N* and *S* to *E* and *W*.

At the first view of this subject, it has surprised many to find *W* for *west* on the right hand toward the east, and *E* in the direction toward the west. The reason of this is explained by the next figure.



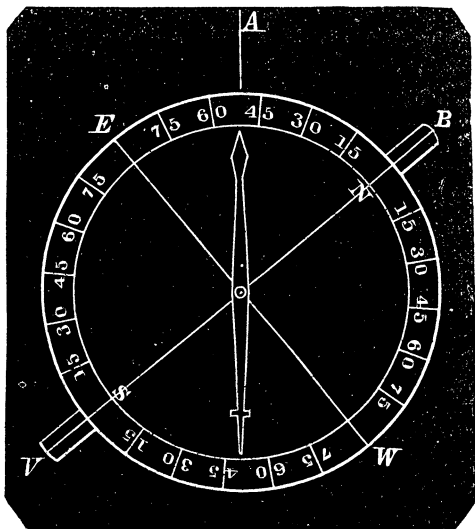
Suppose we wished to find the direction of a line from the center of the compass to the object *B*. We set the compass, that is place it horizontal on its staff or tripod, the needle will take the same direction as in the first figure, parallel to the margin of the paper.

The sight-vanes *Vv* are turned toward the object *B* which turns the whole plate, but the needle retains its position.



We now read the degree pointed out by the north end of the needle, and we find it to be about  $50^\circ$  on the arc between *N* and *E*, showing that the course or direction from the center of the compass to *B* is North about  $50^\circ$  toward the East — a result obviously true.

Turning the sight-vanes toward the north west will bring



the arc between *N* and *W* to the north point of the needle. For example, if it were required to run a line North  $31^\circ$  West, from a certain point, all we have to do is to set the compass over that point, level the plate, see that the needle is free to move on its pivot, and so turn the plate that the north end of the needle will settle at  $31^\circ$  between *N* and *W*, the range of the sight-vanes will then show the required line. Proceed in the same manner to find any other line.

Care should be taken that no iron or steel comes near the compass while operating with it. To insure a correct position of the needle is the principal difficulty, but if it settles with a free motion, describing nearly equal arcs, slowly decreasing on each side of a given point and finally rests at that point, it operates well, and may be relied upon.

In whatever direction we run, the north point of the needle should always lie on *some part* of the north side of the plate, that is, nearer to *N* than to *S* and this can always be except when we run due east or west per compass.

Lines should be tested by taking *back sights* or reverse bearings, which will be *exactly in the opposite* point of the compass, in case there is no local attraction to disturb the needle. If the line just

run over does not correspond to the exact opposite point of the compass, it shows carelessness in running or some local attraction of the needle. We shall show how to overcome this last difficulty further on.

Compasses are usually marked to half degrees, some of them are subdivided to one fourth degrees, but by the aid of a *vernier scale* we can *theoretically* read the arc to *one minute* of a degree.

#### DESCRIPTION OF THE VERNIER.

The vernier to a compass is on the outer edge of the graduated limb. *It is a slip of metal made to fit the graduated limb of an instrument, and the equal divisions upon it are so made that  $n$  divisions on the vernier will cover  $n \pm 1$  divisions on the limb.*

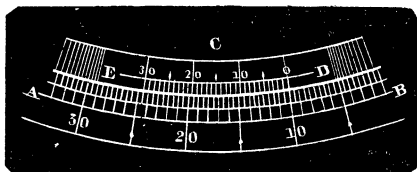
The vernier of the compass is on the outside of the dial plate and it is firmly attached to the bar that holds the sight-vanes. The dial plate can be moved to and fro along it by means of a screw.

The vernier is used when the needle points between two divisions on the limb: the dial plate is then gently moved by the screw until the needle points exactly to the preceding division on the limb, this being done, that division on the vernier which makes a right-line, that is, coincides with a division on the arc is the number of small divisions (minutes) to be added to the division on the limb now pointed out by the needle.

*Practical and experienced men, never use the vernier of the compass, because they can read the compass without it to greater accuracy than they can really run a line.*

But as the vernier scale is of the greatest importance attached to several other instruments, which will be referred to in this work, we now make an effort to give the learner a clear comprehension of it.

Let *AB* represent a portion of an arc and *ED* the vernier which is conceived to be attached to an index bar and made to revolve with it.



In case 0 on the vernier makes a right line with  $10^\circ$  on the arc as is represented in the figure, then the index marks  $10^\circ$ . But if 0 on the vernier is a little beyond  $10^\circ$  we then look along the vernier

scale to see what division of it makes a right line with some division on the limb, suppose the division 8 on the vernier coincided with a division on the limb then the index would mark  $10^{\circ} 8'$ .

To understand the philosophy of this: Let  $x$  represent the value of a division on the vernier and  $n$  the number of them which cover  $(n-1)$  divisions on the limb, then

$$nx = n - 1$$

$$x = 1 - \frac{1}{n}$$

$$\frac{1}{n} = 1 - x$$

$$\frac{2}{n} = 2 - 2x$$

$$\frac{3}{n} = 3 - 3x$$

&c. &c. to  $n$ , number, from which it appears that one division on the vernier beyond a division on the limb corresponds with the  $n$ th part of the unit of graduation, two divisions of the vernier above two divisions on the limb, correspond with  $2n$ th division of the unit of graduation, &c.

In our figure  $n=30$ , the graduation of the limb is to half degrees or 30 minutes, and this vernier measures minutes. Verniers on many instruments measure as small as *ten seconds* of arc and on some very large instruments as low as *four seconds*.

## CHAPTER II.

Having shown in the preceding chapter how to use a compass—to run lines, and to measure them; the next step is to keep a proper record of all the lines run, and compute the areas they enclose.

A line traced on the ground, is called a *course*, the angle that it makes, with the meridian passing through the point of beginning, is called its *bearing*.

A course written *N 42° E*, indicates that the line runs between the north and the east, and makes an angle of  $42^\circ$  with the meridian; when between the north and west, we write *N. W.*, putting the number of degrees and minutes between.

Lines from the south point, are also written *S. E.* and *S. W.*; that is, bearings are reckoned from the north and south points, east and west, as the case may be.

Hence, to make a record of a survey, all we have to do is to write the *bearing* and distance of each course, and if the last side runs to the point of beginning, it is a complete survey; otherwise it is not.

Of course, no *area* can be attached to any un-enclosed space. To complete a partial survey, to enclose a space, to find an *area*, or to test the accuracy of a complete survey; the most satisfactory method of investigation, is that known as

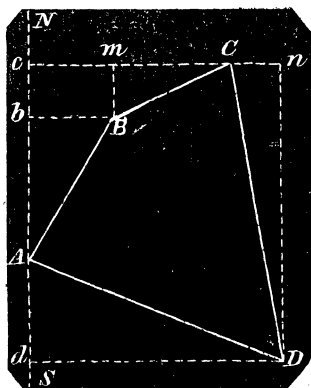
## LATITUDE AND DEPARTURE.

Latitude is the distance of the end of a line north or south of its beginning, measured on a meridian, and it is called either *northing* or *southing*, according as the line runs north or south. *Departure* is the distance of the end of a line east or west of its beginning, measured perpendicular to a meridian, and it is called *easting* or *westing*, according as the line runs east or west.

For example, suppose that we have the following bearings and distances, which enclose a space represented by *ABCD*.

|           | Bearings.        |   |   | Distances. |
|-----------|------------------|---|---|------------|
| <i>AB</i> | <i>N. 23° E.</i> | - | - | 17         |
| <i>BC</i> | <i>N. 83° E.</i> | - | - | 11         |
| <i>CD</i> | <i>S. 14° E.</i> | - | - | 23         |
| <i>DA</i> | <i>N. 77° W.</i> | - | - | 23.66      |

Let  $NS$  represent the meridian running through  $A$ , the most western point of the field; make the angle  $NAB=23^\circ$ , and  $AB=17$ ; then  $Ab$  is the latitude, and  $Bb$  is the departure, corresponding to the course  $AB$ . By means of the right angled triangle  $ABb$ , having the hypotenuse  $AB$ , and the angles, we can compute  $Ab$ , and  $Bb$ , 15.65, and 6.64, or we can turn to the traverse table, and under  $23^\circ$  and opposite 17, we shall find the value of these lines at once; and this is the utility of having the traverse table.



In the same manner we find  $Bm$  and  $mC$ , the latitude and departure corresponding to the bearing and distance of the line  $BC$ . We find  $Bm=1.34$ , and  $mC=10.92$ .

Thus we go round the field, taking the latitude and departure of each side, and arrange the whole in a table as follows:

| Bearings. |                  | Dist. | N.    | S.    | E.    | W.    |
|-----------|------------------|-------|-------|-------|-------|-------|
| $AB$      | $N. 23^\circ E.$ | 17    | 15,65 |       | 6,64  |       |
| $BC$      | $N. 83^\circ E.$ | 11    | 1,34  |       | 10,92 |       |
| $CD$      | $S. 14^\circ E.$ | 23    |       | 22,32 | 5,56  |       |
| $DA$      | $N. 77^\circ W.$ | 23,66 | 5,33  |       |       | 23,05 |
|           |                  |       | 22,32 | 22,32 | 23,12 | 23,05 |

When the several operations are performed with perfect accuracy, the sum of the northings will be equal to that of the southings, and the sum of the eastings to that of the westings. This necessarily follows from the circumstance of the surveyor's returning to the place from which he set out; and it affords a means of judging of the correctness of the work. But it is not to be expected that the measurements and calculations in ordinary surveying will strictly bear this test. If there is only a small difference, as in the above example, between the northings and southings, or between the eastings and westings, it may be imputed to slight imperfections in the measurements.

Here the northings and southings agree, but the eastings are a little greater than the westings; we will therefore decrease the

eastings by half the error, and increase the westings by the same amount; the sums will then agree.

We do this without any formal statement, but the operation is strictly that of proportion; the greater the line the greater the correction to be applied.

When the errors are considerable, a re-survey should be made, and if the errors are still great, and in the same direction, there is reason to suspect that some local attraction disturbs the free action of the needle; and then, if the importance demands it, a survey can be taken *without* the compass, by methods we shall explain in some following chapter.

We shall make use of this example and this figure to illustrate the

#### TAKING OF ANGLES BY THE COMPASS.

For this, and for several other operations in practical mathematics, the learner must not expect a written rule: original principles are far more simple and reliable.

We now require the angle  $ABC$ ; conceive the  $AB$  to be produced, then the angle between  $BC$  and the produced part, is  $83^\circ$  less  $23^\circ$ , or  $60^\circ$ . Now  $60^\circ$  taken from  $180^\circ$  gives  $120^\circ$  for the angle  $ABC$ .

Again the line  $BC$  makes an angle with the meridian toward the north of  $83^\circ$ , therefore toward the south it must be  $97^\circ$  on the east side of it. The line  $BA$  makes an angle of  $23^\circ$  with the meridian on the west side of it; therefore, the angle  $ABC = 97 + 23 = 120$ , the same as before.

To find the angle  $BCD$ , we add  $83^\circ$  and  $14^\circ$ . Why?

To find the angle  $CDA$ , we subtract  $14^\circ$  from  $77^\circ$ . Why?

To find the angle  $DAB$ , we add  $14^\circ$  to  $77^\circ$ .

The sum of these 4 angles must equal 4 right angles.

Suppose now that the surveyor runs the lines  $AB$ ,  $BC$ ,  $CD$ , and then wishes

#### TO CLOSE THE SURVEY.

To close a survey is to run the last side so as to strike the first point, when we are not able to see it.

To accomplish this, we sum up the latitude and departure as far

as the point  $D$ , the result will show  $Dd$  and  $dA$ . Having then two sides of the right angled triangle, the angle  $dAD$  will be the bearing for  $dAD = nDA$ , because  $nD$  and  $NS$  are parallel. The side  $DA$  can also be computed, but it should be measured also, as a test to the accuracy of the whole survey. If, on running  $DA$ , according to computation, we actually strike the point  $A$ , or very near to it, and there is little or no difference between actual measure and computation, then we may be sure that all the sides have been run correctly; but if on running  $DA$ , we do not strike  $A$ , or the distance does not correspond to computation, we may be sure of errors somewhere — either in want of skill or care in the operation, or the action of the compass has not been uniform at all the angular points. In case of material errors a re-survey should be made.

In case that we have no means of making computations in the field, we may take a course as near the true one as our judgment will permit, and run it. This line must bring us near the point of beginning, if it does not strike it, and when we get opposite to that point we must measure to it at right angles from the line run; then we shall have data to correct our course. Running a line thus by guess work, is called *running a random line*, from which the true line can be found as follows:

Suppose that when we arrive at  $D$ , we judge the course to the first point to be  $N. 75^\circ W.$ , and run that course, and after measuring 28.65 chains we find that we are passing the first point, which is 83 links in perpendicular distance toward the south; what course should have been taken?

By the following investigation we draw out a rule that will apply to *all such cases*.

Let  $AB$  represent a true course,  $AD$  a *random line*, and  $DB$  its amount of deviation.

Also let  $DAE$  equal one degree, and take  $Ad=1$ , then the deviation at  $d$  will be the natural sine of one degree, and may be taken from the table of natural sines (which is .01745).



By proportional triangle we have

$$1 : .01745 :: AD : DE;$$

Whence  $DE = .01745(AD)$ .

Now  $DE$  is contained in  $DB$  as often as  $1^\circ$  is contained in the

number of degrees in the angle  $DAB$ . Let  $x$  represent the number of degrees in  $DAB$ , then

$$\frac{x}{1} = \frac{(DB)}{.01745(AD)}.$$

That is,  $x = \frac{(DB)(57.3)}{(AD)}$ , because  $\frac{1}{.01745} = 57.3$ .

Hence, to correct a course, we have the following

**RULE.**— *Multiply the deviation by 57.3, and divide that product by the distance, and the quotient will be the number of degrees and parts of a degree to add to, or subtract from, the random course.*

This rule, applied to the present example, gives

$$\frac{.83(57.3)}{23.65} = 2^{\circ}.$$

Hence, the true course is  $75^{\circ} + 2^{\circ} = 77^{\circ}$ .

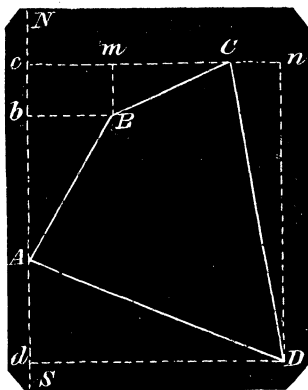
Had the deviation of the *random line* been toward the south, we should have subtracted the correction; but for this the operator must rely on his judgment.

We now come to the

#### COMPUTATION OF AREAS.

By inspecting the figure we perceive that  $cCDd$  is a *trapezoid*, from which, if we subtract the triangles  $ADd$ ,  $ABb$ , and the *trapezoid*  $bBCc$ , the area of the field  $ABCD$  will be left.

**OBSERVATION.**— To preserve uniformity of expression, and clearness and brevity in forming a rule, we shall call *triangles*, *trapezoids*, while discussing this subject. A trapezoid becomes a triangle, when its smallest parallel side is so small as to call it zero — conversely, then, a triangle is a trapezoid, whose smallest parallel side is zero.



We observe that  $C$  is the most northern point of the field, and  $D$  is the most southern. In traversing from  $C$  to  $D$ , from the north to the south, we pass along the oblique sides of trapezoids that we shall call *south areas*, and in traversing from  $D$  to  $A$ ,  $B$ , and  $C$ ,



from the south to the north, we pass along the oblique sides of trapezoids, which we shall call *north areas*. Now it is obvious *that if we subtract the sum of the north areas from the sum of the south areas, we shall have a remainder equal to the area of the field.*

We now require a systematic method of finding these *areas*, or the area of these several trapezoids. In the first place, we must have latitudes and meridian distances.

Latitude and departure have already been defined and explained.

#### MERIDIAN DISTANCES.

*Meridian distances* are the distances of the angular points of the field from the meridian which runs through the most westerly point of the field; thus,  $bB$ ,  $cC$ ,  $dD$ , are meridian distances.

*Double meridian distances* are the bases of triangles, or the sum of the parallel sides of the trapezoids.

Thus  $bB$  is the *double meridian distance* of the side  $AB$ , or it is the double meridian distance of the middle point of the line  $AB$ .  $bB + cC$  is the *double meridian distance* of the line  $BC$ , or double the meridian distance of the middle point of  $BC$ . This double meridian distance ( $bB + cC$ ) multiplied by  $bC$ , or  $Bm$ , the latitude corresponding to  $BC$ , will give the double area of the trapezoid  $bB, Cc$ .

We are now prepared to give the following summary or rule for finding the area of any field bounded by any number of right lines:

**RULE.**—1. *Prepare a table headed as in the example, namely: Bearings, Distance, North, South, East, West, Meridian distance, Double meridian distance, North areas, South areas.*

2. *Begin at the most western point of the field, and conceive a meridian to pass through that point.*

*Find, by the traverse table or by trigonometry, the northings, southings, eastings, and westings of the several sides of the field, and set them in the table opposite their respective stations, under their proper letters N., S., E., or W.*

3. *For the first meridian distance take the departure of the first line; for the second, take the first meridian distance and add to it the departure of the second line, if the departure is east, or subtract if west, &c.*

4. Add each two adjacent meridian distances, and set their sum opposite the last of the two in the column of double meridian distances.

5. Multiply each double meridian distance by the latitude to which it is opposite, and set the product in the column of *N.* areas, if the latitude is north, and in that of *S.* areas, if the latitude is south.

6. Subtract the sum of the *N.* areas from that of the *S.* areas, and take half the remainder, which will be the area of the field in square chains. Dividing this by 10 gives the acres; and the roods and rods are found by multiplying the decimal parts by 4 and by 40.

|    | Bearings. | Dis.  | N.    | S.    | E.               | W.               | M. D. | D. M. D. | N. areas.       | S. areas |
|----|-----------|-------|-------|-------|------------------|------------------|-------|----------|-----------------|----------|
| AB | N 23° E   | 17    | 15.65 |       | 6.64<br>(6.63)   |                  | 6.63  | 6.63     | 103.76          |          |
| BC | N 83° E   | 11    | 1.34  |       | 10.92<br>(10.90) |                  | 17.53 | 24.16    | 32.37           |          |
| CD | S 14° E   | 23    |       | 22.32 | 5.56<br>(5.55)   |                  | 23.08 | 40.61    |                 | 906.42   |
| DA | N 77° W   | 23.66 | 5.33  |       |                  | 23.05<br>(23.08) | 0.00  | 23.08    | 123.02          |          |
|    |           |       |       |       |                  | 23.08            |       |          | 259.15          | 906.42   |
|    |           |       |       |       |                  |                  |       |          |                 | 259.15   |
|    |           |       |       |       |                  |                  |       |          | Diff,           | 647.27   |
|    |           |       |       |       |                  |                  |       |          | Half,           | 323.63   |
|    |           |       |       |       |                  |                  |       |          | Dividing by 10, | 32.363   |

Hence the field contains 323 square chains and 63 hundredths, or thirty two acres and a little more than 36 hundredths of an acre.

The numbers in parentheses, as (6.63), and all others in parentheses are the numbers corrected to make the eastings and westings agree, — the numbers above them are taken from the table.

Before we give any more examples, it is proper to give some examples to show the practical utility of the

#### TRAVERSE TABLE.

This table is computed to every half degree, but if a course is between two courses in the table, the operator can use his judgment and take out the proper intermediate numbers.

Those who are not satisfied with this method, can use the table of natural sines and cosines, as we shall subsequently explain.

The distances are consecutive to 30, then 35, 40, &c., to 100. But a little thought in the operator will enable him to use the table for *any distance whatever*.

The following examples will illustrate.

1. A course is *N. 22° 30' E.* distance 62.43 ; what is the corresponding latitude and departure, as found in the table ?

We shall regard the distance as 6243 links, and separate it into parts.

|             | Lat.          | Dep.          |
|-------------|---------------|---------------|
| Thus : 6000 | 5543          | 2296          |
| 240         | 221.7         | 91.8          |
| 3           | 2.77          | 1.15          |
| <hr/> 6243  | <hr/> 5767.47 | <hr/> 2388.95 |

If we now return to chains and links, the latitude is 57.67 and the departure 23.89.

We entered 240 in the table, as 24 chains, and took the numbers corresponding.

2. A course is *N. 48° 30' W.*, distance 187.61 ; what is the corresponding latitude and departure ?

| Dis.         | Lat.           | Dep.           |
|--------------|----------------|----------------|
| 180.00       | 119.30         | 134.80         |
| 7.60         | 5.036          | 5.692          |
| 1            | 066            | 075            |
| <hr/> 187.61 | <hr/> 124.3426 | <hr/> 139.4995 |

If we now take the distance as 187 chains and 61 links, the Lat. is 124 ch. 34 links, and the Dep. is 139 ch. and 50 links.

3. A course is *S. 81° W.*, distance 76.87 ; what is the corresponding latitude and departure ?

| Dis.        | Lat.         | Dep.          |
|-------------|--------------|---------------|
| 75.00       | 1173.        | 7408.         |
| 1.80        | 28.2         | 177.8         |
| 7           | 1.10         | 6.91          |
| <hr/> 76.87 | <hr/> 1202.3 | <hr/> 7592.71 |

If 76 ch. 87 lin., Lat. 12 ch. 2 lin., Dep. 75 ch. 93 links.

Thus we can find the latitude and departure for any distance corresponding to any degree and half degree.

We can find it to any degree and minute of a degree by the table of natural sines and cosines.

*The common tables containing natural cosines and sines are nothing more than latitude and departure corresponding to unity of distance.*

Therefore, a double distance will correspond to a double distance in latitude and departure, a treble distance will give a treble amount of latitude and departure, and so on in proportion. Lat. = Nat. cosine. Dep. = Nat. sine.

Hence : *The natural cosine of any course taken as a decimal, multiplied by any distance, will give the latitude corresponding to that course and distance. Also, the natural sine taken as a decimal, multiplied by a given distance, will give the departure corresponding to that course and distance.*

N. B. Nat. sines and cosines are found in table II., pages 21-65 of tables. For common purposes, four places of decimals are sufficient.

1. The bearing of a certain line is *N. 35° 18' E.* ; distance 12 chains ; what is the corresponding latitude and departure ?

|                   |                |                |
|-------------------|----------------|----------------|
| Angle 35° 18'     | N. cos. .81614 | N. sin. .57786 |
| Dis. (multiplier) | 12             | 12             |
| Diff. Lat=        | 9.79368        | Dep. 6.93432   |

2. A certain line runs *S. 4° 50' E.* ; distance 74.40 ; what is the corresponding latitude and departure ?

|              |               |               |
|--------------|---------------|---------------|
| Angle 4° 50' | N. cos. .9964 | N. sin. .0842 |
| Distance     | 74.4          | 74.4          |
|              | <u>39856</u>  | <u>3368</u>   |
|              | 39856         | 3368          |
|              | <u>69748</u>  | <u>5894</u>   |
| Lat.         | 74.13216      | Dep. 6.26448  |

3. A line makes an angle with the meridian of *75° 41'*, at a distance of 89.75 chains ; what is the latitude and departure ?

|                  |               |                    |
|------------------|---------------|--------------------|
| 75° 47'          | cos. .2456    | sin. .9694         |
| Distance         | 89.75         | 89.75              |
| Prod. Diff. Lat. | <u>22.042</u> | Dep. <u>87.001</u> |

4. A line bearing *N. 7° 40' W.* ; distance 31.20 chains ; required the difference of latitude and departure.

|            |              |                  |
|------------|--------------|------------------|
| 7° 40'     | cos. .98106  | sin. .13341      |
| Multiplier | 31.2         | 31.2             |
| Diff. Lat. | <u>30.92</u> | Dep. <u>4.16</u> |

5. A line running *S. 80° 10' E.* distance 35.25 chains ; what is the difference of latitude and departure ?

|            |      |              |      |              |
|------------|------|--------------|------|--------------|
| 80° 10'    | cos. | .17078       | sin. | .9853        |
| Multiplier |      | <u>35.25</u> |      | <u>35.25</u> |
| Diff. Lat. |      | 6.02         | Dep. | 34.72        |

In the last three examples, we have given only the results of the multiplications to two places of decimals ; that is, to the nearest link, which is a degree of accuracy sufficient for all practical purposes.

We are now prepared to estimate the areas of the following general surveys, given as

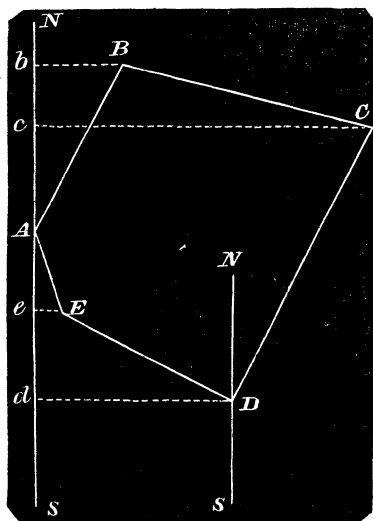
#### EXAMPLES.

1. In May, 1845, the following measures of a field were taken. Beginning at the *western-most* point of the field ; thence *N. 20° 30' E.* 5 chains 83 links ; thence *S. 79° 45' E.* 10 chains 15 links ; thence *S. 27° 30' W.* 9 chains 45 links ; thence *N. 63° 15' W.* 8 chains 28 links ; thence *N. 15° 30' W.* 1 chain and 4 links, to the place of beginning ; required the area.

It is not absolutely necessary to make a plot or figure of the field, but for the sake of perspicuity, it is best to do so ; yet no reliance is placed on the accuracy of the constructed figure.

We perceive by the figure, that there are two *south areas*, *bBCc*, and *cCDd* ; and three *north areas*, *eEdD*, *AeE*, and *ABb*.

Let the reader observe that all the *north areas* are on the outside of the field.



|    | Bearings.    | Dis.  | N.    | S.    | E.    | W.    | M. D. | D.M.D. | N. areas. | S. area. |
|----|--------------|-------|-------|-------|-------|-------|-------|--------|-----------|----------|
| AB | N.20° 30' E. | 5.83  | 5.46  |       | 2.04  |       | 2.04  | 2.04   | 11.1384   |          |
| BC | S.79° 45' E. | 10.15 |       | 1.81  | 9.99  |       | 12.03 | 14.07  |           | 25.4667  |
| CD | S.27° 30' W. | 9.45  |       | 8.38  |       | 4.36  | 7.67  | 19.70  |           | 165.0860 |
| DE | N.63° 15' W. | 8.28  | 3.73  |       |       | 7.39  | 0.28  | 7.95   | 29.6535   |          |
| EA | N.15° 30' W. | 1.04  | 1.00  |       |       | 0.28  | 0.00  | 0.28   | 0.2300    |          |
|    |              |       | 10.19 | 10.19 | 12.03 | 12.03 |       |        | 41.0719   | 190.5527 |

41.0719

2)149.4808

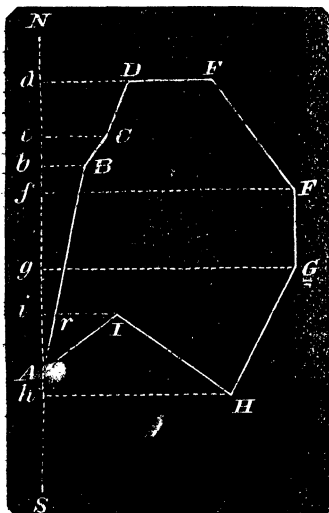
10)74.7204

Area in acres,

7.472

The operator can rely on the rule used in the last operation, whatever be the number of sides, or whatever be the shape of the figure, provided that the lines are right lines, from one angular point to another.

In case of *re-entering* angles, like *HIA*, represented in the adjoining figure, a portion of the figure *AIr*, is reckoned twice. But this is corrected by the subtractive space *HIh*, which includes not only the exterior portion *hHIA*, but also the whole additive triangle *AIi*, belonging to the last side of the figure *IA*.



In the following example are several such re-entering angles.

2. Find the area of a lot of land, of which the following are the field notes.

Beginning at the south west corner, the ancient land mark; thence

1. N. 27° 15' E. distance 9.42 chains.
2. S. 80° 00' E. " 1.15 "
3. S. 69° 00' E. " 12.73 "
4. S. 15° 45' W. " 5.00 "
5. N. 66° 45' W. " 1.05 "
6. S. 31° 00' W. " 2.90 "
7. N. 70° 45' W. " 8.92 "
8. S. 41° 45' W. " 2.08 "
9. N. 63° 00' W. " 4.18 "

We can contract the operation in reference to the space it will occupy, by putting the difference of latitude in one column, and all the departures in another column.

Marking all the northings by the sign +, and all the southings by the sign —. Also, all the eastings by the sign +, and all the westings by the sign —.

This being understood, the work will appear as follows :

|   | Bearings.     | Dis.  | Lat.  | Dep.   | M. D. | D.M.D. | N. areas    | S. areas |
|---|---------------|-------|-------|--------|-------|--------|-------------|----------|
| 1 | N. 27° 15' E. | 9.42  | +8.37 | + 4.31 | 4.31  | 4.31   | 36.0747     |          |
| 2 | S. 80° 00' E. | 1.15  | -0.20 | + 1.13 | 5.44  | 9.75   |             | 1.9500   |
| 3 | S. 69° 00' E. | 12.73 | -4.57 | +11.89 | 17.33 | 22.77  |             | 104.0589 |
| 4 | S. 15° 45' W. | 5.00  | -4.81 | - 1.36 | 15.97 | 33.30  |             | 160.1730 |
| 5 | N. 66° 45' W. | 1.05  | +0.41 | - 0.96 | 15.01 | 30.98  | 12.7018     |          |
| 6 | S. 31° 00' W. | 2.90  | -2.49 | - 1.49 | 13.52 | 28.53  |             | 71.0397  |
| 7 | N. 70° 45' W. | 8.92  | +2.94 | - 8.42 | 5.10  | 18.62  | 54.7428     |          |
| 8 | S. 41° 45' W. | 2.08  | -1.55 | - 1.38 | 3.72  | 8.82   |             | 13.6710  |
| 9 | N. 63° 00' W. | 4.18  | +1.90 | - 3.72 | 0.00  | 3.72   | 7.0680      |          |
|   | Sum           |       | 0.00  | 0.00   |       |        | 110.5873    | 350.8926 |
|   |               |       |       |        |       |        | 110.5873    |          |
|   |               |       |       |        |       |        | 2)240.3053  |          |
|   |               |       |       |        |       |        | 10)120.1526 |          |
|   |               |       |       |        |       |        |             | 12.01526 |

12 acres, and a small fraction over.

3. Having the following field notes, it is required to find the closing side and the area of the field.

|   | Bearings.     | Dis.  | N.    | S.    | E.    | W.    |
|---|---------------|-------|-------|-------|-------|-------|
| 1 | S. 75° W.     | 13.70 |       | 3.54  |       | 13.24 |
| 2 | S. 20° 30' W. | 10.30 |       | 9.65  |       | 3.60  |
| 3 | W.            | 16.20 |       |       |       | 16.20 |
| 4 | N. 33° 30' E. | 35.30 | 29.51 |       | 19.49 |       |
| 5 | N. 76° E.     | 16.00 | 3.87  |       | 15.52 |       |
| 6 | South         | 9.00  |       | 9.00  |       |       |
|   |               |       | 33.38 | 22.19 | 35.01 | 33.04 |
|   |               |       | 22.19 |       | 33.04 |       |
|   |               |       | 11.19 |       | 1.97  |       |

This result shows, that if we commence at the first station, and traverse round to the sixth, we shall then be 11.19 chains to the north of the place of beginning, and 1.97 chains east of it. This is sufficient data to compute the course and distance.

To compute the area, however, it is not necessary to find either the course or the distance.

We do know, however, by merely inspecting the traverse table,

that the course to the place of beginning, is south about  $10^{\circ} 20'$  west, and distance near 12 chains.

We are now prepared to compute the area, and as we wish to commence at the western-most point of the field, we shall begin at the 4th station, calling it the first: thus,

|   | Bearings.              | Dis.  | Lat.   | Dep.   | M. D. | D.M.D. | N. area.    | S. area.  |
|---|------------------------|-------|--------|--------|-------|--------|-------------|-----------|
| 1 | N. $33^{\circ} 30'$ E. | 35.30 | +29.51 | +19.49 | 19.49 | 19.49  | 575.1499    |           |
| 2 | N. $76^{\circ}$ E.     | 16.00 | +3.87  | +15.52 | 35.01 | 54.50  | 210.9150    |           |
| 3 | South                  | 9.00  | —9.00  | 0.0    | 35.01 | 70.02  |             | 630.1800  |
| 4 | S. W.                  | —     | —11.19 | —1.97  | 33.04 | 68.05  |             | 761.4795  |
| 5 | S. $75^{\circ}$ W.     | 13.70 | —3.54  | —13.24 | 19.80 | 42.84  |             | 151.6536  |
| 6 | S. $20^{\circ} 30'$ W. | 10.30 | —9.65  | —3.60  | 16.20 | 36.00  |             | 347.4000  |
| 7 | W.                     | 16.20 | 0.0    | —16.20 | 0.0   | 16.20  |             |           |
|   |                        |       |        |        |       |        | 786.0649    | 1890.7131 |
|   |                        |       |        |        |       |        | 786.0649    |           |
|   |                        |       |        |        |       |        | 2)1104.6482 |           |
|   |                        |       |        |        |       |        | 10)552.3241 |           |
|   |                        |       |        |        |       |        | 55.23241    |           |

This result shows that the field contains 55 acres, and a little more than 23 hundredths of an acre.

4. What is the area of a survey, of which the following are the field notes.

| Stations. | Bearings.              | Distances. |
|-----------|------------------------|------------|
| 1         | S. $46^{\circ} 30'$ E. | 80 rods.   |
| 2         | S. $51^{\circ} 45'$ W. | 34.16      |
| 3         | West.                  | 85.00      |
| 4         | N. $56^{\circ}$ W.     | 110.40     |
| 5         | N. $33^{\circ} 15'$ E. | 75.20      |
| 6         | S. $74^{\circ} 30'$ E. | 123.80     |

Ans. 104.35 acres.

5. Required the contents and plot of a piece of land, of which the following are the field notes.

| Stations. | Bearings.                     | Distances. |
|-----------|-------------------------------|------------|
| 1         | S. $34^{\circ}$ W.            | 3.95 ch.   |
| 2         | S.                            | 4.60       |
| 3         | S. $36\frac{1}{2}^{\circ}$ E. | 8.14       |
| 4         | N. $59\frac{1}{2}^{\circ}$ E. | 3.72       |
| 5         | N. $25^{\circ}$ E.            | 6.24       |
| 6         | N. $16^{\circ}$ E.            | 3.50       |
| 7         | N. $65^{\circ}$ W.            | 8.20       |

Ans. 10A. 0R. 5P.



6. Required the contents and plot of a piece of land, from the following field notes.

| Stations. | Bearings.        | Distances. |
|-----------|------------------|------------|
| 1         | <i>S. 40° W.</i> | 70 rods.   |
| 2         | <i>N. 45° E.</i> | 89         |
| 3         | <i>N. 36° E.</i> | 125        |
| 4         | <i>N.</i>        | 54         |
| 5         | <i>S. 81° E.</i> | 186        |
| 6         | <i>S. 8° W.</i>  | 137        |
| 7         | <i>W.</i>        | 130        |

*Ans. 207A. 3R. 33P.*

7. Given the following bearings and distances of the several sides of a field, namely,

|    |                      |        |
|----|----------------------|--------|
| 1. | <i>N. 58° E.</i>     | 19 ch. |
| 2. | <i>E. 6° S.</i>      | 20     |
| 3. | <i>S. 17° W.</i>     | 20     |
| 4. | <i>W.</i>            | 20     |
| 5. | <i>N. 42° 35' W.</i> | 15.10  |

to find the area.

*Ans. 54.9 acres.*

8. Given the following bearings and distances, namely,

|    |                  |        |
|----|------------------|--------|
| 1. | <i>N. 45° E.</i> | 40 ch. |
| 2. | <i>S. 30° W.</i> | 25     |
| 3. | <i>S. 5° E.</i>  | 36     |
| 4. | <i>W.</i>        | 29.60  |
| 5. | <i>N. 20° E.</i> | 31     |

to find the corrected difference of latitude and departure, and the area.

N. B.—In this last example, as in most others, the northings and southings will not exactly balance; nor will the eastings and westings balance. This arises from inaccuracies in the data. In such cases (if the errors are but trifling) we balance off the errors.

When a course is east or west, as the 4th in this example, some operators have expressed doubts, whether any correction should be applied to latitude in that course.

We reply, that errors do really exist, and, therefore, we cannot

say that the course marked west, in the example, was really west, or not; the probability is, that the course was not exactly due west, and it is therefore proper to put a correction in the latitude column, as shown in the following results :

CORRECTED LATITUDES AND DEPARTURES.

|    | N.    | S.    | E.    | W.    |
|----|-------|-------|-------|-------|
| 1. | 28.30 |       | 28.30 |       |
| 2. |       | 21.63 |       | 12.49 |
| 3. |       | 35.84 | 3.16  |       |
| 4. | 0.02  |       |       | 29.59 |
| 5. | 29.15 |       | 10.62 |       |
|    | 57.47 | 57.47 | 42.08 | 42.08 |

On inspecting these latitudes and departures, we perceive station 5 is the most westerly point of the field, therefore to find the area, we will arrange these results in the following order :

|   | Lat.   | Dep.   | M. D. | D. M. D. | N. areas. | S. areas.          |
|---|--------|--------|-------|----------|-----------|--------------------|
| 5 | +29.15 | +10.62 | 10.62 | 10.62    | 309.5730  |                    |
| 1 | +28.30 | +28.30 | 38.92 | 49.54    | 1400.9820 |                    |
| 2 | -21.63 | -12.49 | 26.43 | 65.35    |           | 1403.5205          |
| 3 | -35.84 | + 3.16 | 29.59 | 56.02    |           | 2007.7568          |
| 4 | +00.02 | -29.59 | 0. 0  | 29.59    | 0.5918    |                    |
|   |        |        |       |          | 1711.1468 | 3410.2773          |
|   |        |        |       |          |           | 1711.1468          |
|   |        |        |       |          |           | 2)1699.1305        |
|   |        |        |       |          |           | 10)849.5652        |
|   |        |        |       |          |           | Ans, areas, 84.956 |

9. What is the area of a survey of which the following are the field notes.

From the place of beginning, *N.*  $31^{\circ} 30'$  *W.*, distance 10 chains :  
thence *N.*  $62^{\circ} 45'$  *E.*, 9.25 chains : thence *S.*  $36^{\circ}$  *E.*, 7.60 chains :  
thence *S.*  $45^{\circ} 30'$  *W.*, 10.60 chains, to the place of beginning.

Ans.  $8\frac{54}{100}$  acres.

10. Do the following bearings and distances enclose a space ?  
If not, give an additional bearing and distance that will, then determine the area so enclosed.

| Stations. | Bearings.            | Distances. |
|-----------|----------------------|------------|
| 1         | <i>S. 40° 30' E.</i> | 31.80 ch.  |
| 2         | <i>N. 54° 00' E.</i> | 2.08       |
| 3         | <i>N. 29° 15' E.</i> | 2.21       |
| 4         | <i>N. 28° 45' E.</i> | 35.35      |
| 5         | <i>N. 57° 00' W.</i> | 21.10      |

Ans. These bearings and distances *do not* enclose a space. A line run from the further extremity of the 5th to the first station will bear south  $46^{\circ} 43' W.$ , distance 31.21 chains, and the area thus enclosed will contain 92.9 acres.

11. Do the following bearings and distances enclose a space? If not, determine the additional line that will, and the area of the space so enclosed.

| Stations. | Bearings.            | Distances. |
|-----------|----------------------|------------|
| 1         | <i>S. 85° 00' W.</i> | 46.4 rods. |
| 2         | <i>N. 53° 30' W.</i> | 46.4 "     |
| *3        | <i>N. 36° 30' E.</i> | 76.8 "     |
| 4         | <i>N. 22° 00' E.</i> | 56.0 "     |
| 5         | <i>S. 76° 30' E.</i> | 48.0 "     |

Ans. These bearings and distances do not enclose a space. A line run from the last station to the first would bear *S. 30° 25' W.*, distance 128.6 rods. Area 56.86 acres nearly.

The operation for the area is as follows:

We commence at station 3, for reasons that have been several times explained. Station 6 is the one we supplied.

| Sta. | Lat.    | Dep.   | M. D.  | D. M. D. | N. area.  | S. area.             |
|------|---------|--------|--------|----------|-----------|----------------------|
| 3    | + 61.73 | +45.65 | 45.65  | 45.65    | 2817.9745 |                      |
| 4    | + 51.92 | +20.98 | 66.63  | 112.28   | 5829.5776 |                      |
| 5    | - 11.21 | +46.67 | 113.30 | 179.93   |           | 2017.0153            |
| *6   | -125.18 | -29.85 | 83.45  | 196.75   |           | 24629.1650           |
| 1    | - 4.85  | -46.15 | 37.30  | 120.75   |           | 585.6375             |
| 2    | + 27.59 | -37.30 | 0. 0   | 37.30    | 1029.1070 |                      |
|      |         |        |        |          | 9676.6590 | 27231.8178           |
|      |         |        |        |          |           | 9676.6590            |
|      |         |        |        |          |           | 2)17555.1588         |
|      |         |        |        |          |           | 160)8777.5794(54.86. |

\* The stations marked with a \* are those supplied.

12. What is the area of a survey of which the following are the field notes.

| Stations. | Bearings.            | Distances. |
|-----------|----------------------|------------|
| 1         | <i>N. 75° 00' E.</i> | 54.8 rods. |
| 2         | <i>N. 20° 30' E.</i> | 41.2 "     |
| 3         | <i>E.</i>            | 64.8 "     |
| 4         | <i>S. 33° 30' W.</i> | 141.2 "    |
| 5         | <i>S. 76° 00' W.</i> | 64.0 "     |
| 6         | <i>N.</i>            | 36.0 "     |
| 7         | <i>S. 84° 00' W.</i> | 46.4 "     |
| 8         | <i>N. 53° 15' W.</i> | 46.4 "     |
| 9         | <i>N. 36° 45' E.</i> | 76.8 "     |
| 10        | <i>N. 22° 30' E.</i> | 56.0 "     |
| 11        | <i>S. 76° 45' E.</i> | 48.0 "     |
| 12        | <i>S. 15° 00' W.</i> | 43.4 "     |
| 13        | <i>S. 16° 45' W.</i> | 40.5 "     |

In this survey 4 is the most easterly and 8 the most westerly station. The area is equal to 110*A.* 2*R.* 23*P.* It may vary a little, on the account of the way in which the balancing is done.

### CHAPTER III.

#### ON THE MERIDIAN LINE AND THE VARIATION OF THE COMPASS.

THE meridian is an astronomical line, having no necessary connection with the magnetic needle. Meridians would be primary lines to which we would refer all surveys, if there were no such thing as magnetism, or a magnetic needle.

It is only a coincidence, that the magnetic needle settles near the meridian, so near, that for a long time it was considered the meridian itself, but accurate observations have shown that the needle does not point rigorously north and south, but has a *variation* which is not the same at all times in the same place, therefore a line run by the compass is still unknown in respect to the meridian,

unless we know the variation of the compass, that is, the declination of the needle.

In the year 1657 the needle at London pointed due north, since that time its variation has been west, previous to that time the variation was east.

In the Atlantic ocean, between Europe and the United States, the variation is from  $12^{\circ}$  to  $18^{\circ}$  west.

The needle seems to point to the region of greatest cold, which is in the northern part of America, and not the north pole, and if this be true, if the point of *minimum* heat changes its position, there will be a corresponding change in the direction of the magnetic needle.

The needle has a small annual and also a diurnal variation, corresponding to the temperature of the different seasons of the year, and of the different times of the day, but these variations are too small to trouble the common operations of surveying.

Some of the variations of the compass are regular, others irregular; some amount to many degrees and require a long period of time, others are small in amount and require but short intervals to pass through all their changes.

The daily variation consists of an oscillation eastward and westward of the mean position, and is different in different places. Generally the greatest oscillation eastward is between six and nine in the morning, and westward about one in the afternoon, gradually returning toward the east until eight P. M. At night it is stationary.

On the subject of magnetism we know nothing, beyond facts drawn from observation, but there is no doubt that the earth is a great magnet, made so by the action of the sun, and the poles of this great magnet are near the poles of the equator. Indeed, all observations made correspond to this hypothesis, for changes of the weather, clouds, and storms, all have an influence on the needle.

These facts are sufficient to convince any reader that to survey correctly we must know the

#### VARIATION OF THE COMPASS.

As the true meridian is an astronomical line, we must find it by astronomical observations, and then by comparing the meridian of

the compass with it, we shall have the variation of the compass.

When the sun is on the equator, it rises due east, and sets directly in the west. Should we then observe the direction of its center, just as it was rising or setting, at the time it had no declination, and trace that line a short distance on the ground, we should then have a due east and west line.

If from any point in that line we draw another line at right angles, we should then have the true meridian.

If we now put the compass on this meridian, and make the sight-vanes range with it, the needle will also range with it, if there is no variation, but if the north point of the needle is to the west of the sight-vane, the variation is westerly, if to the east, easterly, and the number of degrees and parts of a degree that the needle deviates from the direction of the sight-vanes shows the amount of the variation.

But it is not to be supposed that any particular observer can be at the points and places, where the sun is either rising or setting just at the time the sun is on the equator. We must have a broader basis, and in fact by means of the latitude of the observer and the declination of the sun, any observer has the means of knowing the precise direction in which the sun will rise or set, any day in any year.

Let us suppose that the sun on a certain day, observed from a certain place, must have arisen *S. 81° E.*, but by the compass it was observed to rise *S. 79° E.*, the variation of the compass was therefore 2° west.

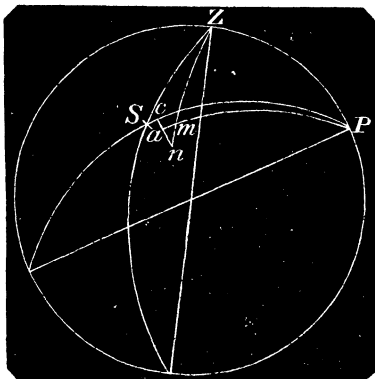
These observations are called taking an azimuth. Azimuths are often taken at sea to determine the variation of the compass.

On land, however, the horizon is rarely visible, and very few observations on sun rise or sun set can be made, besides there are other objections arising from atmospherical refraction; it is therefore best, most convenient, and more conducive to accuracy, to take the sun when up 10, 15 or 25° above the horizon, and observe its direction per compass, and compare the result to the computed bearing for the same moment, and if the two results agree the compass has no variation; if they disagree the amount of such disagreement is the amount of the variation of the compass.

By means of spherical trigonometry the true bearing of the sun can be determined at any time, on the supposition that the observer knows his latitude, the declination of the sun, and its altitude ; these three conditions furnish a triangle like  $PZS$ .

The altitude subtracted from  $90^\circ$ , gives  $ZS$ , the latitude from  $90^\circ$ , gives  $ZP$ , and  $PS$  is found by adding or subtracting the sun's declination to  $90^\circ$ , according as it is north or south.

$ZS$  is co-altitude,  $ZP$  is co-latitude, and  $PS$  is the sun's polar distance ; the angle  $PZS$  is required, and it can be found by the following rule.



1. Add the three sides of the triangle together and take the half sum. From the half sum, subtract the sun's polar distance, thus finding the remainder.

2. Add the sin-complement of the co-altitude, the sin-complement of the co-latitude, the sine of the half sum, and the sine of the remainder. The sum of these four logarithms divided by 2, will be the cosine of half the azimuth angle.

N. B. This rule is the application of equations on page 204 Robinson's Geometry. The sin-complement is the logarithmic sine of an arc, subtracted from 10.

#### EXAMPLES.

In latitude  $39^\circ 6' 20''$  north, when the sun's declination was  $12^\circ 3' 10''$  north, the true altitude of the sun's center was observed to be  $30^\circ 10' 40''$ , rising. What was the true bearing of the sun, or its azimuth ?

|         |          |         |            |      |           |
|---------|----------|---------|------------|------|-----------|
|         | 90°      |         | 90         |      | 90        |
| Lat.    | 39 6 20  | Alt.    | 30. 10. 40 | Dec. | 12° 3' 10 |
| co-Lat. | 50 53 40 | co-Alt. | 59. 49. 20 | PD   | 77. 56 50 |

|                 |                    |           |                    |
|-----------------|--------------------|-----------|--------------------|
| <i>P. D.</i>    | 77° 56. 50         |           |                    |
| co-Lat.         | 50. 53. 40         | sin. com. | 0.110146           |
| co-Alt.         | 59. 49. 20         | sin. com. | 0.063295           |
|                 | <u>2)188 39 50</u> |           |                    |
| $\frac{1}{2}S.$ | 94 19 55           | sin.      | 9.997758           |
|                 | <u>77 56 50</u>    |           |                    |
| Rem.            | 16 23 5            | sin.      | 9.450376           |
|                 |                    |           | <u>2)19.622575</u> |
|                 | 49° 38' 30''       | cosine    | 9.811287           |

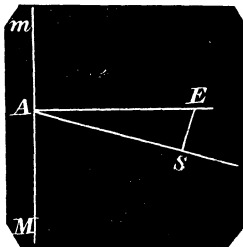
Bearing, 99° 17' 0 from the north, or 80° 43' from the south.

If at the time of taking the altitude of the sun, another observer had taken its bearing by the compass, and found it to be *S. 80° 43' E.*, then the compass would have no variation, and whatever it differed from that would be the amount of variation.

If a line were run along the ground, direct toward the center of the sun, at the time the altitude was taken, and sufficiently marked, that would be a standing line of known direction; and if from any point in that line, we could draw another line, making an angle with it of 99° 17' on the north, or 80° 43' on the south, such a line definitely marked, would be a permanent meridian line, for all time to come; on which we could at any time place a compass, and observe its variation.

Let *AS* be the line toward the sun, along the ground, *AE* a line due east, and *Mm* a true meridian line. The angle *SAE* must equal 9° 17'.

To make that angle, take *AS*, one chain or 100 links; from *S*, draw the line *SE* at right angles to *AS*, by means of a surveyor's cross.\*



From *S* take *SE*, of such a value as will make *SAE* 9° 17', which is determined by trigonometry; as follows,

\*Surveyor's cross is nothing more than a pair of sight vanes, set at right angles with each other, for the purpose of making right angles.



$$\begin{array}{rcl}
 \text{As } 100. : SE = \text{Rad.} : \tan. 9^\circ 17' & & \\
 \text{Whence } SE = \frac{100. (\tan. 9^\circ 17')}{R.} = & \frac{9.213405}{2} & \\
 SE = 16.347 \text{ links.} & \frac{1.213405}{2} &
 \end{array}$$

That is, from *S* measure off 16 and a little more than  $\frac{1}{3}$  of a link, and there is the point *E*. A line drawn from *A* to *E*, is a due east and west line.

If we put the surveyor's cross on this line, at any point as *A*, and range one branch of it along the line *AE*, the other branch will mark out the line *Mm*, a true meridian, if everything has been done to accuracy.

In the afternoon, or some other day, another meridian may in like manner be drawn near this one, and if they are both true meridians, they will be parallel. If not parallel, other observations should be made until some two or three are obtained, that are parallel or very nearly so; and the mean direction then, may be regarded as the true meridian.

A true meridian will always be a test line for a compass; and by placing any compass upon it, the declination\* of the needle can be determined.

Again. In the triangle *PZS*, if we compute the angle *ZPS* (as is done on page 211 Robinson's Geometry), we shall have the sun's distance from the meridian or the apparent time; then if we have a time piece that can be relied upon, for three or four hours, we can determine the time within a few seconds, when the sun will be on the meridian. A line at that time, run direct toward the center of the sun, will define the meridian.

The objections to these methods are,

1. The sun is a large body, and its center cannot be exactly defined.
2. The sun changes position so rapidly that, unless we are in an observatory, where every thing is prepared and in order, it is difficult to get observation upon it.

---

\*Declination of the needle, in common language, is called the variation of the compass; and, as a general thing, we adhere to common language.

3. The sun is so bright an object that it cannot be viewed without prepared glasses.

4. The majority of persons that have been, and probably will be practical surveyors, have not the instruments to take altitudes of the sun, and they are not and cannot be at home in astronomical observations and computations.

Some of these objections are deserving of little respect, and others can be partially removed.

For instance, if the sun is too large, and too brilliant to be accurately and deliberately observed, we can take the planet Venus, Jupiter, or Saturn, and, by proper observations, determine their directions, during the twilight of evening, when we can see the planet distinctly, and at the same time that other objects are sufficiently distinct to run lines.\*

But the method most known and most in favor among practical men, is that of taking the direction of the north star.

The north star is a star of the second magnitude (Polaris), whose right ascension, Jan. 1, 1851, was 1h 5m 18s ( at present increasing at the rate of 17s 71 per annum ), and declination was then  $88^{\circ} 30' 55''$ , with an annual increase of  $19'' 8$ , it is therefore, but  $1^{\circ} 29' 5''$  from the pole, and it is called the pole star or north star because it is so near the pole.

If the star were situated directly at the polar point, a line toward it would be the true meridian line, but being  $1^{\circ} 29' 5''$  distant, the star apparently makes a circle round the pole in a sidereal day, making two transits across the meridian, one above and the other below the pole, — a direction to it, at these times, would be a true meridian line.

To find these times, *subtract the right ascension of the sun from the right ascension of the star*; increasing the latter by 24h, to render the subtraction possible, when necessary.

---

\* For example, in the year 1853, from the 25th of July to the 5th of August, the planet Jupiter will pass the meridian in the evening twilight. On the first of August, Jupiter will pass the meridian of New York, at 8h 11m 53s, and it will pass the meridian of Cincinnati, at 8h 11 39s, mean local time; and, of course, whoever is able to designate that time within a few seconds, and is also prepared to mark the direction of the planet, will have a true meridian line.

The moon is not a good object for this purpose; it changes its place too rapidly.

The difference will be the time of the upper transit, and 11h and 59 minutes from that time will be the time of the lower transit. The right ascension of the sun is to be found in the Nautical Almanacs, for every day in the year; and it is nearly the same, for the same day, in every year.

For example. At what times will the north star make its transits over the meridian on the first day of July, 1853.

|   |       |   |     |   |   |   | H.    | M. | S. |
|---|-------|---|-----|---|---|---|-------|----|----|
| ✱ | R. A. | + | 24h | - | - | - | 25    | 6  | 0  |
| ☉ | R. A. | - | -   | - | - | - | 6     | 41 | 16 |
|   |       |   |     |   |   |   | <hr/> |    |    |
|   |       |   |     |   |   |   | 18    | 24 | 44 |

This result shows that the upper transit will occur about 6h 24m, in the morning of the 2d of July. I say *about*, because I took the sun's right ascension for the morning of July 1, and from that time to 6, next morning, is 18 hours: and during this time the right ascension of the sun will increase full 3 minutes,—therefore the upper transit will take place 6h 21m in the morning, and the previous lower transit 11h 59m previous, or at 6h 22m, evening.

But neither of these transits will be visible, as they both occur in broad day light, from any place where the north star is ever distinctly visible.

In summer, then, when most surveying is done, the meridian transits of the north star are not visible, nor is this important: for the transits are seldom used, by reason of two objections:

1. The star changes its direction most rapidly while passing the meridian.

2. Observers, generally, have not the means of knowing the time to sufficient accuracy.\*

To obviate these objections, observations may be taken on the star at its greatest elongations; for, about those points and for full

---

\* NOTE.—Very few persons consider that their clocks and watches, however good and valuable, do not give the exact time, but only approximations to the time.

For any astronomical purpose, like the one under investigation, the character of the time piece should be well tested—its rate of motion known—and its errors established by astronomical observations.

15 minutes before and after, the star does not scarcely change its direction ; hence the observer has a sufficient interval to be deliberate, and he can be sufficiently exact as to time without any extra trouble.

The following tables show the times of the greatest eastern and western elongations, which occur in the night season. These tables are not perpetual, but they will serve without correction for 20 years or more to come.

## EASTERN ELONGATIONS.

| Days. | April. | May.  | June. | July. | August. | Sept. |
|-------|--------|-------|-------|-------|---------|-------|
|       | H. M.  | H. M. | H. M. | H. M. | H. M.   | H. M. |
| 1     | 18 18  | 16 26 | 14 24 | 12 20 | 10 16   | 8 20  |
| 7     | 17 56  | 16 03 | 14 00 | 11 55 | 9 53    | 7 58  |
| 13    | 17 34  | 15 40 | 13 35 | 11 31 | 9 30    | 7 36  |
| 19    | 17 12  | 15 17 | 13 10 | 11 07 | 9 08    | 7 15  |
| 25    | 16 49  | 14 53 | 12 45 | 10 43 | 8 45    | 6 53  |

## WESTERN ELONGATIONS.

| Days. | Oct.  | Nov.  | Dec.  | Jan.  | Feb.  | March. |
|-------|-------|-------|-------|-------|-------|--------|
|       | H. M. | H. M. | H. M. | H. M. | H. M. | H. M.  |
| 1     | 18 18 | 16 22 | 14 19 | 12 02 | 9 50  | 8 01   |
| 7     | 17 56 | 15 59 | 13 53 | 11 36 | 9 26  | 7 38   |
| 13    | 17 34 | 15 35 | 13 27 | 11 10 | 9 02  | 7 16   |
| 19    | 17 12 | 15 10 | 13 00 | 10 44 | 8 39  | 6 54   |
| 25    | 16 49 | 14 45 | 12 34 | 10 18 | 8 16  | 6 33   |

It will be observed that these times are astronomical ; the day commencing at noon, and 12h 40 means 40m after midnight, etc.

Now, admitting that the direction of the star can be observed, the next step is to find how much that direction deviates from the meridian — and this is a problem in spherical trigonometry.

A great circle passing through the zenith of the observer to the star, when the star is at one of its greatest elongations will touch the apparent small circle made by the apparent revolution of the star about the pole, and will therefore, with the star's polar distance, form a right angle — and we shall have a right angled spherical triangle, of which the observer's co-latitude is the hypotenuse, the star's polar distance one side, and the angle opposite to this side is the angle required.

## EXAMPLE.

What will be the bearing of the north star observed from latitude  $42^{\circ} N$ . in the year 1860, when the star's polar distance will be  $1^{\circ} 26' 12''$  ?

Ans.  $1^{\circ} 56'$ .

|                                 |   |   |   |   |           |
|---------------------------------|---|---|---|---|-----------|
| As cos. Lat. $42^{\circ}$       | - | - | - | - | 9.871073  |
| is to radius -                  | - | - | - | - | 10.000000 |
| So is sin. $1^{\circ} 26' 12''$ | - | - | - | - | 8.399183  |
| To sin. $1^{\circ} 56'$         | - | - | - | - | 8.628110  |

In this manner the following table was computed. The mean angle only is put down, being computed for the first of July in each year.

AZIMUTH TABLE.

| Years. | Lat. $30^{\circ}$<br>Azimuth. | Lat. $35^{\circ}$<br>Azimuth. | Lat. $40^{\circ}$<br>Azimuth. | Lat. $45^{\circ}$<br>Azimuth. | Lat. $50^{\circ}$<br>Azimuth. |
|--------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 1852   | $1^{\circ} 42' 30''$          | $1^{\circ} 48' 21''$          | $1^{\circ} 55' 52''$          | $2^{\circ} 5' 32''$           | $2^{\circ} 18' 5''$           |
| 1854   | $1^{\circ} 41' 45''$          | $1^{\circ} 47' 39''$          | $1^{\circ} 55' 2''$           | $2^{\circ} 4' 30''$           | $2^{\circ} 17' 6''$           |
| 1856   | $1^{\circ} 41' 2''$           | $1^{\circ} 46' 49''$          | $1^{\circ} 54' 12''$          | $2^{\circ} 3' 44''$           | $2^{\circ} 16' 9''$           |
| 1858   | $1^{\circ} 40' 27''$          | $1^{\circ} 46' 11''$          | $1^{\circ} 53' 30''$          | $2^{\circ} 3' 2''$            | $2^{\circ} 15' 12''$          |
| 1860   | $1^{\circ} 39' 43''$          | $1^{\circ} 45' 24''$          | $1^{\circ} 52' 32''$          | $2^{\circ} 2' 4''$            | $2^{\circ} 14' 16''$          |
| 1862   | $1^{\circ} 38' 50''$          | $1^{\circ} 44' 29''$          | $1^{\circ} 51' 44''$          | $1^{\circ} 1' 2''$            | $2^{\circ} 13' 18''$          |

This table is given for those who may wish to use it, but we would recommend each observer to follow the example which precedes the table, and compute the azimuth corresponding to his latitude and time.

## THE PRACTICAL DIFFICULTY.

The north star is not brilliant, it cannot be seen until it is so dark that all minute terrestrial objects are totally invisible, it is therefore difficult to draw a line and accurately mark it. All these night operations are, at best, perplexing and inaccurate, yet, by the means of lights and artificers, lines can be drawn.

If the observer have a theodolite and an assistant, there will be no difficulty. Let them be at the place from which they wish to take the observation in time, adjust the instrument and direct the telescope to the north star. Now sufficient light must be reflected into the telescope to enable the observer to see the cross hairs, and this may be done by the assistant holding a light before a stiff sheet of white

paper, so as to throw the reflected light from the paper into the telescope, or this may be done by means of a stand to hold both the light and the paper.

When the vertical spider's line becomes visible, let the star be brought directly upon it, and if it is near the time of greatest elongation it will appear to remain so, for some time. But if the star has not reached its greatest elongation, it will move from the line more to the east, if the elongation is easterly, and more to the west, if westerly.

The telescope must be continually directed to the star, by means of the tangent screw of the horizontal plate, but for some time the spider line and star will coincide without moving the screw, and then the star will depart from the line in the contrary direction to its former motion, *but the telescope must no longer follow the star*, its position will now show the direction to the star, when the star had its greatest elongation, *and thus it should be left until morning*.

In the morning, carefully range and mark a line through the telescope.

If we now make an angle with this line equal to the azimuth, by means of the theodolite, or by means of measuring a triangle as explained in the former part of this chapter, and mark this new line either to the right or left, as the case may require, we shall then have a permanent meridian line for all future use.

By placing a compass on any well defined and true meridian we can determine its variation by simple observation.

If we have not a theodolite, we can obtain a tolerably accurate direction to the north star by means of illuminated plumb lines suspended in vessels of water, so placed as to range to it.

## CHAPTER IV.

### TO SURVEY WITHOUT A COMPASS.

THE inquiry is sometimes made, whether lands could be surveyed without a compass; we reply in the affirmative. The compass is only a convenience, and if it had never been discovered, it is probable

that surveys would have been more accurately made. Too much reliance has been placed on the accuracy of the compass, and in consequence little attention has been paid to defining any astronomical lines.

Were it not for the compass, it is probable, that every country-town, and even every large land holder, would have meridian lines well defined about his premises.

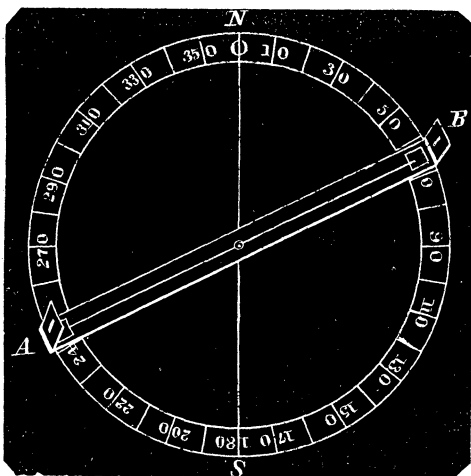
Having a meridian line to start upon, we can find angles and define the position of lines very accurately by means of a

#### CIRCUMFERENTOR.

The circumferentor consists of a horizontal circular plate divided into 360 degrees, over which an index bar, or another circular plate, is made to revolve. This index bar carries sight-vanes or a telescope. The index bar or the revolving circular plate also carries a vernier scale, which will enable the operator to make an angle to one minute of a degree. The whole instrument is placed on a tripod, and by the aid of spirit levels attached to the lower plate, the horizontal position is attained with a sufficient degree of accuracy.

The figure before us, represents the essential parts of a circumferentor. *NS* is considered as the primitive or meridian line, and *AB* is the index bar, which turns horizontally on the common center.

Vernier scales are fitted into the index bar, and revolve over the graduated arc. At *A* and *B* are openings, to receive cross hairs or a telescope.



When a vertical semicircle is made to revolve vertically through

the plane  $AB$ , and the diameter of that circle a telescope, then we have all the essentials of a *theodolite*.

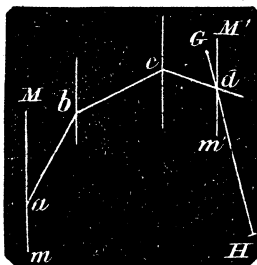
To most theodolites, a magnetic needle is attached, but the magnetic needle is, properly speaking, no part of the instrument.

To show the manner of finding the direction between two given points, by means of the circumferentor, we propose the following problem.

*Mr. T. H. Jones wishes me to run the east line of his lot, in the town of A, and give the true bearing, the corners being known. In the public square of the town, about one and a quarter miles distant, a meridian line has been established.*

Let  $Mm$  be the established meridian in the public square, and  $GH$  the direction of the line required.

Place the circumferentor on the meridian line  $Mm$ , so that  $NS$  of the instrument will coincide with it, the center of the instrument being at  $a$  in a road.



The general direction of the road is  $ab$ , and the index bar  $AB$  is made to revolve over the plate, which is firmly fixed, until the index bar or sight vanes point out the line  $ab$ . The line is run by means of ranging objects; such as flag staffs, if the line is long; or if short, by sending on a flag, and stationing it at  $b$ .

Now clasp the index bar on the plate, by the clamp screw under it (made for the purpose). Leave a flag at  $a$ ; take the instrument to  $b$ , and there place it, so that the sight vanes will range back to  $a$ ; then the position of  $NS$  on the instrument will show a meridian line through that point.

Here the road bends a little, unclasp the index (being careful that  $NS$  rigidly retains its position), and direct it to the general direction of the road  $bc$ . Mark the point  $c$ , by an object as before, and mark some other point, so as to secure the line (the other point may or may not be  $b$ ). Now clamp the index again, and remove the instrument to  $c$ . Place the instrument firmly as before, and make the index range along the line  $bc$ ; the line  $NS$  of the instrument,



will mark out a meridian line at the point *c*, and thus we can transfer the meridian line *Mm* to any other point whatever.

Thus we may go to any point *d*, in the given line ; no matter, theoretically speaking, how many angles we have made during the traverse. Placing the instrument at *d*, with its index to range along the last line, the line *NS* of the instrument gives the meridian *M'm'*.

Now unclamp the instrument, and direct its index along the required line *GH* ; the position of the index, on the graduated plate, will give the angle from the north, which, by means of the vernier, can be determined with great exactness.

In this manner we may go to any point, and place a meridian there, and then run any required line whatever ; therefore, we can survey any field, farm, or tract of land, without a compass, if we have a circumferentor, and a meridian line.

It would not be safe to transfer meridians, as we have just done, over any very great extent of country, for at every angle, small errors might be made, and the accumulation of many small errors may produce too great inaccuracies to be tolerated or overlooked. When using the magnetic needle, no errors accumulate, for every setting of the compass is primary, and independent of every other.

Therefore, in case no compasses were in existence, primary meridians, astronomically established, would be necessary in every town ; and it would be better to have several of them in the same town.

From the foregoing illustrations, we perceive that surveying can be done, and well done, without a compass, yet the compass is an inestimable blessing to mankind ; for it is the only index to direction over the wild waste of waters, when the heavens are obscured, and no mariner would dare brave the ocean without it.

## CHAPTER V.

### ORIGINAL AND SUBSEQUENT SURVEYS.— DIFFICULTIES AND DUTIES OF A SURVEYOR.

In this country, lands were ceded to States, or sold to companies in large tracts, without any definite surveys ; the boundaries

described, were mountains, rivers, or a certain number of miles along the shores of a lake, and then a certain number of miles back.

The land companies hired surveyors from time to time, to survey off their lands, into lots of 100, 200, and 500 acres; and wherever these surveyors left monuments for the corner of lots, established the corners for all time to come, whether *correctly placed or not*.

These surveys were very loose and inaccurate; it could not be otherwise, for a company of surveyors would frequently run 15 miles in a day; when to run a line accurately, and measure it, four miles is a good day's work.

But, notwithstanding inaccuracies, these surveys are legal and cannot be changed; "thou shalt not move thy neighbor's ancient land mark," and it is right it should be so; for any attempt at correction, would create more trouble, confusion, and injustice, than it could remedy.

Lots originally sold for 100 acres in the state of New York, generally contain from 101 to 106 acres, in consequence of the original surveyors having directions to *have their lots hold out*. Where the lots thus overrun in one portion of the tract, they fall short on another, for the surveyors were probably desirous to show to the company, that their grant actually contained as much land as was anticipated.

The author surveyed one of these lots, that originally sold for 100 acres, and found that it contained but a little over 76 acres.

Some of the companies had their grants laid off into townships 6 miles square, or 6 by 8 miles; then each township into four sections, each section divided off into lots, and the lots numbered, generally beginning at the south-west corner.

The description of the lots in the deeds given, were very loose and indefinite, stating the township, section, and number of the lot, containing 100 acres, "*be the same more or less*," and in some lots it was more, and in other lots it was less.

As we before remarked, any land mark to the corner of a lot laid down by these original surveyors, must remain; subsequent surveyors can straighten lines between point and point, and decide what the true courses are, and how many acres the lot contains.

When a surveyor is called to survey any farm or estate that has

been previously surveyed, he must find some corner as a place of commencing, and from thence run a *random* line, as near the true line as his judgment permits ; and if he strikes another corner he has run the true course, if not, he corrects his course, as taught in chapter II. Thus, he must go round the field from corner to corner. He has a right to *establish corners* only where no corners are to be found, and no evidence can be obtained as to the existence and locality of a former land mark.

It may be the case, that a surveyor is called to survey a lot where no corners are to be found. If a fence or line exists, which has been the undisputed boundary for a long time, that boundary cannot be changed, and the surveyor must establish a corner by ranging some other line to meet the first. Sometimes corners may be found to some neighboring lot, from which lines can be run, to establish a corner to the lot we wish to survey.

Lines of lots in the same town, are generally parallel, and a surveyor who offers his services to the public, must make himself acquainted with the general directions of the lines of lots, over that section of country where his services are required.

When a surveyor is called to divide a piece of land, he is then an original surveyor, and not liable to be embarrassed by old lines and old traditions, he has then only his mathematical problem before him.

Owing to the inaccuracies of original surveys, and the impossibility of leaving proper land marks, in consequence of the great haste in which lands were originally surveyed ; great confusion has followed, in some sections of our country, in respect to lines, and it has been no uncommon thing to have whole neighborhoods at variance, if not in law, in reference to the boundaries of their lands.

In cases of this kind, one, and then another of the disaffected, have successively employed surveyors, and surveyors thus employed, are apt to act the part of advocates, rather than arbitrators, and survey too much according to the direction of their employer ; but all such efforts to settle difficulties, but aggravate them more and more.

On the contrary, however, if the surveyor clearly understands his duties, and can rise above being a special advocate for any one of the parties concerned, he can do more than judges or juries to restore

harmony and peace. To illustrate these views, and possibly to give some valuable instruction to some readers, we give a history of a case of this kind, which occurred in the year 1837, in the county of Ontario, in the State of New York. A tract of land consisting of about 670 acres, of an irregular shape, was divided *on paper* into five equal parts, and sold to five different individuals.

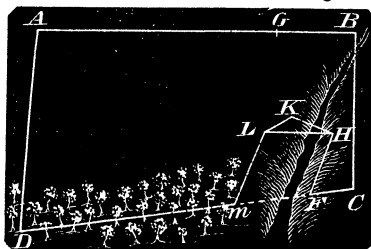
The whole 670 acres was bounded by four lines, no two of them were equal, and neither of the angles was a right angle. The largest boundary line could not be directly measured on account of an impassable ravine; and the banks of this ravine was so thickly set with hemlocks, that it was impossible even to sight across.

In consequence of the irregular shape of the whole, and the impossibility of directly measuring the principal boundary, they had never been able to agree on their division lines.

Each one imagined that his neighbor was inclined to crowd upon him, and although permanent fences were desirable, none could be made until lines were agreed upon. They had employed several surveyors, but they had not been able to agree on their divisions. In this state of things a surveyor was called upon to go and make a division of this land, but the difficulties of so doing were carefully concealed from him.

When he arrived on the ground, ready for operations, the whole neighborhood was present, and by unmistakable signs he soon learned that an unusual degree of interest was taken in the survey.

He also found that the chief difficulty arose from not being able to measure the line  $CD$ . All the corners,  $A$ ,  $B$ ,  $C$ , and  $D$ , were established. The surveyor commenced at  $C$  to run a *random* line as near  $CD$  as possible. After going a few chains, he came to the bank of the ravine at  $F$ , where it was impossible to pass or sight across. Driving a stake at  $F$ , he took a direction  $FH$  along the bank of the ravine, carefully noting the angle, and measuring the line to  $H$ , a point where objects were clearly to be seen on the other side of the ravine. The surveyor then sent a man over with a flag, stationing his staff, first at  $K$ ,



then at  $L$ , carefully noting the direction of each, and being careful to have the angle  $KHL$  greater than  $30^\circ$ . He then passed over and set the compass at  $K$ , took the direction of  $KL$  and measured it. Having now  $KL$  one side, and all the angles of the triangle  $HKL$ , he computed  $HL$ . He now set the compass at  $L$  and took a definite direction  $Lm$ ; this definite direction gave him the angle  $HLm$ , and he now had all the angles of the quadrilateral  $LmFH$ , and two of its sides. Whence he computed the exact distance to  $m$ , to strike the line  $CF$  produced.

He measured that distance and drove a stake at  $m$ , and computed  $mF$ . The company now ran through the random line, driving stakes at the end of every third chain, and the random line came out within a few feet of the established corner at  $D$ . The surveyor measured the perpendicular distance to  $D$ , and corrected the course by the rule in chapter II. He also computed how far each stake that had been placed on the random line must be moved to transfer it to the true line; this, the reader will perceive, was done by proportional triangles.

At  $D$  he set the compass, and carefully noted the course and distance to  $A$ . He then returned to  $C$ , *taking care not to pass along the line  $AB$* .

At  $C$  he set the compass, and carefully noted the course and distance to  $B$ . He now computed the course and distance from  $B$  to  $A$ . The line lay in the open fields, over tolerably smooth ground, and it could be directly and accurately measured.

The surveyor now called all the parties interested, including the sour and the belligerent, and told them that the distance from  $B$  to  $A$  was a certain number of chains and links, and that they would now measure it, and if they found it to correspond without any material error, they must then be convinced that he had obtained the true length of  $CD$ , and that he could then divide the land into five equal parts, as required.

To this test they all cheerfully assented; the line was measured and corresponded to the computation within three links; all parties were satisfied, and thus ended a neighborhood quarrel of six years' standing.

Previous surveyors commenced at the point  $D$  and run  $DA, AB$ ,

and  $BC$ , and then computed  $CD$ . This was more direct, simple, and proper, than the method just described, but it left no test behind it, and it is vain to expect that the mass of men will receive theoretical computation as actual measurement.

Here, and in most other cases that involve contention, the surveyor must not only convince himself that the survey is correctly made, but he must, if possible, show others that his conclusions are not only right, but cannot be wrong; hence judicious surveyors must often measure lines, where there is no *mathematical necessity* for so doing.

The next duty of this surveyor was to divide the land into five equal parts. Each one had previously purchased his part, and he knew its locality, but not his exact boundary line on the division.

As  $CD$  was not parallel to  $AB$ , and  $AD$  not exactly parallel to  $CB$ , to divide this *mathematically exact* was a problem of considerable difficulty, and this will be explained in the next chapter; but practically we need not apply all mathematical rigor, the surveyor can divide this more strictly conformable to justice without, than with the mathematical rigor.

The persons who had the two most eastern lots, had the worthless part of the land in the ravine, and of course if any one had an excess of *area* it should be these.

To find where or nearly where the divisions come, divide the line  $AB$  into five parts and suppose  $G$  one of those parts. Now  $BG$  is not quite long enough, because the field is a little narrower at this end than at the other; the surveyor took a distance  $BG$  a few links greater than one fifth of  $AB$ , and from that point run a line in a medium direction between  $BC$  and  $AD$ , and then computed its area, the result would show whether the area was too great or too small, and if it were within a very small fraction of the area required, the line is left as the true one, otherwise it is moved as the case requires. In the same manner the other division lines were run.

#### UNITED STATES' LANDS.

Soon after the organization of the present government, several of the States ceded to the United States large tracts of unoccupied land, and these, with other lands, since acquired by treaty and purchase, constitute what is called the public lands.

Previous to 1802, there was no general plan for surveying the public lands, or in fact, no surveys were made, and when grants were made the titles often conflicted with each other, and in some cases different grants covered the same premises.

In the year 1802, Colonel I. Mansfield, then Surveyor General of the north-western territory, adopted the following method :

Through the middle, or about the middle of the tract to be surveyed, a meridian is to be run, called the *principal meridian*. At right angles to this, and near the middle of it, an east and west line is to be run, and called the *principal parallel*.

Other meridians are to be run, six miles distant from the principal meridian, both east and west.

Also, parallels of latitude are to be run, six miles from the principal parallel, both north and south.

When this was done ( and it has been on all the public lands east of the Mississippi river ), the whole country is divided into squares, six miles on a side, called *townships*.

Each township contains 36 square miles. Each square mile is called a section, and it contains 640 acres. Sections are divided into half sections, quarter sections, and eighths. But these divisions are only made on paper.

When a person makes a purchase of a half or quarter section, it is supposed that he will find it himself, or employ a surveyor to mark it out.

Townships which lie along a meridian, are called a *range*, and numbered to distinguish them from each other.

Sections are regularly numbered in every township, and to designate any particular one, we say, section 13, in township number 4 north, in range 3 east.

This shows that the third range of townships east of the principal meridian, in township No. 4 north of the principal parallel, is the township, and the thirteenth section of this township is the one sought.

Not more than ten townships north or south of a principal parallel should be drawn, before a new principal parallel should be designated, and new measures made between meridians: *because* meridians tend toward the pole, and the north lines of townships

will be theoretically shorter than south lines, if the meridians are run by the compass.

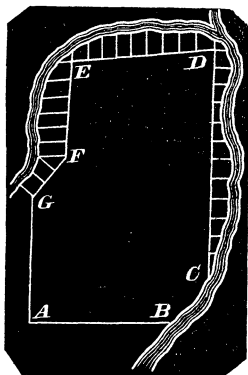
Where the public lands extend to rivers and lakes, there will be fractional townships along the shores.

Where the locality of a particular number is found to be occupied by a lake or pond, the sale is void.

## CHAPTER VI.

### METHODS OF SURVEYING IRREGULAR FIGURES AND OF DIVIDING LANDS.

FARMS and tracts of lands, wholly or partially bounded by water, as represented in the figure before us, are surveyed and there areas determined by drawing right lines within the tract as near the real boundaries as possible, and from these right lines, at *equal intervals*, measuring the *off-sets* to the real boundary. These off-sets form the parallel sides of trapezoids, and as they are all equally distant from each other, the computation of the areas they occupy will be very easy. A summary rule for finding the united area of all these trapezoids that are bounded by one line, is to be found in Prob. VIII, Mensuration. The area of the right lined figure *ABCDEFG*, is found as directed in Chapter IV, to which add the area of all the trapezoids, and we shall have the area of the whole.



We have now investigated every possible case of computing areas, and we are now prepared to divide them. Commencing with the most simple case of the most simple figure, the triangle, or rather the figure that has the least number of sides.

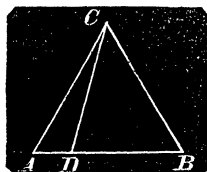


## PROBLEM I.

*To divide a triangle into two parts, having a given ratio of  $m$  to  $n$ .*

CASE 1. By a line drawn from one angle to its opposite side.

Let  $ABC$  represent the triangle; divide its base into two parts, corresponding to the given ratio, and let  $AD$  be one of the parts; then we shall have the following proportion.



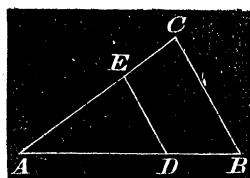
$$AD : AB :: m : m+n$$

$$\text{Whence, } AD = \frac{m}{m+n}(AB) \text{ and } BD = \frac{n}{m+n}(AB)$$

Now the two parts are numerically known, and are to each other as  $m$  to  $n$ . Triangles, having the the same altitudes, are to one another as their bases. Therefore,  $ADC : CDB :: m : n$  as required.

CASE 2. By a line parallel to one of its sides.

Let  $DE$  divide the triangle as required, and as similar triangles are to one another as the squares of their homologous sides, therefore :



$$(AB)^2 : (AD)^2 :: m+n : n$$

$$\text{Whence, } AD = AB \sqrt{\frac{n}{m+n}}$$

Which shows that if we have the numerical value of  $AB$ , and of  $n$  and  $m$ , we can find that of  $AD$ , and from  $D$  draw  $DE$  parallel to  $BC$ , and the triangle is divided as required.

CASE 3. By a line parallel to a given line, or by a line running in a given direction.

To make this case clear, we commence by giving a definite example :

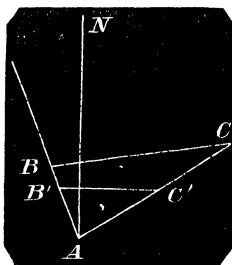
*There is a triangular piece of land, from one of the angular points,  $A$ , one line runs  $N. 25^\circ W.$ , distance 12 chains ; another from the same point runs  $N. 42^\circ E.$ , distance 15 chains. It is required to divide this*

triangle into two parts in the ratio 2 to 3, by a line running due east and west.

Let  $ABC$  be the given triangle, and  $B'C'$  the required division line. It is required to find the numerical value of  $AC'$  or  $AB'$ , to make the area  $AB'C'$   $\frac{2}{5}$  of the area  $ABC$ .

Let  $b$  represent the side of the triangle opposite  $B$ , and  $c$  the side opposite  $C$ .

Let  $AC' = x$ . As  $AC'$  and  $C'B'$  have definite directions, the angle  $AC'B'$  is given, also  $AB'C'$  is given.  $AC'B' = 48^\circ$ ,  $AB'C' = 65^\circ$ ,  $BAC = 67^\circ$ .



In the triangle  $AB'C'$  we have

$$\sin. 65^\circ : x :: \sin. 48^\circ : AB$$

$$\text{Whence } AB' = \frac{\sin. 48^\circ}{\sin. 65^\circ} x \quad (1)$$

Now, by Prob. III, Mens., area  $ABC = \frac{1}{2}bc \sin. A$ .

$$\text{Also, " " " area } AB'C' = \frac{1}{2} \left( \frac{\sin. 48^\circ}{\sin. 65^\circ} \right) x^2 \sin. A$$

By the conditions of our problem, we have the following proportion.

$$\frac{1}{2} bc \sin. A : \frac{1}{2} \left( \frac{\sin. 48^\circ}{\sin. 65^\circ} \right) x^2 \sin. A :: 5 : 2$$

$$\text{Or, } bc : \frac{\sin. 48^\circ}{\sin. 65^\circ} x^2 :: 5 : 2 \quad (2)$$

We may here stop, and make the problem general.

If  $B'C'$  is given in direction, the angles  $B'$  and  $C'$  will be given.

We now require the division of the triangle  $ABC$  into two parts, in the ratio of  $m$  to  $n$  by a line opposite to the angle  $A$ , running in a given direction.

Represent the sides of the given triangle adjacent the angle  $A$  by  $b$  and  $c$ ,  $b$  extending from  $A$  to  $B$ , and  $c$  extending from  $A$  to  $C$ .

Put  $x$  = the distance from  $A$  to the division line, on the side  $CA$ .

Then, by the preceding proportion we have,

$$bc : \frac{\sin. C'}{\sin. B} x^2 :: m+n : m$$

Whence, 
$$x = \left( \frac{m}{m+n} \right)^{\frac{1}{2}} \left( \frac{bc \sin. B'}{\sin. C'} \right)^{\frac{1}{2}}$$

Observe that  $x$  is opposite the angle  $B'$ , the sine of which stands in the numerator of the second fraction. Had  $x$  represented  $AB'$ ,  $\sin C'$  would have been the numerator.

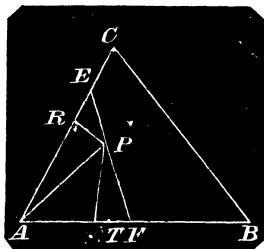
Drawing out the result for (2), we find that

$$\frac{5 \sin 48^\circ}{\sin 65^\circ} x = 2bc = 2.15.12$$

Or, 
$$x = \sqrt{\frac{72 \sin 65^\circ}{\sin 48^\circ}} = 9.446 \text{ chains.}$$

CASE 4. By a line that shall pass through a given point within the triangle.

A point in a triangle cannot be given, unless the perpendicular distances from that point to the sides are given, and if these perpendicular distances are given, then we can readily find the three distances from the angular points of the triangle, and the angles which these lines make with the sides of the triangle are known. For instance, if the point  $P$ , in the triangle  $ABC$ , is known,  $PR$  and  $PT$  are known, and all the angles of the quadrilateral  $ATPR$  are known. These are sufficient data to compute the line  $AP$ , and the angles  $RAP$  and  $TAP$ .



REMARK. — We may now require a triangle to be cut off, by a line running through  $P$ , which shall contain any definite portion of the triangle  $ABC$ , *not involving an impossibility*.

For instance, if the point  $P$  is near the center of the triangle, it would not do to require us to cut off a tenth part of the triangle, or any smaller portion, for it would be impossible to do so. When it is required to cut off a very small portion of the whole triangle, the point  $P$  must be near one of the sides, or near one of the angular points. Sometimes the required quantity can be cut off from one angular point, sometimes from another, and sometimes from all three.

Let us now require one-third of the triangle cut off, by a line passing through

$P$ , taking the angular point  $A$ , and let  $EF$  be that line. We are to determine the value of  $AF$ .

In the triangle  $ABC$ , the angles  $A$ ,  $B$ , and  $C$ , are known, and the sides opposite to them,  $a$ ,  $b$  and  $c$ , are also known.  $AP$  is known, and call it  $h$ . Put the angle  $EAP=p$ ,  $PAF=q$ . Then,  $A=p+q$ . Put  $AF=x$ ,  $AE=y$ .

By Prob. III. Mens. area  $ABC = \frac{1}{2}bc \sin. A$

Also " " area  $AEF = \frac{1}{2}xy \sin. A$

By the conditions of the problem,

$$\frac{3}{2}xy \sin. A = \frac{1}{2}bc \sin. A$$

$$\text{Whence,} \quad 3xy = bc \quad (1)$$

The triangle  $AFE$  consists of two parts,  $AFP$ ,  $APE$ ; therefore,  
 $\frac{1}{2}hx \sin. q + \frac{1}{2}hy \sin. p = \frac{1}{2}xy \sin. A$

$$\text{Or} \quad x \sin. q + y \sin. p = \frac{xy \sin. A}{h} \quad (2)$$

If we had required the  $n$ th part of the triangle  $ABC$ , in place of the 3rd part, equation (1) would have been  $nxy = bc$ .

Making this supposition, to make the problem more general, we have  $xy = \frac{bc}{n}$  and  $y = \frac{bc}{nx}$ . By the aid of these last two equations,

(2) becomes

$$x \sin. q + \frac{bc \sin. p}{nx} = \frac{bc \sin. A}{np}$$

$$\text{Or,} \quad x^2 - \frac{bc \sin. A}{nh \sin. q} x = -\frac{bc \sin. p}{n \sin. q} \quad (3)$$

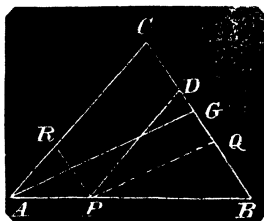
$$\text{Whence,} \quad x = \frac{bc \sin. A}{2nh \sin. q} \pm \left( \frac{b^2 c^2 \sin.^2 A}{4 n^2 h^2 \sin.^2 q} - \frac{bc \sin. p}{n \sin. q} \right)^{\frac{1}{2}}$$

In case  $n$  is large, that is the part to be cut off small, the value of  $x$  may be imaginary, corresponding to the preceding remark.

CASE 5. When the given point is on one side of the triangle.

The two parts must be equal, or one of them will be less than half of the whole.

We always compute the less part. Let  $P$  be the point in the line  $AB$ ,  $PQ$  and  $PR$  perpendiculars to the other sides, are known;  $BC$  and  $AG$  are both known. Now, through the given point  $P$ , it is



required to draw  $PD$ , so that the triangle  $BPD$ , shall be the  $n$ th part of  $ABC$ . That is

$$PQ. BD = \frac{BC. AG}{n}$$

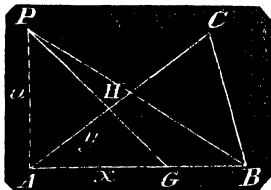
Whence, 
$$BD = \frac{BC. AG}{n PQ}$$

CASE. 6. When the given point is without the triangle.

Let  $ABC$  be the given triangle, and  $P$  any given point without it. It is required to run a line from  $P$ , to cut off a given portion of the triangle  $ABC$ , or (which is the same thing) to divide the triangle into two parts having the ratio of  $m$  to  $n$ . Let  $PG$  be the line required.

As  $P$  is a given point,  $AP$  is a line given in distance and position; therefore, the angle  $PAH$  is known.

*Solution.*—Put the angle  $PAH=u$ ,  $CAB=v$ ; then  $PAG=(u+v)$ . Also put  $AG=x$ ,  $AH=y$ ,  $AP=a$ , and the area of the triangle  $AHG=mc$ ,  $mc$  being a known quantity.



Now, (by Prob. III. Mens.)  $\frac{1}{2}xy \sin. v = mc$  (1)

Also “ “  $\frac{1}{2}ax \sin. (u+v) = \text{area } APG$

And “ “  $\frac{1}{2}ay \sin. u = \text{area } PAH$

Therefore,  $\frac{1}{2}ax \sin. (u+v) - \frac{1}{2}ay \sin. u = mc$  (2)

Or,  $x \sin. (u+v) - y \sin. u = \frac{2mc}{a}$  (3)

From (1), we find  $y = \frac{2mc}{x \sin. v}$  which value substituted in (3) gives

$$x \sin. (u+v) - \frac{2mc \sin. u}{x \sin. v} = \frac{2mc}{a}$$

Whence,  $x^2 \sin. (u+v) - \frac{2mc}{a} x = \frac{2mc \sin. u}{\sin. v}$

Or,  $x^2 - \frac{2mc}{a \sin. (u+v)} x = \frac{2mc \sin. u}{\sin. v \sin. (u+v)}$

Therefore,  $x = \frac{mc}{a \sin. (u+v)} \pm \sqrt{\frac{m^2 c^2}{a^2 \sin.^2 (u+v)} + \frac{2mc \sin. u}{\sin. v \sin. (u+v)}}$

## EXAMPLES.

1. In the triangle  $ABC$ , the side  $AB=23.645$  chains,  $AC=17.51$  chains, and  $BC=12.575$  chains.

The given point  $P$  from the angle  $A$ , is distant 10 chains, at an angle of  $40^\circ$  from the line  $AC$ .

It is required to draw a line from this given point  $P$ , through the triangle, so as to divide it into two equal parts.

Whereabouts on  $AB$  will  $PG$  intersect?

The angle  $BAC=31^\circ 17' 19''=v$ .  $PAH=40^\circ=u$ . Therefore  $PAG=71^\circ 17' 10''=(u+v)$ .

The area of the triangle  $ABC$  is 107.52 square chains. The part to be cut off by the triangle  $AHG$  is therefore  $=53.76=m$ .

We must use the natural sines, or the logarithmic sines if we omit them in the index.

|                                          |            |                                                  |
|------------------------------------------|------------|--------------------------------------------------|
| $mc$                                     | log. - - - | 1.730464                                         |
| $a \sin.(u+v)$                           | log. - - - | 0.976406                                         |
| $\frac{mc}{a \sin.(u+v)}$                | $=5.676$   | log. 0.754053                                    |
|                                          |            | <hr style="width: 100px; margin-left: auto;"/> 2 |
| $\frac{m^2 c^2}{a^2 \sin.^2(u+v)}$       | $=32.22$   | log. 1.508116                                    |
| $2mc$                                    | log. - - - | 2.031494                                         |
| $\sin. u$                                | log. - - - | <u>—1.808067</u>                                 |
| $2mc \sin. u$                            | - - -      | 1.839561                                         |
| $\sin. v \sin.(u+v)$                     | - - -      | <u>—1.691866</u>                                 |
| $\frac{2mc \sin. u}{\sin. v \sin.(u+v)}$ | $=702.56$  | log. 2.147695                                    |

The part of the formula under the radical is therefore  $(32.22 + 702.56)$  or 734.78

Whence  $x=5.676 \pm \sqrt{734.78}=32.7829$  or  $-21.43$ .

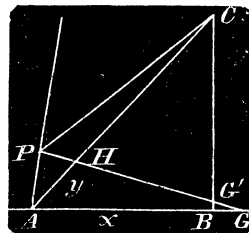
REMARK. — In geometry plus and minus generally indicate opposite directions, here the sign  $\pm$  means, sum and difference of the two numbers, the sum is 32.7829, the difference is 21.43; in fact there can be no such thing as minus a line

The value of  $AG$  is 21.43; the other value is not admissible.

2. We have a right angled triangle whose base is 48.87 chains, and perpendicular 54.46 chains. From a given point without it, we are required to run the center of a straight road, to leave one third of the triangle on one side and two thirds on the other.

From the acute angle at the base, the distance to the given point is 20 chains, and the line to it makes an angle with the hypotenuse of  $30^\circ$ .

REMARK.—If  $P$  is a given point, its distance and direction from one of the angular points must be given; and if the distance and direction from one of the angular points is given, the distances and directions from all of them are virtually given; thus, if we have  $AP$ ,  $AC$ , and the angle  $PAC$ , we have  $CP$ , and the angle  $ACP$ , and we may theorize on the triangles  $PCH$ ,  $CHG'$  as well as on  $APH$  and  $AHG$ .



The area of the triangle  $ABC=1304.94$  square chains.

One third of this is  $mc=434.97$ .

$BAC=v=49^\circ 12' 20''$ .  $PAC=u=30^\circ$ .  $(u+v)=79^\circ 12' 20''$ .  
 $AP=a=20$ .  $AH=y$ .  $AG=x$ .

|                                           |      |   |          |      |           |
|-------------------------------------------|------|---|----------|------|-----------|
| $mc$                                      | log. | - | -        | -    | 2.638459  |
| $a=20$                                    | log. | - | 1.301030 |      | 1.293268  |
| $\sin. (u+v)$                             | log. | - | 1.992238 |      |           |
| $\frac{mc}{a \sin. (u+v)}$                |      |   |          |      |           |
|                                           |      |   |          | log. | 1.345191  |
|                                           |      |   |          |      | 2         |
| $\frac{m^2 c^2}{a^2 \sin.^2 (u+v)}$       |      |   |          |      |           |
|                                           |      |   |          | log. | 2.690382  |
| $2mc$                                     | log. | - | -        | -    | 2.939489  |
| $\sin. u$                                 | -    | - | -        | -    | -1.698970 |
| $2mc \sin. u$                             | -    | - | -        | -    | 2.638459  |
| $\sin. v \sin. (u+v)$                     | -    | - | -        | -    | -1.871339 |
| $\frac{2mc \sin. u}{\sin. v \sin. (u+v)}$ |      |   |          |      |           |
|                                           |      |   |          | log. | 2.767120  |

Whence,  $x=22.142 \pm \sqrt{1075.32}=54.932$  or  $-10.648$ .

Here  $AG=54.932$ .  $AB=47.87$ . Hence  $BG=7.062$ . Having  $AP$ ,  $AG$ , and the angle  $PAG$ , we can compute the angle  $APG$ .

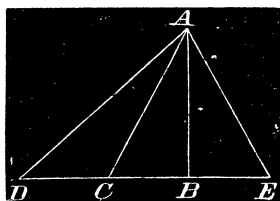
## PROBLEM II.

To divide a triangle into THREE PARTS having the ratio of the three numbers  $m, n, p$ .

CASE 1. By lines drawn from one angle of the triangle to the opposite side.

Let  $ADE$  be the triangle and  $A$  the angle from which the lines are to be drawn.

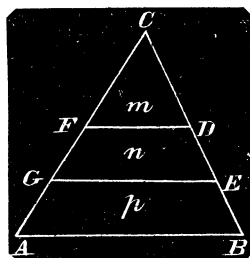
Divide  $DE$  the opposite side into parts in the ratio of  $m, n$ , and  $p$ , and from the points of division  $C$  and  $B$ , draw  $AC, AB$ , and the triangle is divided as required.



*Demonstration.* — The areas of triangles are their bases multiplied into their altitudes, but here all the triangles have the *same altitude*; therefore multiplying the bases into that altitude gives the same proportional product, and the areas of the triangles are as  $m, n, p$ .

CASE 2. By lines parallel to one of the sides.

Let  $ABC$  be the triangle. Divide its numerical area into three parts in the ratio of  $m, n, p$ . Conceive the problem solved and  $DF, EG$ , the division lines parallel to  $AB$ . We are to determine the numerical values of  $CD$ , and  $CE$ .  $CB$  is known, put it equal to  $a$ . Put  $CD=x$ .  $CE=y$ .



Now as similar triangles are to one another as the squares of their homologous sides, therefore

$$x^2 : a^2 :: m : m+n+p.$$

$$\text{Whence, } x = a \sqrt{\frac{m}{m+n+p}}$$

In the same manner,

$$y = a \sqrt{\frac{m+n}{m+n+p}}$$

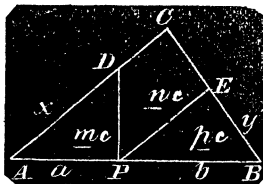
In this manner we might divide the triangle into any proposed number of parts, having given ratios.

CASE 3. By lines drawn from a given point on one of the sides of the triangle.



Let  $ABC$  be the given triangle, and  $P$  the given point on the side  $AB$ .

It is required to draw lines from  $P$ , as  $PD$  and  $PE$ , dividing the triangle into three parts  $mc$ ,  $nc$ ,  $pc$ , that is, assume the given numerical area to be  $(mc+nc+pc)$ , then the required parts will be  $mc$ ,  $nc$ , and  $pc$ .\* Put  $AD=x$ , then (by Prob. III, Mens.)



$$\frac{1}{2} ax \sin. A = mc \text{ or } x = \frac{2mc}{a \sin. A}$$

By comparison,

$$y = \frac{2pc}{b \sin. B}$$

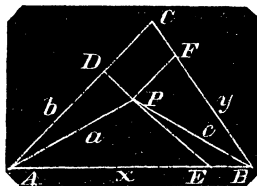
When  $mc$  and  $pc$  are cut off,  $nc$  is left. Having  $a$  and  $x$ , the angle  $ADP$  is easily determined.

In a similar manner we can divide the triangle into any proposed number of parts, by lines drawn from the given point  $P$ .

CASE 4. By lines drawn from a given point within the triangle.

Let  $ABC$  be the given triangle, and  $P$  the given point within it.

A variety of lines may be drawn from  $P$ , to divide the triangle into the parts required. Conceive  $PD$ ,  $PF$ , and  $PE$  to make the requisite division.



As  $P$  is a given point,  $AP$ ,  $PB$ , and  $PC$  are known lines, and the angles  $DAP$ ,  $PAE$  are known angles.

Take  $AD=b$ , any convenient assumed value. Take  $AE=x$ . Put  $AP=a$ . Then,

$$\frac{1}{2} ab \sin DAP = \text{area } \triangle DAP$$

$$\text{Also, } \frac{1}{2} ax \sin PAE = \text{area } \triangle PAE$$

\* Suppose we had a triangular piece of ground containing 320 square rods, and we wished to divide it into three parts in the ratio of 2, 3, and 5, what is the area of each of the parts?

We decide it thus :  $m=2$ .  $n=3$ .  $p=5$ .  $c$  is at present unknown, but,  
 $mc+nc+pc=320$

That is,  $10c=320$  or  $c=32$ .

Whence,  $mc=64$ .  $nc=96$ .  $pc=160$ .

The quantity  $c$  becomes known on dividing the area, and the parts separately  $mc$ ,  $nc$ , and  $pc$ , are *always known*.

Conceive the triangle  $ABC$ , divided into three parts, in the ratio of  $m, n, p$ ; and conceive  $ADPE$  to be one of these parts represented by  $mc$ . Then

$$ab \sin. DAP + ax \sin. PAE = 2mc$$

$$\text{Whence, } x = \frac{2mc - ab \sin. DAP}{a \sin. PAE}$$

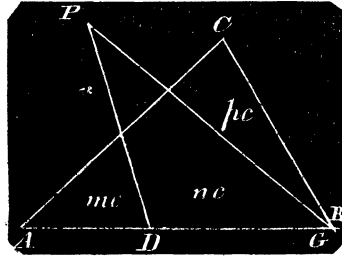
Had we taken  $b$  greater than we did,  $x$  would have been less, and a variety of lines could be drawn as well as  $PD$  and  $PE$ , and the same area cut off.

Having  $x$ , we have  $EB$  as a known quantity, and by the two triangles,  $PEB$  and  $BPF$ , we determine  $y$  in precisely the manner as we found  $x$ : thus, we have two parts of the triangle  $mc$  and  $pc$ , and, consequently, the remainder  $DPEC$  corresponds to  $nc$ .

CASE 5. By lines drawn from a given point without the triangle.

Let  $ABC$  be the triangle, and  $P$  the given point without it.

Divide the numerical area of the triangle into the three required parts,  $mc$ ,  $nc$ , and  $pc$ , as in former cases. Draw  $PD$ , cutting off the portion  $mc$ , as in Case 6 of the last problem; then cut off the two portions ( $mc + nc$ ) by the line  $AG$ : and the portion  $pc$  will be left. Or, we may cut off  $pc$ , and the portion  $nc$  will be left.



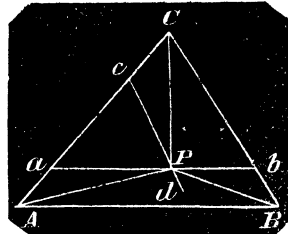
### PROBLEM III.

To divide a triangle into three parts, having the ratio of  $m, n$ , and  $p$ , by three lines drawn from the three angular points to some point within.

Divide any side, as  $AC$ , into three parts in the proportion of  $m, n$ , and  $p$ .

Let  $Aa$  represent the portion corresponding to  $m$ , and  $Cc$  the part corresponding to  $p$ .

Through  $a$ , draw  $ab$  parallel to  $AB$ ,



and through  $c$ , draw  $cd$  parallel to  $CB$ . Where these two lines intersect is  $P$ , and the triangle  $ABC$  is divided into three triangles,  $APB$ ,  $CPB$ , and  $APC$ .

*Demonstration.*—Any triangle having  $AB$  for its base, and its vertex in the line,  $ab$  will have the same ratio to the triangle  $ABC$ , as  $Aa$  has to  $AC$ , that is, as  $m$  to  $m+n+p$ .

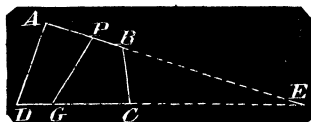
Also, the triangle  $CPB$  is to  $ABC$ , as  $Cc$  is to  $CA$ , that is, as  $p$  to  $m+n+p$ , for triangles on the same base are to one another as their altitudes. If these two triangles,  $APB$  and  $CPB$ , are in due proportion, the third one,  $APC$ , is in due proportion, of course.

#### PROBLEM IV.

*To divide a quadrilateral into two parts, having any given ratio  $m$  to  $n$ .*

**CASE 1.** By a line drawn from a given point in the perimeter.

Let  $ABCD$  be the given quadrilateral, and  $P$  the given point in the side  $AB$ .



It is required to determine the magnitude, and the direction of the line  $PG$ , which divides the figure into parts in the ratio of  $m$  to  $n$ . All the sides and angles of the quadrilateral are known, and its *area* is known.  $AB$  and  $DC$  are, or are not parallel; if they are parallel the figure is a *trapezoid*, then the method of finding  $G$ , in the opposite side, is easy and obvious. If  $AB$  and  $CD$  are not parallel, we can produce them and form a triangle in the one direction or the other; by this figure we form the triangle  $BCE$ , whose *area* we may represent by  $t$ .

As  $BC$ , and the angles as  $B$  and  $C$  are all known, the triangle  $EBC$  is determined in all respects.

As  $PB$  is known,  $PE$  is known, and designate  $EG$  by  $x$ . Let  $cm$  and  $cn$  designate the portions of the quadrilateral after it is divided, and let  $cm$  represent the part  $BPGC$ .

Put  $PE=a$ .

Now (by Problem III, Mensuration), we have

$$\frac{1}{2}ax \sin. E = t + cm.$$

Whence, 
$$x = \frac{2t + 2cn}{a \sin. E}$$

We now have the numerical value of  $x$ , from which we subtract  $EC$ , and we have  $CG$ , which being measured from  $C$  will give the point  $G$ , through which to draw the line from  $P$ , to divide the figure as required.

CASE 2. By a line making a given angle with one of the sides.

If the division line makes a given angle with one side, it must also make a known angle with the opposite side.

Taking the last figure, conceiving  $PG$  to take a given direction across  $AB$  and  $CD$ , so as to cut off the area  $mc$ .

As in the former case, let  $t$  represent the area of the triangle  $EBC$ , to this add  $mc$ , and we have the area of the triangle  $PGE$ . But in this case  $P$  is not a given point, and  $EP$  is not known.

Put  $EG = x$ . Let  $P$  represent the given angle at  $P$ , and  $G$  the given angle at  $G$ . Now, by trigonometry,

$$\sin. P : x :: \sin. G : EP$$

Or, 
$$EP = \frac{\sin. G}{\sin. P} x$$

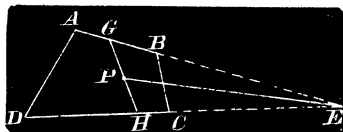
(Prob. III. Mens.) 
$$\frac{\sin. E \sin. G}{2 \sin. P} x^2 = t + mc$$

Whence, 
$$x = \sqrt{(2t + 2mc) \frac{\sin. P}{\sin. G \sin. E}}$$

From  $x$  we take  $EC$ , measure off the remainder along  $CD$ , to the point  $G$ , there making the given angle, and the figure will be divided as required.

CASE 3. By a line drawn through a given point within the quadrilateral.

Let  $ABCD$  be the quadrilateral as before, and  $P$  the given point within it; and as  $P$  is the given point,  $EP$  is a known line, and the angles  $PEH$ ,  $PEG$  are known.



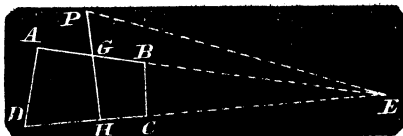
Let  $t$  equal the area of the triangle  $EBC$ , as before, and  $mc$  the area  $GHCB$ . Put  $EH = x$ , and  $EG = y$ .

Now we have a problem *precisely like* Case 4, Problem I, of this chapter; therefore, further explanations would be superfluous.

CASE 4. By a line drawn through a given point without the quadrilateral.

Let  $ABCD$  be the quadrilateral as before, and  $P$  the given point *without* it.

By producing the two sides  $AB$   $CD$ , we form the triangle  $ADE$ . Let the area of the triangle  $BCE$  be represented by  $t$ , and the part  $GHCB$  by  $mc$ , then from a given point  $P$ , without a triangle, we are required to draw a line  $PH$ , to divide the triangle into two given parts, and this is Case 6, of Problem I of this chapter, which has been fully investigated.



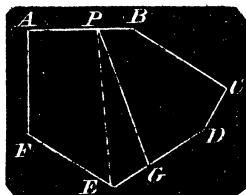
REMARK. — By extending the principles of these several cases we may divide a quadrilateral into three or more parts.

#### PROBLEM V.

To divide any polygon (regular or irregular) into two parts having a given ratio,  $m$  to  $n$ , by a line drawn through a given point.

CASE 1. When the given point is on one of the sides of the polygon.

Let  $ABCDEF$  be the polygon, and  $(mc + nc)$  express its numerical area. Let  $P$  be the given point on the side  $AB$ . Let the surveyor run a *random* line as near to the line required as his judgment permits, and generally it will be best to run a line from  $P$  to one of the opposite angular points.



In this figure, let  $PE$  represent such a random line, and let the surveyor compute the area of the figure  $PAFE$ , thus cut off, which area will be *equal to*, or *greater*, or *less* than one of the required parts. We will suppose it *less*; then subtract it from the required portion  $mc$ , and let the triangle  $PEG$  represent that *known difference*, which we shall designate by  $t$ .

$PE$  is known, the angle  $PEG$  is known; and put  $EG = x$

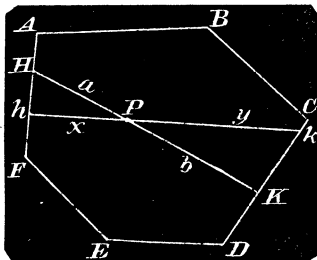
Then,  $\frac{1}{2} PE x \sin. PEG = t$

Whence, 
$$x = \frac{2t}{PE \sin. PEG}$$

This determines the point  $G$ , and  $PG$  divides the polygon as required.

CASE 2. When the given point is within the polygon.

Let  $ABCDEF$  be the polygon as before, and  $(mc+nc)$  express its numerical area; also, let  $P$  be the given point within. Through  $P$  let the surveyor run the *random* line,  $HPK$ , measuring from  $H$  to  $P$ , and from  $P$  to  $K$ , let him also observe the angles that this line makes with the sides of the polygon  $AF$  and  $CD$ , and compute the area  $HABCK$ , and note the *difference* between it and  $mc$ , the required portion of the polygon; call this difference  $d$ , a known quantity.



Let  $hPk$  represent the true line through  $P$ , which divides the polygon as required; but this line diminishes the area  $HABCK$ , by the triangle  $PKk$ , and increases it by the triangle  $PHh$ .

*Therefore the difference of these triangles must equal  $d$ .*

The sine of the angle  $AHP$ , has the same numerical value as the sine of  $PHh$ , and the sine of the angle  $PKD$  has the same numerical value as the sine of the angle  $PKk$ .

Put the angle  $AHP=u$ , and the angle  $PKD=v$ ; let the acute verticle angles at  $P$  be designated by the letter  $P$ .

Let  $HP=a$ ,  $PK=b$ ,  $hP=x$ ,  $Pk=y$

In the triangle  $PhH$ , we have

$$\sin. u : x :: \sin. (u-P) : a$$

$$\text{Whence,} \quad x = \frac{a \sin. u}{\sin. (u-P)} \quad (1)$$

$$\text{Similarly,} \quad y = \frac{b \sin. v}{\sin. (v-P)} \quad (2)$$

The area of the triangle  $PhH = \frac{1}{2}ax \sin. P$

Also,  $PkK = \frac{1}{2}by \sin. P$

$$\text{Whence,} \quad b \sin. Py - a \sin. Px = 2d \quad (3)$$

By substituting the values of  $x$  and  $y$ , taken from (1) and (2), we have,

$$b^2 \frac{\sin. P \sin. v}{\sin. (v-P)} - a^2 \frac{\sin. P \sin. u}{\sin. (u-P)} = 2d \quad (4)$$

Equation (4) contains only one unknown quantity  $P$ , the value of  $P$ , or the angle  $HPH$  can therefore be deduced.

$$b^2 \left( \frac{\sin. P \sin. v}{\sin. v \cos. P - \cos. v \sin. P} \right) - a^2 \left( \frac{\sin. P \sin. u}{\sin. u \cos. P - \cos. u \sin. P} \right) = 2d.$$

Dividing the numerator and denominator of the first fraction by  $(\sin. P \sin. v)$ , and of the second fraction by  $(\sin. P \sin. u)$ , recollecting that *cosine* divided by *sine* gives *cotangent*. Thus we shall obtain

$$b^2 \left( \frac{1}{\cot. P - \cot. v} \right) - a^2 \left( \frac{1}{\cot. P - \cot. u} \right) = 2d \quad (5)$$

This last equation shows the surveyor that if he can make it convenient to run his *random* line from  $P$ , perpendicular to one of the sides, his equation will be less complex. For instance, if  $AHP = 90^\circ$ , its cotangent will be 0, and  $\cot. u$  would then  $= 0$ , and equation (5) would become.

$$\frac{b^2}{\cot. P - \cot. v} - \frac{a^2}{\cot. P} = 2d \quad (6)$$

For the sake of convenience put  $\cot. P = z$ , and  $\cot. v = c$ .

Then

$$\frac{b^2}{z-c} - \frac{a^2}{z} = 2d$$

Or,

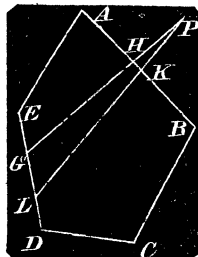
$$z^2 + \left( \frac{a^2 - b^2}{2d} - c \right) z = \frac{a^2 c}{2d}$$

The numerical value of  $z$  will be the numerical value of  $\cot. P$ ; its logarithm taken, and 10 added to the index will be logarithmic  $\cot.$  in our table. The same remarks will apply to  $\cot. v$  or  $c$ .

CASE 3. When the given point is without the polygon.

Let  $ABCDE$  be the polygon, and  $P$  the given point without it.

From the last case we learn that the surveyor had better run his *random* line perpendicular to one of the sides, therefore let  $PHG$  be the *random* line, perpendicular to  $AB$ .



As before, compute the area,  $AEGH$ , subtract it from  $mc$ , the difference is the difference between the triangles  $PGL$  and  $PHK$ . Draw  $PKL$  the line that divides the polygon as required.

Put  $PH=a$ ,  $PG=b$ ,  $PK=x$ ,  $PL=y$ , angle  $H=90^\circ$ , angle  $PGE=u$ , and the angle at  $P$ , designated by  $P$ .

$$\begin{aligned}\text{The triangle } PHK &= \frac{1}{2}ax \sin. P \\ \text{" } PGL &= \frac{1}{2}by \sin. P\end{aligned}$$

$$\text{Whence, } b \sin. Py - a \sin. Px = 2d. \quad (1)$$

Here (2) represents a similiar quantity as in the last case.

In the triangle  $PHK$ , we have

$$1 : x :: \cos. P a, \text{ or } x = \frac{a}{\cos. P} \quad (2)$$

In  $PGL$ ,  $\sin. u : y :: \sin. (u-P) : b$ .

$$y = \frac{b \sin. u}{\sin. (u-P)} \quad (3)$$

When the values of  $x$  and  $y$  are substituted in (1) we have

$$b^2 \frac{\sin. P \sin. u}{\sin. (u-P)} - a^2 \frac{\sin. P}{\cos. P} = 2d \quad (4)$$

$$\text{Or, } b^2 \left( \frac{\sin. P \sin. u}{\sin. u \cos. P - \cos. u \sin. P} \right) - a^2 \frac{\sin. P}{\cos. P} = 2d \quad (5)$$

$$\text{Or, } \frac{b^2}{\cot. P - \cot. u} - \frac{a^2}{\cot. P} = 2d \quad (6)$$

This equation is exactly similar to equation (6) of the last case, and it is reduced in the same manner.

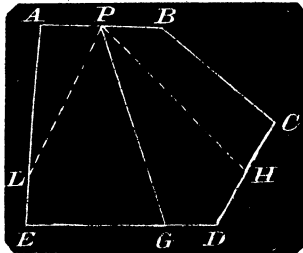
### PROBLEM VI.

*To divide a polygon into three or more parts, having a given ratio,  $m, n, p, q$ , by lines passing through a given point.*

This problem admits of three cases.

CASE 1. When the given point is on one side of the polygon.

Divide the numerical area of the whole into parts,  $mc, nc, pc, qc$ , corresponding to the given ratio. Unite these into two parts ( $mc+nc$ ) and ( $pc+qc$ ).





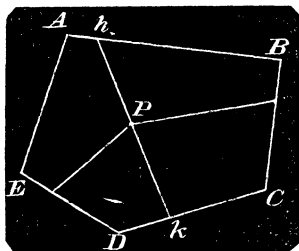
From the given point  $P$ , draw  $PG$ , by Case 1, Problem V, so as to divide the polygon into the two parts  $(mc+nc)$  and  $(pc+qc)$ .

We have now to divide the polygon  $PAEG$  into two parts,  $mc$ ,  $nc$ , by the line  $PL$ , and the polygon  $PBCDG$  into two parts,  $pc$  and  $qc$ , by the line  $PH$ .

CASE 2. When the given point is within the polygon.

Let  $ABCDE$  be the given polygon and  $P$  the given point within.

Draw  $hk$  through  $P$ , by Case 2, Problem V, so that the area  $AhkDE$  shall equal  $mc$ , and the area  $hkCB$  shall equal  $(nc+pc)$ , when the whole is required to be divided into three parts in the ratio of  $m, n, p$ .



When the whole is to be divided into four parts, in the ratio of  $m, n, p, q$ , then draw  $hk$ , so that one portion shall be  $(mc+nc)$  and the other  $(pc+qc)$ .

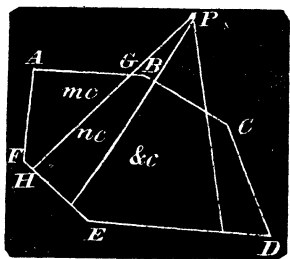
Then we have  $P$  as a given point, in one side of the polygon,  $AhkDE$ , to divide it into two parts, in the ratio  $m$  to  $n$ , and  $P$  a given point on one side of the polygon,  $hkCB$ , to divide it into two parts, in the ratio of  $p$  to  $q$ , and this is done by Case 1, Problem V.

CASE 3. When the given point is without the polygon.

Let  $ABCDEF$  be the given polygon, and  $P$  the given point without it.

Divide the numeral area into the required proportional parts,  $mc$ ,  $nc$ ,  $pc$ , &c., as many as required.

From the point  $P$  draw the line  $PH$ , as directed in Case 3, Problem V, dividing the polygon into two parts,  $mc$  and  $(nc+pc+\&c.)$ .



Then divide the polygon,  $GHEDCB$ , into two parts, one of which is  $nc$ , and the other  $(pc+qc, \&c.)$ , and thus we can proceed and cut off one portion after another, as many as may be required.

The application of the foregoing principles will meet any case that

can occur in the division of lands ; and we now close this subject with the following practical

## EXAMPLES.

1. *A triangular field, whose sides are 20, 18, and 16 chains, is to have a piece of 4 acres in content fenced off from it, by a right line drawn from the most obtuse angle to the opposite side. Required the length of the dividing line, and its distance from either extremity of the line on which it falls ?*

Ans. Length of the dividing line, 13 chains, 95 links, if run nearest the side 16. Distance it strikes the base from the next most obtuse angle is 5.55 chains.

2. *The three sides of a triangle are 5, 12, and 13. If two-thirds of this triangle be cut off by a line drawn parallel to the longest side, it is required to find the length of the dividing line, and the distance of its two extremities from the extremities of the longest side.*

Ans. Distance from the extremity on 5, is  $5(\sqrt{3}-\sqrt{2})$ ; on the side of 12, it is  $12(\sqrt{3}-\sqrt{2})$ ; both divided by  $\sqrt{3}$ .

The division line is  $13\sqrt{\frac{2}{3}}$ .

3. *It is required to find the length and position of the shortest possible line, which shall divide, into two equal parts, a triangle whose sides are 25, 24, and 7 respectively.*

REMARK. — It is obvious that the division line must cut the sides 25 and 24, and to make it the shortest line possible, the triangle cut off must be Isosceles.

Ans. The division line makes an angle with the sides 25 and 24 of  $81^{\circ} 52' 31''$ , and its length is 4.896.

4. *The sides of a triangle are 6, 8, and 10. It is required to cut off nine-sixteenths of it, by a line that shall pass through the center of its inscribed circle.*

Ans. The division line cuts the side of 10, at the distance of 7.5 from the most acute angle, and on the side of 8, at the distance 6 from the most acute angle.

5. *Two sides of a triangle, which include an angle of  $70^{\circ}$ , are 14 and 17 respectively. It is required to divide it into three equal parts, by lines drawn parallel to its longest side.*

Ans. The first division line on the side 17, cuts that side at the distance  $\frac{17}{\sqrt{2}}$ ; the second division line  $\frac{17\sqrt{2}}{\sqrt{3}}$ . The side 14 is cut at  $\frac{14}{\sqrt{3}}$  and  $\frac{14\sqrt{2}}{\sqrt{3}}$ .

6. *Three sides of a triangle are 1751, 1257.5, and 2364.5. The most acute angle is  $31^{\circ} 17' 19''$ . This triangle is to be divided into three equal parts by lines drawn from the angular points to some point within. Required the lengths of these lines.*

Ans. The line from the most acute angle is 1322.42, and from the next most acute angle 1119

7. *The legs of a right-angled triangle are 28 and 45. Required the lengths of lines drawn from the middle of the hypotenuse, to divide it into four equal parts.*

Ans. A line drawn from the middle of the hypotenuse to the right angle, divides the triangle into two equal parts.

8. *In the last example, suppose the given point on the hypotenuse at the distance of 13 from the most acute angle, whereabouts on the other sides will the division lines fall to divide the triangle into three equal parts?*

N. B. The sine of an acute angle to any right-angled triangle is equal to the side opposite that angle divided by the hypotenuse.

Ans. Both division lines fall on the side 28, distance of the first from the acute angle  $12\frac{1}{3}$ , or the second  $24\frac{2}{3}$

9. *There is a farm containing 64 acres, commencing at its south westerly corner, the first course is North  $15^{\circ}$  E., distance 12 chains; the second is N.  $80^{\circ}$  E. (distance lost), the third S. (distance lost), the fourth is N.  $82^{\circ}$  W. (distance lost), to the place of beginning. It is required to determine the distances lost.*

OBSERVATION. — Extend the northern and southern boundary westward, and thus form a triangle on the west side of 12.

Ans. The 2nd side is 35.816 ch. 3rd, 23.21 ch. 4th, 38.76 ch.

The two following problems are from GUMMERE'S *Surveying*, and are considered very difficult.

1. *There is a piece of land bounded as follows :*

Beginning at the south-west corner ; thence,

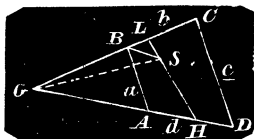
1. *N. 14° 00' W., Distance 15.20 chains = a ;*
2. *N. 70° 30' E., " 20.43 " = b ;*
3. *S. 6° 00' E., " 22.79 " = c ;*
4. *N. 86° 30' W., " 18.00 " = d.*

*Within this lot there is a spring ; the course to it from the second corner is S. 75° E., distance 7.90 chains. It is required to cut off ten acres from the west side of this lot, by a line running through the spring. Where will this line meet the fourth side, that is, how far from the first corner ?*

Ans. 4.6357 chains.

First make a plot of the field. It is as here represented.

Produce the sides  $b$  and  $d$ , the second and fourth, until they meet at  $G$ . Let  $S$  be the position of the spring, and join  $SG$ . We may or may not find the contents of the field\* : it is not necessary for the location of the line  $LSH$ .



It is necessary to find the area of the triangle  $ABG$ . Conceive a meridian line run through  $B$  ; then we perceive that the angle  $ABG = 14^\circ + 70^\circ 30' = 84^\circ 30'$ . Conceive also a meridian line to be run through  $A$ , and then we perceive that the angle  $BAG = 86^\circ 30' - 14^\circ = 72^\circ 30'$  ; whence  $AGB = 23^\circ$ . With the angles and the side  $AB = 15.20$ , we readily find  $AG = 38.72$ ,  $BG = 37.10$ , and the area  $AGB = 280.65$  square chains.

It is necessary to find the line  $GS$  and the angle  $BGS$ . From the given direction of the lines  $BC$  and  $BS$ , we find the angle  $GBS = 145^\circ 30'$  ; and then from the triangle  $GBS$ , we find  $BGS = 5^\circ 51' 30''$ , and  $GS = 43.83$ . Also we have the angle  $SGA = 17^\circ 8' 30'$ .

To the area of the triangle  $AGB = 280.65$ , add 10 acres, or 100 square chains : then the area of the triangle  $GLH$  must equal 380.65

\* When we have *four sides only*, and all the angles, as in this field, the best method of finding the contents is by conceiving it to be two triangles. Thus in this case the area is represented by

$$\frac{1}{2} ab \sin. ABC + \frac{1}{2} dc \sin. CDA.$$

square chains ; but  $GL$  and  $GH$  are both unknown. Put  $GL=y$ ,  $GH=x$  : then we shall have the equation.

$$xy \sin. 23^\circ = 2(380.65). \quad (1)$$

It is obvious that the sum of the two triangles  $LGS$ ,  $SGH$  is equal to the triangle  $GLH$ .

But  $GS=m=43.83$ ,  $\sin. 23^\circ=P$ ,  $\sin. (17^\circ 8' 30'')=Q$ ,  $\sin. (5^\circ 51' 30'')=R$ , and  $2(380.65)$  or  $761.3=a$  : then we have

$$Pxy=a, \quad (1)$$

$$\text{and} \quad Rmy+Qmx=a, \quad (2)$$

From (1),  $y=\frac{a}{Px}$ . This value put in (2), gives

$$\frac{aRm}{Px} + Qmx=a; \quad (3)$$

$$\text{whence,} \quad x^2 - \frac{a}{mQ}x = -\frac{aR}{PQ}. \quad (4)$$

We now find the numerical values of  $\frac{a}{mQ}$  and  $\frac{aR}{PQ}$  by logarithms, as follows :

As our radius is unity, we diminish the indices of the logarithmic sines by 10.

|                      |                     |                      |           |
|----------------------|---------------------|----------------------|-----------|
|                      | log. $a$ , 2.881556 | log. $a$ , 2.881556  |           |
| log. $m$ , 1.641771  |                     | log. $R$ , -1.008880 |           |
| log. $Q$ , -1.469437 |                     |                      |           |
|                      |                     |                      | 1.790436  |
| 1.111208             | 1.111208            |                      |           |
| 58.932               | 1.770348            | log. $P$ , -1.591878 |           |
|                      |                     | log. $Q$ , -1.469437 |           |
|                      |                     |                      |           |
|                      |                     | -1.061315            | -1.061315 |
| 674.72               | - - - - -           |                      | 2.829121  |

Equation (4) now becomes

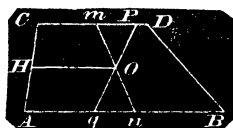
$$x^2 - 58.932x = -674.7;$$

whence  $x=GH=43.366$  : from which subtract  $GA=38.72$ , and we have  $AH$  the distance required  $=4.646$ , which differs from the given answer about *one link* of the chain.

**LEMMA.** Find the point in any trapezoid, through which any straight line which meets the parallel sides will divide the trapezoid into two equal parts.

Let  $ABCD$  be the trapezoid.

Bisect the parallel sides  $AB$  and  $CD$  in the points  $n$  and  $m$ . Join  $mn$ , and bisect  $mn$  in  $O$ , and  $O$  is the point required.



Any line meeting the parallel sides, and passing through  $O$ , will divide the trapezoid into two equal trapezoids. It is obvious that the line  $mn$  divides the figure into two equal parts, because the sum of the parallel sides is the same in each. Now draw any other line through  $O$ , as  $pOq$ : the trapezoid  $pqBD = mnBD$ ; because the triangle  $Omp = qOn$ , and one triangle is cut off and the other is put on at the same time. The triangles are equal, because  $mO = On$ , the angle  $pmO = Onq$ , and the opposite angles at  $O$  are equal: therefore  $pm = nq$ ; and whatever more than  $Cm$  is taken on one side, the equal quantity  $qn$  less than  $An$  is taken on the other side.

Another method of finding the point  $O$ , is to bisect  $AC$  in  $H$ , and draw  $HO$  parallel to  $AB$  or  $CD$ , and equal to one-fourth the sum of  $AB$  and  $CD$ .

2. There is a piece of land bounded as follows :

Beginning at the westernmost point of the field ; thence,

1.  $N. 35^\circ 15' E.$ , 23.00 chains ;
2.  $N. 75^\circ 30' E.$ , 30.50 “
3.  $S. 3^\circ 15' E.$ , 46.49 “
4.  $N. 66^\circ 15' W.$ , 49.64 “

It is required to divide this field into four equal parts, by two lines, one running parallel to the third side, the other cutting the first and third sides. Find the distance of the parallel line from the first corner measured on the fourth side, and the bearing of the other line.

Ans. Distance to the parallel, 32.50 chains ; Bearing of the other side,  $S. 79^\circ 48' E.$

## CHAPTER VII.

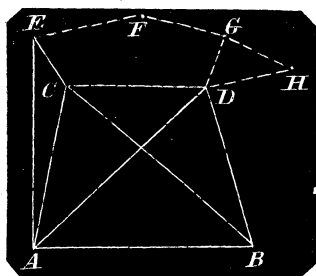
TRIANGULAR SURVEYING: THE PLANE  
TABLE—ITS DESCRIPTION AND USES:  
MAPPING: MARINE SURVEYING.

TRIANGULAR SURVEYING, as here understood, requires the actual measurement of only one line, and all other lines can be deduced from this by means of observed angles forming triangles, of which this measured line forms a base of the first triangle in the series or chain of triangles.

Some of the other lines however should be measured after being computed, as a test to the accuracy or inaccuracy of the operations.

Let  $AB$  represent a base line which must be very accurately measured, for any error on  $AB$  will cause a proportional error in every other line.

If at  $A$  we measure the angles  $BAC$ ,  $BAD$ , and at  $B$  we measure or observe the angles  $ABC$ ,  $ABD$ , we then have sufficient data to determine the points  $C$  and  $D$ , and the line  $CD$ .



With equal facility that we determine the point  $C$ , we can determine the point  $E$ , or  $F$ , or  $G$ , or any other *visible* point.

Thus we may determine all the sides and angles of the figure  $CEFGHD$ , or any *visible* part of it, by triangulating from the base  $AB$ .

The lines forming the triangles are not drawn, except those to the points  $C$  and  $D$ ; we omitted to draw others to *avoid confusion*.

After any line, as  $FG$ , has been computed, it is well to measure it, and if the measurement corresponds with computation, or nearly so, we may have full confidence of the accuracy of the work as far as it has been carried.

We may take  $CD$  as the base, and determine any visible number of points, as  $A$ ,  $B$ ,  $H$ ,  $F$ ,  $G$ , &c., trace any figure and determine its area, or show the relative positions and distances of objects from each other, such as buildings, monuments, trees, &c.

But to make the computation, triangle after triangle, for the sake of making a map, would be very tedious, and to measure every side and angle would be as tedious, and to facilitate this kind of operation we may have an instrument called the

## PLANE TABLE.

The plane table is exactly what the name indicates ; it is a plane board table, about two feet long, and twenty inches wide, resting on a tripod, to which it is firmly screwed, yet capable of an easy motion on its center, having a ball and socket like a compass staff.

Directly under the table is a brass plate, in which *four milled screws* are worked, for the purpose of adjusting the table, the screws pressing against the table.

To level the table, a small detached spirit level may be used. The level being placed on the table over two of the screws, the screws are turned contrary ways, until the level is horizontal ; after which it is placed over the other two screws, and made horizontal in the same manner.

The table has a clamp screw, to hold it firmly during observations, and also a tangent screw, to turn it minutely and gently, after the manner of the theodolite.

The upper side of the table is bordered by four brass plates, about an inch wide, and the center of the table is marked by a pin.

About this center, and tangent to the corners of the table, *conceive* a circle to be described. Suppose the circumference of this circle to be divided into degrees and parts of a degree, and radii to be drawn through the center, and each point of division.

The points in which these radii intersect the outer edge of the brass border, are marked by lines on the brass plates ; these lines of course show degrees and parts of degrees ; they are marked from right to left, from 0 to 180° on both sides, but on some tables the numbers run all the way round, from 0 to 360°.

Near the two ends of the table are two grooves, into which are fitted brass plates, which are drawn down into their places by screws coming up from the under side. The object of these grooves and corresponding plates, is to hold down paper firmly and closely to the table.



The paper before being put on, should be moistened to expand it, then carefully drawn over the table, and fastened down by the plates that fit into the grooves ; on drying, it will fit closely to the table.

A delicate fine edged ruler is used with the plane table, it has vertical sights ; the hairs of the sights are in the same vertical plane as the edge of the ruler.

A compass is sometimes attached to the table, to show the bearings of the lines ; but the most practical mathematicians prefer each instrument by itself.

The plane table may be used to advantage for three distinct objects.

1. For the measurement of horizontal angles.
2. For the determination of the shorter lines of a survey, both as to extent and position.
3. For the purpose of mapping down localities, harbors, water-courses, &c.

1. *To measure a horizontal angle.*

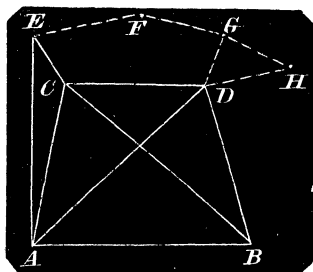
Place the center of the table over the angular point, by means of a plumb. Level the table, then place the fine edge of the ruler against the small pin at the center ; direct the sights to one object, and note the degree on the brass plate ; then turn the ruler to the other object, and note the degree as before.

*The difference of the degrees thus noted, is the angle sought.*

If the ruler passed over  $O$ , in turning from one object to the other, subtract the larger angle from  $180^\circ$ , and to the remainder add the smaller angle, for the angle sought.

2. *To determine lines in extent and position.*

Let  $CD$  be a base line; having the paper on the table, all dried and ready for use. Place the table over  $C$ , so that the point on the table, where we wish  $C$  to be represented, shall fall directly over  $C$ ; and place the position of the table so that  $CD$  shall take the desired direction on the table.



Now level the instrument, and clamp it fast ; it is then ready for use.

Sight to the other end of the base line, and mark it along the fine edge of the ruler.

In the same manner sight along the direction of  $CE$ , and mark that direction in a fine lead line, that can be easily rubbed out, the point  $E$  is *somewhere* in that line.

Sight in the direction of  $F$ , and mark the line on the paper ;  $F$  is somewhere in that line. In this manner sight to as many objects as desired, as  $G$ ,  $H$ ,  $B$ ,  $A$ , &c.

Now the base *on the paper*, may be as long, or as short as we please ; suppose the *real base* on the ground to be 1200 feet ; this may be represented on the table by 3, 4, 5, 10, or 12 inches, more or less ; suppose we represent it by 6 inches, then one inch on the paper, will correspond with 200 feet on the ground (*horizontally*).

Take  $CD$  six inches, and place a pin at  $D$ , remove the instrument to the other end of the base, and place  $D$  of the table right over the end of the base, by the aid of a plumb, and give the table such a position as will cause  $CD$  on the table, to correspond with the direction of the base.

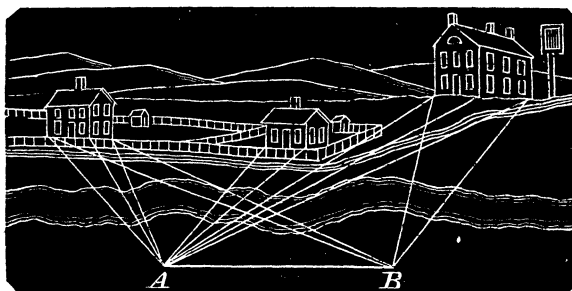
Level the table and clamp it. Now, if  $CD$  on the table, does not exactly correspond with the direction of the base on the ground, make it correspond by means of the tangent screw.

Now from  $D$ , by means of the ruler and its sight vanes, draw lines on the paper, in the direction of the points  $E$ ,  $F$ ,  $G$ ,  $H$ ,  $B$ ,  $A$ , &c.; and where these lines intersect those from the other end of the base, to the same points, is the real localities of those points, *in proportion* to the base line. Lines drawn from point to point, where these lines intersect, as  $EF$ ,  $FG$ ,  $GH$ , &c., will determine the distances from point to point, at the rate of 200 feet to the inch.

Lines drawn from the center of the table, parallel to  $FE$  and  $FG$ , will determine the angle  $EFG$ , in case the angle is required. After the points  $E$ ,  $F$ ,  $G$ , &c., have been determined, the light pencil lines to them, from the ends of the base, may be rubbed out, except those that we may wish to retain.

*Here we perceive the utility of the plane table ; we have a multitude of results, as soon as the observations are made.*

The plane table will give us at once, the relative distances of buildings from the base, and from each other, and if we are careful and particular, we can obtain the magnitudes of the buildings, as is obvious by the adjoining figure.



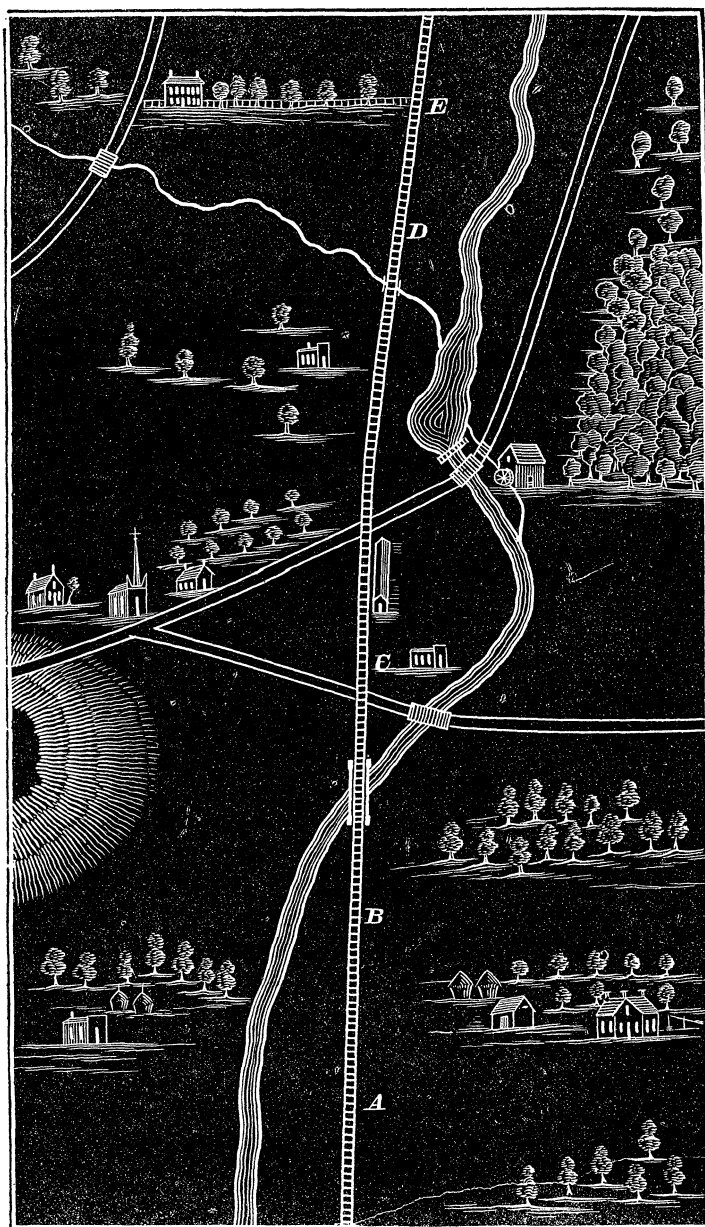
This is most useful to an officer or a spy, who wishes as exact knowledge of an enemy's locality as possible. Or from a distant place  $AB$ , we may examine and measure any objects whatever, on the other side of a river, or give a correct delineation of the river itself.

The plane table can be made very useful to civil engineers, for mapping the localities through which a canal or railroad passes. Take, for example, a railroad line,  $ABCDE$ , represented in the next figure. The lines being all measured and marked in distances of 100 or 200 feet, the bases are all ready where the line is straight.

Set the table as at  $A$ , and draw lines to all objects that you wish to appear on the map, both to the right and to the left,—then move the instrument to  $B$ , drawing lines to the same object *from a corresponding base on the paper*, and also draw lines to other objects further in advance on the line that may be seen from another base.

The intersections of the lines from the extremities of any base to the same object will locate the object.

When all the objects are thus located, both to the right and to the left, we pass on to new objects, taking care to keep at least two of the old objects in sight, to connect one new observation with those previously taken. We now commence a series of observations from a new base, which base must take its proper relative position on the



paper, and if the paper on the table is not large enough, it must be taken off and new paper put on, and two of the objects on the old paper must appear on the new, and then these two papers can be put together so that the objects which are on both papers will coincide, and then the two papers will be the same as one, and thus we may put any number of papers together and form as large a map as we please.

If the different bases are not in the same direction, the objects which are on two different papers on being put together will show it, and several papers put together may make a very inconvenient figure; but they must be put together and then a square sheet of tissue paper put over the whole, and the map taken off. From the tissue paper the map can be put on any other paper.

The engineers of Napoleon's army frequently made maps of the localities they were about to pass; indeed it is a military principle never to go into an unknown locality, except in cases of absolute necessity.

This subject naturally leads us to

#### MARINE SURVEYING.

Marine Surveying is too extensive a subject to be fully investigated in any work like this. We shall only explain how to find shoals, rocks, and turning points in a channel, by ranging to objects on shore.

In trigonometrical surveying, on shore, the observer is supposed to take his angles from the extremities of a base line, but in trigonometrical surveying on water, the observer can take his angles only from single points which may be connected together by distant base lines on the shore.

Important points along the shore are determined by taking latitude and longitude, and intermediate places, by regular land surveying.

The localities of rocks and shoals are also determined by astronomical observations, establishing latitude and longitude, in case no land is in sight, or they are far from the shore; but in the vicinity of the land, the determination of a point is commonly effected by the *three point problem*.

The three point problem is the determination of any point from observations taken at that point, on *three* other distant points where the distances of these three points from each other are known.

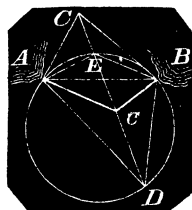
It is immaterial how those points are situated, provided the three points and the observer are not in the same right line, the middle one may be nearest or most remote from the observer, or two of them may be in one right line with the observer, or all three may be in one right line, provided the observer be not in that line. The following example will illustrate the principle.

Coming from sea, at the point *D*, I observed two headlands, *A* and *B*, and inland *C*, a steeple, which appeared between the headlands. I found, from a map, that the headlands were 5.35 miles from each other; that the distance from *A* to the steeple was 2.8 miles, and from *B* to the steeple 3.47 miles; and I found with a sextant, that the angle *ADC* was  $12^{\circ} 15'$ , and the angle *BDC*  $15^{\circ} 30'$ . Required my distance from each of the headlands, and from the steeple.

If the direction of *AB* is known, the direction of *AC* is equally well known.

The case in which the three objects, *A*, *C*, and *B*, are in one right line may require illustration.

At the point *A*, make the angle *BAE* = the observed angle *CDB*, and at *B*, make the angle *CBE* = the observed angle *ADC*.



Describe a circle about the triangle *ABE*, join *E* and *C*, and produce that line to the circumference in *D*, which is the point of observation. Join *AD*, *BD*. The angle *ADB* is the sum of the observed angles, and *AEB* added to it must make  $180^{\circ}$ .

*The Trigonometrical Analysis.*—In the triangle *ABE*, we have the side *AB* and all the angles, *AE* and *EB* can therefore be computed.

In the triangle *AEC*, we now have *AC*, *AE*, and the angle *CAE*, from which we can compute *ACE*, then we know *ACD*.

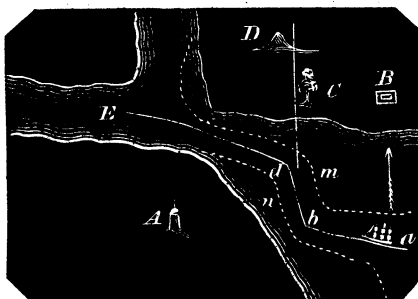
Now in the triangle *ACD*, we have *AC* and all the angles, whence we can find *AD* and *CD*.

When the bearing of the base line on shore is known, as it generally is, and the bearings to its extremities, or even to one extremity, are taken, the triangle is known at once.

A pilot guides a vessel in and out of a port by ranging lines on the shore, minutely or approximately, as the case may require.

We will illustrate this by a figure. Let the deep shaded lines represent the shores, *A* a light house on a rocky promontory. *B* another prominent object on the opposite shore; the position of the arrow indicates the direction north and south.

The faint dotted lines represent the boundaries of shoal water, over which it would not be prudent, if even possible, to conduct a vessel. The line *a, b, d, E*, the center of the ship channel. All the pilots know that a line from *A* to *a*, which is nearly east south east, runs safe to the open sea, after passing the *shoal coast* near *n*.



Now suppose a pilot boards a ship coming in from sea, sufficiently far from the coast, he directs her sailing so as to bring the light house at *A* to bear west north-west. He then sails toward the light house until he finds the object *B* bearing due north of him. He then knows that the ship must be near *a*, the mouth of the channel.

He continues the same course, and knows when he is about half way from *a* to *b* by the two objects, *C* and *D*, appearing in the same line. When *C* and *D* becomes fairly open, and *C* nearly north, and *B* not quite north-west, he is then at the turning point *b* of the channel. His course is then north, a little to the west, until the ship is nearly in a line between the two objects *A* and *B*. From thence, west north-west takes the ship directly through the proper channel into the harbor.

## CHAPTER VIII.

## LEVELING.

Two or more points are said to be on a level, when they are equally distant from the center of the earth, or when they are equally distant from a tranquil fluid, situated immediately below them. A level surface on the earth, is nearly spherical, *and is not a plane*; it is everywhere *perpendicular to a plumb line*.

Any small portion of a true level surface, cannot be distinguished from a plane; and, therefore, when observations are taken in respect to level, within short distances of each other, the spherical form of the earth is disregarded, and the level treated as a plane. But when any considerable portion of surface is taken into account, the curvature of the earth's surface must be considered.

The apparent level, at any point on the earth, is a tangent plane, touching the earth at that point only, and the true level is below this, and the distance below, depends on the distance from the tangent point.

Let  $T$  be any point on the surface of the earth, at right angles to the plumb line from this point is the plane or apparent level  $ATB$ ; but the true level, or the surface of standing water, would be the curved surface  $GTH$ .

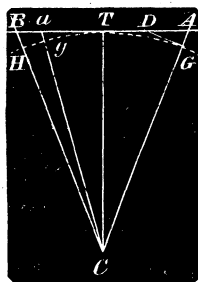
The distance  $AG$ , depends on the distance  $AT$ , and the radius of the earth  $CG$  or  $CT$ .

From  $G$ , draw  $GD$ , at right angles to  $AC$ ; then the two triangles  $ATC$ ,  $AGD$ , are equiangular and similar, and give the proportion

$$CT : TA :: DG : GA.$$

*Practically.*— $TA$  is a very short distance compared to  $CT$ , for  $TA$ , the distance within which we can take observations, is never more than two or three miles, while the distance  $CT$  is near 4000 miles; therefore,  $CT$  is nearly equal to  $CA$ , consequently,  $DA$  is nearly equal to  $DG$ , so near that we shall call it equal.

Observe that  $TD = DG$ ; hence  $DG = \frac{1}{2}TA$ .





Now, in the preceding proportion, in the place of  $DG$ , put its equal  $\frac{1}{2}TA$ , and we shall have,

$$CT : TA :: \frac{1}{2}TA : GA.$$

$$\text{Whence, } GA = \frac{\overline{TA}^2}{2CT} \quad \text{Also, } ga = \frac{\overline{Ta}^2}{2CT} \quad (1)$$

That is, *The square of the distance, divided by the diameter of the earth, is the distance between the apparent and the true level.*

We can arrive at the same result by the direct application of the 36th proposition of the third book of Euclid, or by the application of theorem 18, third book of Robinson's Geometry.

Because  $A$  is a point without a circle, and  $AT$  touches the circle, we must have

$$AG \times (2CG + AG) = \overline{AT}^2$$

But  $2CG$ , which is the diameter of the earth, cannot be *essentially* or *appreciably* increased, by the addition of  $AG$ , which is at most, but a few feet, therefore,  $AG$  within the parentheses, may be suppressed without making any appreciable error. Then divide by  $2CG$ .

$$\text{Whence, } AG = \frac{\overline{AT}^2}{2CG}, \text{ the same as before.}$$

If we take one mile for the distance  $TA$ , the value of  $GA$  will be  $\frac{1}{7918} = 8.001$  inches.

By comparing equations (1) we perceive that,

$$GA : ga :: \overline{TA}^2 : \overline{Ta}^2$$

That is, *The corrections for apparent levels, are in proportion to the squares of the distances.*

*The correction for one mile is 8.001 inches; what is it for 10 miles?*

**Ans.** It is  $x$  inches; then we have the following proportion,

$$8.001 : x :: 1^2 : 10^2 \quad x = 800.1 \text{ inches.}$$

We have seen above, that the correction for *one mile* or 80 chains distance, on an apparent level, is 8.001 inches, what is the correction for the distance of 20 chains?

Let  $x$  = the correction sought, and the solution is thus,

$$\begin{aligned} 8.001 : x :: (80)^2 : (20)^2 \\ :: 4^2 : 1^2 \quad x = 0.500 \text{ inches.} \end{aligned}$$

In this manner, the following table was computed.

*Table showing the differences in inches, between the true and apparent level, for distances between 1 and 100 chains.*

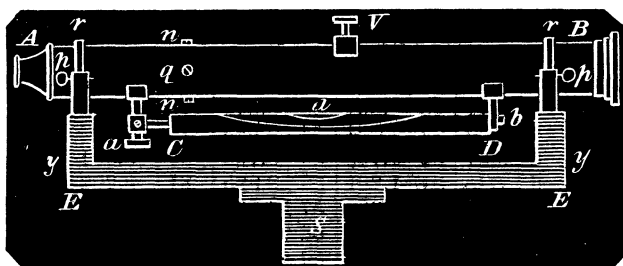
| Ch's. | In's. | Ch's. | In's. | Ch's. | In's. | Ch's. | In's.  |
|-------|-------|-------|-------|-------|-------|-------|--------|
| 1     | .001  | 26    | .845  | 51    | 3.255 | 76    | 7.221  |
| 2     | .005  | 27    | .911  | 52    | 3.380 | 77    | 7.412  |
| 3     | .011  | 28    | .981  | 53    | 3.511 | 78    | 7.605  |
| 4     | .020  | 29    | 1.051 | 54    | 3.645 | 79    | 7.802  |
| 5     | .031  | 30    | 1.125 | 55    | 3.785 | 80    | 8.001  |
| 6     | .045  | 31    | 1.201 | 56    | 3.925 | 81    | 8.202  |
| 7     | .061  | 32    | 1.280 | 57    | 4.061 | 82    | 8.406  |
| 8     | .080  | 33    | 1.360 | 58    | 4.205 | 83    | 8.612  |
| 9     | .101  | 34    | 1.446 | 59    | 4.351 | 84    | 8.832  |
| 10    | .125  | 35    | 1.531 | 60    | 4.500 | 85    | 9.042  |
| 11    | .151  | 36    | 1.620 | 61    | 4.654 | 86    | 9.246  |
| 12    | .180  | 37    | 1.711 | 62    | 4.805 | 87    | 9.462  |
| 13    | .211  | 38    | 1.805 | 63    | 4.968 | 88    | 9.681  |
| 14    | .245  | 39    | 1.901 | 64    | 5.120 | 89    | 9.902  |
| 15    | .281  | 40    | 2.003 | 65    | 5.281 | 90    | 10.126 |
| 16    | .320  | 41    | 2.101 | 66    | 5.443 | 91    | 10.351 |
| 17    | .361  | 42    | 2.208 | 67    | 5.612 | 92    | 10.587 |
| 18    | .405  | 43    | 2.311 | 68    | 5.787 | 93    | 10.812 |
| 19    | .451  | 44    | 2.420 | 69    | 5.955 | 94    | 11.046 |
| 20    | .500  | 45    | 2.531 | 70    | 6.125 | 95    | 11.233 |
| 21    | .552  | 46    | 2.646 | 71    | 6.302 | 96    | 11.521 |
| 22    | .605  | 47    | 2.761 | 72    | 6.480 | 97    | 11.763 |
| 23    | .661  | 48    | 2.880 | 73    | 6.662 | 98    | 12.017 |
| 24    | .720  | 49    | 3.004 | 74    | 6.846 | 99    | 12.246 |
| 25    | .781  | 50    | 3.125 | 75    | 7.032 | 100   | 12.502 |

This table is of *little* or no practical use, for levelers rarely take sight to a greater distance than 10 chains, and at that distance the correction is only *one-eighth* of an inch, and if they put the level midway between two stations, they *annihilate* the corrections altogether.

Suppose a level to be placed at *T*, midway between *A* and *B*; the instrument will show them to be on the *same level*, as so they really are, for they are at equal distances from the *center of the earth*; but if the observations were taken in reference to *A* and *a*, the apparent level would not show equal distances from the center of the earth, and a correction must be applied, if the difference of distances is more than four or five chains.

To comprehend the whole subject, we must now describe the modern

## SPIRIT LEVEL.



The figure before us represents this useful instrument, apart from its tripod.

Its principal parts are the telescope *AB*, to which is attached the leveling tube *CD*; the telescope rests in a bed, which is supported by posts *yy*, called the *y*'s; *EE* is a firm bar, supporting the *y*'s. In *S* is a socket, which receives the central pivot of the tripod (which is not here represented).

When the instrument is put upon its tripod, the tube *S* can be clasped on the outside, and held firmly by a clamp screw, it can then be moved horizontally, as minutely and readily as desired, by means of a tangent screw.

The tripod contains a pair of brass plates, to the lower one the legs of the tripod are firmly attached, the other plate moves in all directions on its center, and is worked by four screws; these are called the leveling screws; these plates are purposely made small as a greater surety against bending: the four leveling screws are placed at the four quadrant points of the circle, and, with the center, form diameters at right angles.

The eye-glass of the telescope is at *A*, the object glass at *B*. The screw *V* runs out the tube which holds the object glass, to adjust it to different distances.

The telescope is fastened into the *y*'s, by the loops *rr*, which are fastened by the pins *pp*. The telescope can be reversed in the *y*'s, by taking out the pins *pp*; opening the loops *rr*; taking up the tube, turning it round, and again placing it in the *y*'s; then *A* will

take the position of  $B$ , and  $B$  of  $A$ . The necessity of this construction will appear when we describe the adjustment.

At  $n$   $n$  are two small screws that are attached to a ring inside of the tube; this ring holds a horizontal spider line; the object of the screw is to elevate and depress that spider line.

At  $q$   $q$  (only one  $q$  can be seen in the figure), are two screws that work another ring, which holds a perpendicular spider line, which can be moved to the right and left by the screws  $q$   $q$ . The two spider lines show a perpendicular and horizontal cross at the focus of the telescope.

Before using the instrument it must be adjusted. The adjustment consists:

1st. *In making the center of the eye-glass and the intersection of the spider's lines coincide with the axis of the telescope, and this line is called the line of collimation.*

2nd. *In making the axis of the attached level,  $CD$ , parallel to the line of collimation, in respect to elevation.*

3rd. *In making the attached level lie exactly in the same direction as the line of collimation.*

To make the *first* adjustment, the telescope is made to revolve in the  $y$ 's.

To make the *second* adjustment, there is a screw  $a$ , which serves to elevate and depress the end of the leveling tube at  $C$ .

To make the *third* adjustment, there is a side screw  $b$ , which drives the end of the tube  $D$  to the right and left, as the case may require.

*First Adjustment.*—Plant the tripod, place the instrument upon it, and direct the telescope to some well defined and distant object.

Draw out the eye-glass at  $A$ , until the spider's lines are distinctly seen, then run out the object glass by the screw  $V$  to its proper focus, when the object and the spider's lines will be distinct. Now note the precise point covered by the horizontal spider's line. Having done this, revolve the telescope in the  $y$ 's half round, when the attached level will be on the upper side. See if in this position the horizontal spider's line appears *above* or *below* the same object.

If this line should appear exactly in the same point of the object

as before, this spider's line is already in adjustment, but if it appears above or below, bring it half way to the same point by means of the screws  $n\ n$ , loosening the one and tightening the other.

Carry back the telescope to its original position, and repeat the observation, and continue to repeat it until the telescope will revolve half round without causing the horizontal line to rise or fall. This will show that the horizontal line is a *diameter* of the circle of revolution, and not a *chord* of it. Make the same adjustment in respect to the vertical hair, and the line of collimation is then adjusted.

*Second Adjustment.*—That is, to make the tube  $CD$  horizontally parallel to the line of collimation. Place the instrument properly on its tripod, and bring the horizontal bar  $EE$  directly over two of the leveling screws; turn these screws until the bubble  $d$  rests in the center of the tube.

Now  $CD$  is on a level, but we are not able to say that the line of sight through the telescope, that is the line of collimation, is on a level also. To test this, take out the pins  $p\ p$ , open the loops  $r\ r$ , and take out the telescope with its attached level, and turn it end for end, put it back in its bed, and put the loops over and pin them down. If the bubble now rests in the middle, no adjustment is required; if not, the bubble will run to the elevated end. In that case the bubble must be brought half way back by the leveling screws, and the other half by the screw  $a$ .

Repeat the operation until the bubble will settle in the middle of the tube after reversing the telescope.

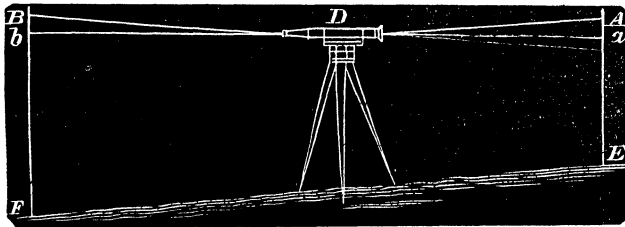
*Third Adjustment.*—The second adjustment being completed, revolve the telescope in the  $y$ 's, and if the bubble continues in the middle, the axis of the telescope and the axis of the tube  $CD$  lie in the same direction, or in the same vertical plane; and if they be not in the same vertical plane the bubble will run to one end or the other; in that case the side screw  $b$  will remedy the defect.

The three adjustments are now made *approximately*, no one of them can be made perfectly while the instrument is greatly out of adjustment in relation to the others; therefore commence anew.—Bring the bar  $EE$  over two of the leveling screws, and level the instrument; then turn it over the two other screws and level it in that direction also. Now, if we can turn the instrument quite round

without removing the bubble from the center it is in pretty good adjustment, but if otherwise, as is to be expected, *make all these adjustments over again*; they can now be made with much less difficulty.

It is important that a level should be in as perfect adjustment as possible, but perfection in all respects is almost, yea, quite impossible. Yet, with a level considerably out of adjustment, we can obtain the *relative* elevation of any two points, provided we can set the level midway between them.

To illustrate this, suppose the instrument placed at  $D$ , midway between two perpendicular rods  $Aa Bb$ .



Let  $ab$  represent the true horizontal line, but suppose that the instrument is so imperfect, or out of adjustment, that when the leveling tube  $CD$  is horizontal, the telescope would point out the *rising line*  $DA$ , and the rise would be  $Aa$ . On turning the instrument round and sighting to  $B$ , the rise must be the same as in the opposite direction: *for the distance is the same*, therefore  $A$  and  $B$  are as truly on a level with each other as  $a$  and  $b$ .

By this problem, practical men complete the second adjustment of the instrument. They make the three adjustments as just explained, as accurately as possible. They then measure, very carefully the distance between two stations, as  $E$  and  $F$ , and set the instrument exactly midway between them as represented in the last figure.

They then level the instrument (that is the tube  $CD$ ), and find the difference of the levels between  $E$  and  $F$  (two pegs driven into the ground).

|                                        |   |             |
|----------------------------------------|---|-------------|
| Now, suppose $AE$ measures on the rod, | - | 4.752 feet. |
| And $BF$ " " "                         | - | 6.327 feet. |
| Then $E$ is above $F$ - - -            | - | 1.575 feet. |

They now bring the level near to one of the stations as *E*, and level it very accurately, and sight to the rod *AE*.

|                                   |   |   |   |              |
|-----------------------------------|---|---|---|--------------|
| Now, suppose the target stands at | - | - | - | 5.137 feet.  |
| To this add                       | - | - | - | 1.575 feet.  |
|                                   |   |   |   | <u>6.712</u> |

The rod man now goes to the station *F*, puts his target on the rod exactly at 6.712, and the telescope is turned upon it, and the horizontal spider's line ought to just coincide with the target, and will, if the instrument is in perfect adjustment; if it is not, the error is taken out by the screws *nn*. If the error was but slight, as in such cases it always is with good instruments, the adjustment is as complete as it can be made.

With the level there must be

#### A R O D .

The rod is commonly ten feet long, and divided into tenths and hundredths, some have also a vernier scale which in effect subdivides to *thousandths*. The target slides up and down the rod, and carries the vernier on the back of the rod; the target has equal alternate portions painted black and white for contrast.

A party to take the necessary levels on the line of a railroad or canal, after the stations are measured off, should consist of a leveler, and assistant leveler, a rod man, and an axe man.

The leveler and assistant leveler both keep book, and sometimes the rod man also. If there is no assistant leveler, the rod man will have an abundance of time to keep book, and under such circumstances always does so. Under all circumstances, two persons keep book, to have a check on each other and guard against mistakes.

In the field the aim is to put the level midway between the two stations, but they are not particular about it if the instrument is in good adjustment; they rather take the most advantageous spot to sight from.

When the ground admits of it, that is, sufficiently level, two or three intermediate stations are observed, as well as the extreme back and fore stations. The extreme back and fore stations are called *changing stations*; at these stations pegs are driven into the ground by the axe man for the rod man to plant his rod, so as to secure

the same point for both the fore and back sight. At the intermediate stations they have no pegs, and are *not particular in any respect*, for all errors cancel each other.

The common railroad chain is 100 feet, divided into 100 links; each link is therefore *one foot*. Levels are commonly taken at intervals of 200 feet, oftener if the ground is very uneven, but a station is considered 200 feet, and the number of the station multiplied by 200 gives the number of feet from the commencement.

The field book is kept thus :

B. S. means back sight, F. S. fore sight, A. ascent, D. descent, T. total elevation above a common base.

N. B.—When the back sight is less than the fore sight, the ground is descending, and the converse.

| Sta. | B. S. | F. S. | A.   | D.   | Total.<br>100 |
|------|-------|-------|------|------|---------------|
| 0    | 4.32  | 7.21  |      | 2.89 | 97.11         |
| 1    | 5.52  | 8.17  |      | 2.65 | 94.46         |
| 2    | *9.18 | 6.27  | 2.91 |      | 99.37         |
| 3    | 6.27  | 6.12  | 0.15 |      | 97.52         |
| 4    | 6.12  | 3.76* | 2.36 |      | 99.88         |
| 5    | 9.81  | 11.62 |      | 1.81 | 98.07         |
| 6    | 8.47  | 9.02  |      | 0.55 | 97.52         |
| 7    | 2.64  | 8.91  |      | 6.27 | 91.25         |
| 8    | 1.07  | 7.38  |      | 6.31 | 84.94         |
| 9    | 4.29  | 5.32  |      | 1.03 | 83.91         |
| 10   | 5.32  | 4.85  | 0.47 |      | 84.38         |
| 11   | 4.85  | 3.17  | 1.68 |      | 86.06         |
| 12   | 8.22  | 1.53  | 7.31 |      | 93.37         |

Thus we go through the whole line. We commenced with the *constant* 100, but this is arbitrary; the object of taking a constant is to avoid the minus sign, that is, getting below our ruled paper in making a profile of the vertical section.

Where the line is to be generally ascending, we assume a small constant, where generally descending a large one; taking care in all cases to have it so large as not to run it out.

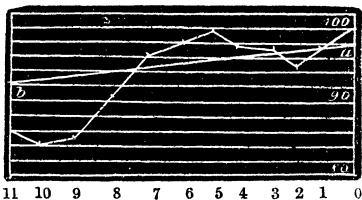
At each and every section we know exactly how much we are above or below the constant base, and the exact ascent or descent from any one station to any other.

The following diagram represents a vertical section of the ground



where these levels were taken, with the exception of the exaggeration of the roughness caused by the difference of scales for the base and perpendicular lines. From one station to another is 200 feet ; we have made 10 feet occupy more space in the perpendicular direction than 400 feet does in a horizontal direction. We do this to show more clearly where any particular grade will enter the ground, and how much it is necessary to cut or fill at any particular point.

The zigzag line from 100 of altitude to 86, represents the surface of ground, and suppose that we wished to reduce it to a regular grade so as to remove as little earth as possible. By the mere exercise of judgment, we conclude to run the grade between 0 station and 11, from 98 to 92 ( but the propriety of this conclusion would depend on the contour of the ground on each side of these stations ). The direct line *ab* drawn, shows that the grade runs out of the ground at station 1, we must fill in about  $2\frac{1}{2}$  feet at station 2, the grade runs into the ground again at about 80 feet before we come to station 3. At section 4 the cutting must be a little over 2 feet, at station 5 a little over 5 feet, at 7, 2 feet, and runs out of the ground midway between stations 7 and 8.



At stations 9 and 10 we must fill in about 8 feet, and so on ; the depth of cutting or filling is obvious at every station.

If the contour of the ground beyond 11 was generally level or descending, we would change the grade at station 7 and render it more descending, so as to make less filling up at stations 9, 10, and 11. In the adoption of grades for a railroad, an engineer has great scope to exercise his judgment.

In the representation here made, *ab* appears like a steep grade, but it is not ; it could scarcely appear on the ground other than a level, for the difference is only 6 feet in 2200 feet of distance, which is at the rate of  $14\frac{4}{5}$  feet to the mile.

An engineer can freely vary his grade, while it keeps under 18 or 20 feet to the mile ; but they submit to a great deal of cutting and filling, before they establish a grade over 40 feet to the mile.

CONTOUR OF GROUND.

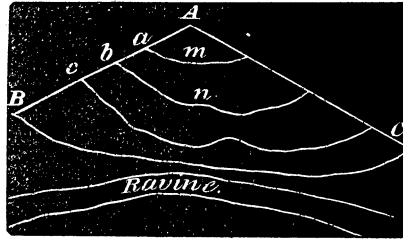
Contour of ground is shown on maps, by marking where parallel planes run out on the surface.

We shall give only the general principle.

Let  $A$  be the top of a hill, whose contour we wish to delineate; measure any convenient line as  $AB$ , up or down the hill, and by the level or theodolite, ascertain the relative elevations of  $a, b, c, d$ , &c., as many planes as we wish to represent.

At *a*, place the level or theodolite, and level it ready for observation; measure the height of the instrument, and put the target on the rod at that height.

Send the rod-man and axe-man round the hill, on the same level as the instrument. Let the rod-man set the rod; the leveler will sight to it through the telescope, and if the target is below the level, he will motion the rod-man up the hill, if too high, down the hill; at length he will get the same level, and there the axe-man will drive a stake. In the same manner we will establish another stake further on; and thus proceed from point to point. To get round the hill, it may be necessary to move the instrument several times. The plane thus established, is represented by the curve *am*.



In the same manner, by placing the instrument at  $b$ , we can establish the next plane  $bn$ .

Then the next, and the next, as many as we please. Where the hill is more steep, two of these parallel planes will be nearer together in the figure; where less steep, they will appear at a greater distance asunder, and this, with the proper shading, will give a true representation of the ground.

ELEVATIONS DETERMINED BY ATMOSPHERIC PRESSURE, AS INDICATED BY THE BAROMETER.

The higher we ascend above the level of the sea, the less is the atmospheric pressure (other circumstances being the same), and therefore we can determine the ascent, provided we can accurately measure the pressure, and know the law of its decrease.

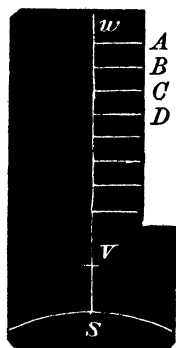
As this work is designed to be educational as well as practical, we shall here make an effort to explain the philosophy of the problem, in such a manner, as to *force* it upon the comprehension of the learner.

The pressure of the atmosphere at any place, is measured by the height of a column of mercury it sustains in the barometer tube.

It is found *by experiment*, that air is compressible, and the amount of compression is always in proportion to the amount of the compressing force.

Now, suppose the atmosphere to be divided into an indefinite number of strata, of the *same thickness*, and so small that the density of each stratum may be considered as uniform.

Commence at an indefinite distance above the surface of the earth, as at *A*, and let *w* represent the weight of the whole column of atmosphere resting on *A*. Let the small and indefinite distances between *AB*, *BC*, *CD*, &c., be equal to each other, and we shall call them *units* of some unknown magnitude.



The weight of the column of atmosphere supposed to rest on *B*, is greater than *w*, by some indefinite part of *w*, say the *n*th part. Then the weight on *B*, must be expressed by  $\left(w + \frac{w}{n}\right)$  or  $\left(\frac{n+1}{n}\right)w$ .

In the same manner, the weight or pressure resting on *C*, must be the weight above *B*, increased by its *n*th part; that is, it must be  $\left(\frac{n+1}{n}\right)w + \left(\frac{n+1}{n^2}\right)w$ , which by addition is  $\left(\frac{n+1}{n^2}\right)^2 w$ .

In the same manner, we find that the weight resting on *D*, must be  $\frac{(n+1)^3}{n^3} w$ , and so on. For the sake of perspicuity, we recapitulate.

The pressure on  $A$  is  $w$ . Units from  $A$  0

“ “ on  $B$  is  $\left(\frac{n+1}{n}\right)w$  “ “ 1

“ “ on  $C$  is  $\frac{(n+1)^2}{n^2}w$  “ “ 2

“ “ on  $D$  is  $\frac{(n+1)^3}{n^3}w$  “ “ 3

“ “ on  $E$  is  $\frac{(n+1)^4}{n^4}w$  “ “ 4

“ “ on  $F$  is  $\frac{(n+1)^5}{n^5}w$  “ “ 5

&c. &c.

&c. &c.

Here we observe the series which represents the pressure of atmosphere, at the different points  $A, B, C, \&c.$ , is a series in *geometrical progression*, and it corresponds with another series in *arithmetical progression*; therefore, by the nature of logarithms, the numbers in the arithmetical series, may be taken as the logarithms of the numbers in the geometrical series.

But this system of logarithms, may not be *hyperbolic* nor tabular, indeed it is neither; the *base of this system* is as yet unknown, but our investigations will soon lead to its discovery.

Now, let the number of units from  $A$  to  $S$  (the surface of the sea), or to the *lower of two stations*, be represented by  $x$ , then the expression for the pressure of the air would be  $\left(\frac{n+1}{n}\right)^x w$ , but this is neither more nor less than the weight of the column of mercury in the barometer, which is sustained by this pressure.

By calling this  $b$ , and designating the logarithms of this unknown system by  $L'$ , we shall have

$$L'b = x \quad (1)$$

Taking  $y$  to represent the number of units from  $A$  to  $V$ , and  $b_1$  to represent the pressure of the air at that point, we shall have

$$L'b_1 = y \quad (2)$$

Subtracting (2) from (1), we shall have

$$L'b - L'b_1 = x - y = SV$$

This is, a certain *peculiar logarithm* of the barometer column at the lower station, diminished by the logarithm of the barometer at the

upper station will give the difference of levels between the two stations. But still all is indefinite and unknown, because we know nothing of these logarithms.

In algebra, we learn that the logarithms of one system can be converted into another by multiplying them by a constant multiplier called the *modulus* of the system, therefore,

Assume  $Z$  to be the modulus or constant that will convert common logarithms into these peculiar logarithms.

$$\text{Then,} \quad Z(\log. b - \log. b_1) = SV \quad (3)$$

Here,  $\log. b$  denotes the common logarithms of the barometer column.

Equation (3) is general, and determines nothing until we know  $SV$  in some particular case.

Taking  $SV$  some *known elevation*, and observing the altitude of the barometer column at both stations, and then equation (3) will give  $Z$  once for all.

Putting  $h$  to represent the *known elevation*, and we have, in general,

$$Z = \frac{h}{\log. b - \log. b_1} \quad (4)$$

*Example.*—At the bottom and top of a tower, whose height was 200 feet, the mercury stood in the barometer as follows.

At the bottom,                    -   -   -   -   -   29.96 inches =  $b$

At the top,                         -   -   -   -   -   29.74 inches =  $b_1$ ,

the temperature of the air being  $49^\circ$  of Fahrenheit's thermometer.

$$\text{Whence, } Z = \frac{200}{\log. 29.96 - \log. 29.74} = \frac{200}{0.003201} = 62500 \text{ nearly.}$$

“ But this multiplier is constant only when the mean temperature of the air at the two stations is the same ; and for a lower temperature the multiplier is less, and for a higher it is greater. A correction, however, may be applied for any deviation from an assumed temperature, by increasing or diminishing ( according as the temperature is higher or lower ) the approximate height by its 449th part for each degree of Fahrenheit's thermometer. We can moreover change the multiplier to a more convenient one by assuming such a temperature as shall reduce this number to 60000 instead of 62500. Now 62500 exceeds 60000 by its 25th part ; and, since  $1^\circ$  causes a change of one 449th part, the proportion

$$\frac{1}{449} : 1^\circ :: \frac{1}{25} : 17.9,$$

gives  $18^\circ$  nearly for the reduction to be made in the temperature of the air at the time of the above observations, in order to change the constant multiplier from 62500 to 60000, or to 10000, by calling the height fathoms instead of feet. Thus, instead of the thermometer standing at  $49^\circ$ , we may suppose it to stand at  $49^\circ - 18^\circ$  or  $31^\circ$ ; and then, we take 10000 as the multiplier, and apply a correction additive for the  $18^\circ$  excess of temperature."

The same observations, for example, being given, to find the height of the tower.

|       |   |   |               |   |   |         |
|-------|---|---|---------------|---|---|---------|
| 29.96 | - | - | log.          | - | - | 1.47654 |
| 29.74 | - | - | log.          | - | - | 1.47334 |
|       |   |   | Diff. of log. | - | - | 0.00320 |
|       |   |   | Multiplier    | - | - | 10000   |
|       |   |   | Product       | - | - | 32      |

Then the height of the tower is 32 fathoms, or  $32 \times 6 = 192$  feet, on the supposition that the temperature of the air was  $31^\circ$  in place of  $49^\circ$ . But it being  $49^\circ$ , we must increase 192 by its  $\frac{1}{4\frac{1}{8}}$  part for each degree above  $31^\circ$ , that is, by  $\frac{1}{4\frac{1}{8}}$  or  $\frac{1}{25}$  nearly of its approximate height, which gives nearly 8 feet to add to 192, making 200 feet for the height of the tower.

The same method is applicable to other cases whatever be the temperature of the air at the two stations, provided it be the same or nearly the same at both stations, or provided we take the mean temperature of the two stations. We can find the difference of levels of two stations to considerable accuracy by the following

**RULE.** — 1st. *Take the difference of the logarithms of the two barometer columns, and remove the decimal point four places to the right. This is the approximate difference of levels in fathoms.*

2d. *Add  $\frac{1}{4\frac{1}{8}}$  of the approximate height for each degree of temperature above  $31^\circ$ , and SUBTRACT the same for each degree below  $31^\circ$ ; the result cannot be far from the truth.*

#### EXAMPLES.

1. The barometer at the base of a mountain stood at 29.47 inches, and taking it to the top, it fell to 28.93 inches.

The mean temperature of the air was  $51^\circ$ . What was the height of the mountain in feet?

Ans. 503.34 feet.

|            |   |   |   |                 |
|------------|---|---|---|-----------------|
| 29.47 log. | - | - | - | 1.469380        |
| 28.93 log. | - | - | - | 1.461348        |
|            |   |   |   | <u>0.008032</u> |

Approximate height in fathoms, 80.32.

*Correction.*—Add  $\frac{2}{4 \frac{1}{2}}$  of 80.32 to itself, that is, add 3.57.

|                     |   |   |          |
|---------------------|---|---|----------|
| Height, in fathoms, | - | - | 83.89    |
| Multiply by         | - | - | <u>6</u> |
| Height in feet,     | - | - | 503.34   |

The average height of the barometer, at the level of the sea, is 30.09 inches; and now if we know the average height of the barometer at any other place, and the average temperature, it is equivalent to knowing the elevation of the latter place above the level of the sea.

For example, the mean height of the barometer at Albany Academy is 29.96, and the mean temperature is 49°. How high is the academy above tide water?

Ans. 117.3 feet.

|                                                |                |           |
|------------------------------------------------|----------------|-----------|
| 30.09 log.                                     | 1.478422       | 49°       |
| 29.96 log.                                     | 1.476542       | <u>31</u> |
|                                                | 0.001880       | 18°       |
| Approximate height                             | 18.80 fathoms. |           |
| Add $\frac{1}{4 \frac{1}{2}}$ or $\frac{1}{2}$ | <u>75</u>      |           |

$19.55 \times 6 = 117.3$  feet.

2. The average height of the barometer at Amenia Seminary in Dutchess, Co., New York, is stated in the Regents' report at 29.81 inches: average temperature 49°. What is the height of that point above tide water?

Ans. 253.32 feet.

3. The mean height of the barometer at Pompey Academy, Onondagua Co., New York, is 28.13, corrected and reduced to 32° Fahr. What then is the elevation of that locality?

Ans. 1755 feet.

Others have made it 1745 feet.

4. From various observations on the summit of Mount Washington, in New Hampshire, the mean height of the barometer there is 24.20, mean temperature at the times of observation was about 65° Fahr.

Now admitting that the mean temperature at the surface of the sea, in the same latitude must have been  $75^{\circ}$ , which would make the mean temperature between the two stations  $70^{\circ}$ , what then is the height of Mount Washington above the sea?

Ans. 6170 feet, nearly.

By some observations the elevation is estimated at 6496 feet, by others at 6290 feet.

5. Lieutenant\* Fremont, in his narrative of the exploring expedition across the Rocky Mountains, page 45, under date of July 13th, 1842, states his latitude at  $41^{\circ} 8' 31''$ , longitude  $104^{\circ} 39' 37''$ , height of the barometer 24.86 inches, *attached* thermometer  $68^{\circ}$ ; what was his elevation above the sea?      Ans. 5389.2 feet.

REMARK.—The author states his elevation at 5449. feet. As he does not state the temperature of the air by the *detached* thermometer, we know not what correction he made. These *solitary* barometrical observations are more or less valuable, according to the settled or unsettled state of the weather. A person of experience and good judgment in such matters, like Fremont, would not of course record the inapplicable observations.

6. Lieut. Fremont, in his journal, page 104, under date of August 15th, 1842, when, as he supposed, on the highest point of the Rocky Mountains, observed the barometer to stand at 18.29 inches, and the attached thermometer at  $44^{\circ}\dagger$ ; what was the elevation above the level of the sea?      Ans. 13522 feet.

Fremont estimates the elevation at      13570 feet.

This shows that he estimated the mean temperature above  $50^{\circ}$ , and no doubt a similar cause made the difference in the result of the previous example.

7. On page 140, of Fremont's Journal, under date of July 12th, 1843, he says; "The evening was tolerably clear, with a temperature at sunset, of  $63^{\circ}$ . Elevation of the camp, 7300 feet."

Taking the mean temperature of the two stations, the sea and his

\* Lieutenant was his proper title at this time.

† If the sea were at the base of the mountain, the temperature at the lower station would no doubt be as high as  $60^{\circ}$ . Making this supposition, the mean temperature of the two stations would be  $50^{\circ}$ . We therefore take  $50^{\circ}$  as the mean temperature.



place of observation, at  $67^{\circ}$ , what must have been the height of his barometer ?

Ans. 23.21 inches.

Represent the approximate elevation by  $y$ , then

$$y + \frac{36y}{449} = 7300 \quad \text{Or, } y = 6738.14$$

Divide  $y$  by 6, which gives 1126.35. Divide this by 10000. Then, let  $x$  represent the altitude of the barometer column.

Whence,  $1.478422 - \log. x = 0.112635$

Therefore,  $\log. x = 1.365787$

In the preceding examples we could only be general and approximate, we had only the observations at one station referred to general observations at the other ; but our results cannot be far from the truth.

When the difference of temperatures at the two stations is considerable, the result must be affected by it.

When the upper station is the coldest, which is generally the case, the mercurial column will be shorter than it otherwise would be, and consequently indicate too great a height.

If the temperature of the upper station is taken for the temperature of the lower, the mercurial column at the lower station would not be high enough, and the deduced result would be too small, as is the case in example 5.

The contraction of mercury being about one 10000th part for each degree of cold, or, 0.0025 in a column of 25 in., it would require  $4^{\circ}$  difference of temperature, to produce an effect amounting to one division on the scale of a common barometer, where the graduation is to hundredths of an inch.

This correction is combined with the former in the following equation, in which  $t$   $t'$  represents the temperature of the air at the two stations ;  $t$  at the lower station,  $q$  and  $q'$  the temperature of the mercury, as indicated by the *attached* thermometer.

The fraction 0.00223, is equal to  $\frac{1}{448}$  nearly ;  $h$  = the height sought,  $b$  and  $b'$  represent the observed height of the mercurial column.

$$h = 10000 \left\{ 1 + 0.00223 \left( \frac{t+t'}{2} - 31 \right) \right\} \log. \frac{b}{b' (1 + 0.001 (q - q'))}$$

Beside the corrections previously considered, regard is sometimes had to the effect of the variation of gravity in different latitudes, and at different elevations above the earth's surface. The latter, however, is too small to require any notice in an elementary work. The former may be found by multiplying the approximate height by  $0.0028371 \times \cos. 2 \text{ lat.}$  It is additive, when the latitude is less than  $45^\circ$ , and subtractive when greater. Or it may be taken from the following table.

| Latitude.        | Correction.          |                     |
|------------------|----------------------|---------------------|
| $0^\circ$ - - -  | $+$ $\frac{1}{352}$  | of the app. height. |
| $5^\circ$ - - -  | $+$ $\frac{1}{358}$  |                     |
| $10^\circ$ - - - | $+$ $\frac{1}{375}$  |                     |
| $15^\circ$ - - - | $+$ $\frac{1}{407}$  |                     |
| $20^\circ$ - - - | $+$ $\frac{1}{460}$  |                     |
| $25^\circ$ - - - | $+$ $\frac{1}{548}$  |                     |
| $30^\circ$ - - - | $+$ $\frac{1}{705}$  |                     |
| $35^\circ$ - - - | $+$ $\frac{1}{1030}$ |                     |
| $40^\circ$ - - - | $+$ $\frac{1}{2030}$ |                     |
| $45^\circ$ - - - | 0                    |                     |
| $50^\circ$ - - - | $-$ $\frac{1}{2030}$ |                     |
| $55^\circ$ - - - | $-$ $\frac{1}{1030}$ |                     |
| $60^\circ$ - - - | $-$ $\frac{1}{705}$  |                     |
| $65^\circ$ - - - | $-$ $\frac{1}{548}$  |                     |
| $70^\circ$ - - - | $-$ $\frac{1}{460}$  |                     |
| $75^\circ$ - - - | $-$ $\frac{1}{407}$  |                     |
| $80^\circ$ - - - | $-$ $\frac{1}{375}$  |                     |
| $85^\circ$ - - - | $-$ $\frac{1}{358}$  |                     |
| $90^\circ$ - - - | $-$ $\frac{1}{352}$  |                     |

Given, the pressure of the atmosphere at the bottom of a mountain, equal to 29.68 in. of mercury, and that at its summit, equal to 25.28 in., the mean temperature being  $50^\circ$ , to find the elevation.

Ans. 727.2 fathoms, or 4363.2 feet.

The following observations being taken at the foot and summit of a mountain, namely,

at the foot, bar. 29.862 attach. therm.  $78^\circ$  detach. therm.  $71^\circ$   
 at the summit, " 26.137 "  $63^\circ$  "  $55^\circ$   
 to find the elevation.

Ans. 618.9 fathoms, or 3713.4 feet.

It is required to find the height of a mountain in latitude  $30^{\circ}$ , the observations with the barometer and thermometer being as follows ; namely,

|                  |            |                             |                              |
|------------------|------------|-----------------------------|------------------------------|
| at the foot,     | bar. 29.40 | attach. therm. $50^{\circ}$ | detach. therm.* $43^{\circ}$ |
| at the summit, " | 25.19      | " $46^{\circ}$              | " $39^{\circ}$               |

Ans. 683.27 fathoms, or 4099.62 feet.

If we assume any temperature, for instance  $45^{\circ}$ , and the height of the barometer at the level of the sea, at 30.09 inches ; we can compute the elevation of the point, where it would be 29.99, 29.89, 29.79, 29.69, &c., inches ; and thus we might form a table, showing the elevations that would correspond to any assumed height of the barometer at that temperature. It will be found, that the first fall of  $\frac{1}{10}$  of an inch will correspond to about 88 feet in elevation, but every subsequent tenth, will require a greater and greater elevation.

---

\* The attached thermometer measures the temperature of the mercury in the barometer, and the detached thermometer, that of the surrounding air.

# NAVIGATION.

---

## CHAPTER I.

### INTRODUCTION.

NAVIGATION is the art of conducting a ship from one place to another.

In most works this art is mixed up with seamanship and elementary science. In this work, navigation will stand by itself — alone ; and we shall presume that the reader is properly prepared in elementary science.

This being the case, it will not be necessary to take up time and space in giving definitions of latitude, longitude, meridian, horizon, &c., &c., the previous indispensable knowledge of geography necessarily gives a knowledge of all these terms.

Navigation, rightly understood, requires an accurate knowledge of the geography of the seas — the winds and currents that here and there prevail, and also a good general knowledge of astronomy.

Running a line in surveying and running a course at sea, are mathematically the same thing, except that the latter is on a larger scale than the former, without its accuracy, and it is for a different object.

In surveying we take no account of the magnitude and figure of the earth. In navigation we are compelled to do so, unless the limits of the operation be very small.

There are two methods of finding the position of a ship.

1st. By tracing her courses and distances, as in a survey. This is called *dead reckoning*.

2d. By deducing latitude and longitude from observations on the heavenly bodies. This is called *nautical astronomy*.

No one expects accuracy from *dead reckoning*, and as a general thing it is only used from day to day, between observations ; or to keep the approximate run during cloudy weather or until observations can again give a new starting point.

Some navigators keep a continuous dead reckoning from port to port, which enables them to judge of drifts, currents, and unknown causes of error.

The earth is so near a sphere that for the practical purposes of navigation it is taken as precisely so. Its magnitude corresponds to  $69\frac{1}{4}$  English miles to one degree of the circumference, but in the early days of navigation 60 miles were supposed to be about a degree, which for the sake of convenience is still retained.

The *sixtieth* part of a degree is called a *nautical mile*, and it is, of course, larger than an English mile.

In an English mile there are        -        -        -        5280 feet.

In a nautical mile there are        -        -        -        6079 feet.

The rate which a ship sails is measured by a line running off of a reel, called the *log line*.

The log is nothing more than a piece of thin board in the form of a sector, of about six inches radius, the circular part is loaded with lead to make it stand perpendicular in the water.

The line is so attached to it that the flat side of the log is kept toward the ship, that the resistance of the water against the face of the log may prevent it, as much as possible, from being dragged after the ship by the weight of the line or the friction of the reel.

The time which is usually occupied in determining a ship's rate is half a minute, and the experiment for the purpose is generally made at the end of every hour, but in common merchantmen at the end of every second hour. As the time of operating is half a minute, or the hundred and twentieth part of an hour, if the line were divided into 120ths of a nautical mile, whatever number of those parts a ship might run in a half minute she would, at the same rate of sailing, run exactly a like number of miles in an hour. The 120th part of a mile is by seamen called a *knot*, and the knot is generally subdivided into smaller parts, called *fathoms*. Sometimes (and it is the most convenient method of division ) the knot is divided into ten parts, more frequently perhaps into eight ; but in either case the subdivision is called a *fathom*.

We shall consider a fathom the tenth of a knot, and as the nautical mile is 6079 feet, the 120th part of it is 50.66, the length of a knot on the line, and a little over 5 feet is the length of a fathom.

The operation of ascertaining the rate of sailing is called by seamen *heaving the log*.

At the end of an hour the *loaded chip*, or log, is thrown over the stern into the sea; a quantity of the line, called the *stray line*, is allowed to run off, then the glass is turned and the number of knots that runs off the reel during the *half* minute is the rate of the ship's motion.

The log is then hauled in and the same operation is repeated at the end of the next hour.

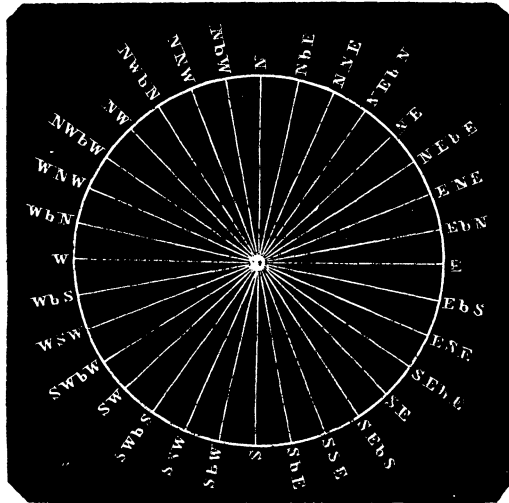
The officer of the watch, who has been on deck during the hour, will mark on the slate or board, called the log board, the number of miles and parts of a mile which the ship has sailed during the last hour, according to the best of his *judgment*; the log was thrown only to help make up that judgment, for the rate at the time the log was thrown may have been considerably more or less than the average motion during the hour.

The course of a ship is marked by the mariner's compass.

The mariner's compass differs from the surveyor's compass only by its construction, that is, the magnetic needle is the motive power in both. In consequence of the motion of a ship at sea, the mariner's compass is suspended in a double box, moving on a double axis, one at right angles with the other, the whole balanced by a central weight which keeps the compass card nearly steady and horizontal, whatever be the motion of the vessel.

The card is attached to the needle, and is moved by the needle. The card is divided into 32 equal parts, called points, and to read over these points in order, is called by seamen, *boxing the compass*, and to know the north star and box the compass is too often the amount of the common sailor's knowledge of navigation.

The figure before us represents the card of the mariner's compass. The four quadrant points are marked by a single letter as *N.* for north, *E.* for east. The midway points between these by two letters, as *N. E.* for north-



east, *N. W.* for north-west, &c. One point either way from any one of these eight points is marked by the word *by*. Thus, *N. by E.* is one point from the north toward the east, and it is read north by east; *S. E. by S.* indicates one point from the south-east more to the south, and it is read *south-east by south*; *W. by N.* means west one point toward the north.

To box the compass we begin at any point, as north, and mention every point in order all the way round, thus :

North; north by east; north north-east; north-east by north; north-east; north-east by east; east north-east; east by north; east, &c.

A point of the compass is  $11^{\circ} 15'$ , which is subdivided into four equal parts. Mariners never take into account a less angle than a quarter point, in running a course.

When the mariner sets his course, he makes allowance for the variation of the needle, and his magnetic courses he reduces to true courses, by the following

**RULE.** — *Make a representation of the compass card on paper, and draw a line through the compass course.*

*Now, conceive the compass card turned equal to the variation to the eastward, if variation is west, and VICE VERSA.*

The line will now pass over the true course.

In the following examples, the true courses are required.

| Compass Course.                                | Variation.                | Answer.<br>True Course.                     |
|------------------------------------------------|---------------------------|---------------------------------------------|
| 1. <i>S. S. E. <math>\frac{3}{4}</math> E.</i> | $2 \frac{1}{4}$ <i>W.</i> | <i>S. E. b E.</i>                           |
| 2. <i>E. <math>\frac{1}{2}</math> N.</i>       | 3 <i>E.</i>               | <i>E. S. E. <math>\frac{1}{2}</math> S.</i> |
| 3. <i>N. W. b W.</i>                           | $3 \frac{3}{4}$ <i>E.</i> | <i>N. b W. <math>\frac{1}{4}</math> W.</i>  |
| 4. <i>W. S. W. <math>\frac{1}{2}</math> S.</i> | 4 <i>W.</i>               | <i>S. b W. <math>\frac{1}{2}</math> W.</i>  |
| 5. <i>S. S. W.</i>                             | $1 \frac{1}{2}$ <i>W.</i> | <i>S. <math>\frac{1}{2}</math> W.</i>       |
| 6. <i>N.</i>                                   | 5 <i>E.</i>               | <i>N. E. b E.</i>                           |
| 7. <i>E. b S.</i>                              | $2 \frac{1}{4}$ <i>E.</i> | <i>S. E. <math>\frac{3}{4}</math> E.</i>    |
| 8. <i>S. 60° E.</i>                            | 18° <i>W.</i>             | <i>S. 78° E.</i>                            |
| 9. <i>N. 24 W.</i>                             | 36 <i>E.</i>              | <i>N. 12 E.</i>                             |
| 10. <i>S. 16 W.</i>                            | 40 <i>E.</i>              | <i>S. 56 W.</i>                             |

## LEEWAY.

The angle included between the direction of the fore and aft line of a ship, and that in which she moves through the water, is called the *leeway*.

When the wind is on the right hand side of a ship, she is said to be on the *starboard* tack; and when on the left hand side, she is said to be on the *larboard* tack; and when she sails as near the wind as she will lie, she is said to be *close hauled*. Few large vessels will lie within less than six points to the wind, though small ones will sometimes lie within about five points, or even less; but, under such circumstances, the real course of a ship is seldom precisely in the direction of her head; for a considerable portion of the force of the wind is then exerted in driving her to leeward, and hence her course through the water, is in general found to be leeward of that on which she is steered by the compass. Therefore, to determine the point toward which a ship is actually moving, the leeway must be allowed *from the wind*, or toward the *right* of her apparent course, when she is on the *larboard* tack; but toward the *left*, when she is on the *starboard* tack.

It is only when a ship crowds to the wind, that leeway is made.

It is seldom that two ships on the same course make precisely the same leeway; and it not unfrequently happens, that the same ship makes a different leeway on each tack. It is the duty of the officer



of the watch, to exercise his best skill in determining, or estimating, how much this deviation from the apparent course amounts to ; and *in the dark*, the chief reliance must be placed on the judgment of the experienced mariner.

---

## CHAPTER II.

### PLANE SAILING.

IN plane sailing, the earth is considered as a plane, the meridians as parallel straight lines, and the parallels of latitude as lines cutting the meridians at right angles. And though it is not strictly correct to consider any part of the earth's surface as a plane, yet when the operations to be performed are confined within the distance of a few miles, no material error will arise, from considering them as performed on a plane surface. And, as we have already seen, in all questions where the *nautical distance, difference of latitude, departure, and course*, are the objects of consideration, the results will be the same, whether the lines are considered as curves drawn on the surface of the globe, or as equal straight lines drawn on a plane.

In all maps, and charts, and constructions, when it is not otherwise stated, it is customary to consider the top of the page as pointing toward the north, the lower part as the south, the right side as the east, and the left as the west. The meridians therefore, in any construction, will be represented by vertical lines, and the parallels of latitude, by horizontal ones.

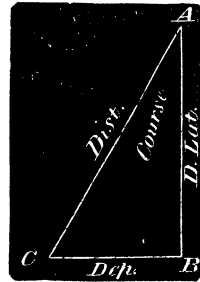
Hence, in constructing a figure for the solution of any case in plane sailing, the difference of latitude will be represented by a vertical line, the departure by a horizontal one, and the distance by the hypotenusal line, which forms, with the difference of latitude and departure, a right-angled triangle ; and the course will be the angle included between the difference of latitude and distance.

With this understanding, the solution of any case that can arise from varying the data in plane sailing will present no difficulty.

## EXAMPLES.

If a ship sail from Cape St. Vincent, Portugal (Lat.  $37^{\circ} 2' 54''$  north), *S. W.*  $\frac{1}{2}$  *S.* 148 miles; required her latitude in, and the departure which she has made?

*By Construction.*—Draw the vertical line *AB*, to represent the meridian; from the point *A*, make the angle  $BAC = 3\frac{1}{2}$  points, the given course; and from a scale of equal parts, take  $AC = 148$  miles, the given distance; from *C* on *AB*, draw the perpendicular *CB*, then *AB* will be the difference of latitude, and *BC* the required departure, and measured on the scale from which *AC* was taken, *AB* will be found 114.4, and *BC* 93.9.



Lat. left - - - -  $37^{\circ} 2' 54''$  *N.*

Diff. lat. - - - -  $1\ 54\ 24$  *N.*

Lat. in - - - -  $35^{\circ} 8' 30''$  *N.* Dep. 93.9

2. If a ship sail from Oporto Bar, in Lat.  $41^{\circ} 9'$  north, *N. W.*  $\frac{1}{2}$  *W.* 315 miles; required her departure and the latitude arrived at?

Ans. Dep. 233.4 miles *W.*; Lat.  $44^{\circ} 41'$  *N.*

3. If a ship sail from lat.  $55^{\circ} 1'$  *N.*, *S. E.* by *S.*, till her departure is 45 miles; required the distance she has sailed and her latitude?

Ans. Dep. 81 miles; Lat.  $53^{\circ} 54'$  *N.*

4. A ship from lat.  $36^{\circ} 12'$  *N.*, sails south-westward, until she arrives at lat.  $35^{\circ} 1'$  *N.*, having made 76 miles departure; required her course and distance.

Ans. Course *S.*  $46^{\circ} 57'$  *W.*; Distance 104 miles.

5. If a ship sail from Halifax, in lat.  $44^{\circ} 44'$  *N.*, *S. E.*  $\frac{1}{2}$  *E.*, until her departure is 128 miles; required her latitude and distance sailed.

Ans. Lat.  $42^{\circ} 51'$  *N.*, and dis. sailed 165.6 miles.

6. A ship leaving Charleston light, in latitude  $32^{\circ} 43' 30''$  north, sails *N.* eastward 128 miles, and is then, by observation, found 93 miles north of the light; required her course, latitude, and departure.

Ans. Lat.  $33^{\circ} 22' 30''$  *N.*; Course *N.*  $72^{\circ} 16'$  *E.*; Dep. 122 miles.

7. A ship from Cape St. Roque, Brazil, in latitude  $5^{\circ} 10'$  south, sails *N. E.*  $\frac{1}{2}$  *N.*, 7 miles an hour, from 3 P. M. until 10 A. M.; required her distance, departure, and latitude in.

Ans. Dis. 133. miles; Dep. 84.4 miles; Lat. in,  $3^{\circ} 27'$  south.

8. A ship from latitude  $41^{\circ} 2' N.$ , sails  $N. N. W. \frac{3}{4} W. 5\frac{1}{2}$  miles an hour, for  $2\frac{1}{2}$  days ; required her distance, departure, and latitude arrived at.

Ans. Dis. 330 miles ; Dep. 169.7 miles ; Lat.  $45^{\circ} 45' N.$

Similar examples, might be given without end, but these are sufficient, for they only involve the principles of the solution of a plane right-angled triangle.

In the preceding examples it will be observed that we traced latitude from latitude, and the distances east and west we called departure — *not difference of longitude*. It now remains to determine *difference of longitude from departure*.

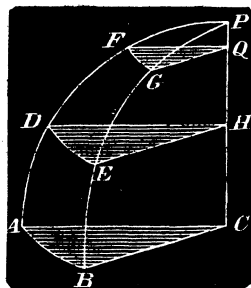
On the equator, 60 miles of departure are equal to one degree of longitude, and the further we are from the equator, north or south, that is, the greater the latitude we are in, the same departure will cover more longitude.

To discover the *law* for changing departure to difference of longitude, we adduce the following figure.

Let  $C$  be the center of the earth,  $P$  the pole,  $PC$  a portion of the earth's axis.

The plane  $PCB$  is the plane of one meridian, and  $PCA$  the plane of another meridian.

$AB$  is a portion of the equator between the two meridians.  $ACB$  is a sector in the plane of the equator, and  $DEH$  and  $FGQ$  are sectors similar to  $ACB$ .



Observe that  $DE$ ,  $FG$ , &c., are parallels of latitude, that is, they are parallel to  $AB$ , the plane of the equator.

The magnitude of  $DE$  is called *departure*, and it corresponds to the difference of *longitude*  $AB$ .

Also,  $FG$  is departure corresponding to the same difference of longitude  $AB$ . The difference of longitude is always greater than any corresponding departure, that is,  $AB$  is obviously greater than any other parallel distance between the same two meridians.

Because the two sectors  $ACB$   $DEH$  are similar, we have the proportion.

$$AC : DH :: AB : DE \quad (1)$$

Observe that  $AC$  is the radius of the sphere,  $DH$  is the sine of the arc  $PD$ , or the cosine of  $DA$ , which is the cosine of the latitude of the point  $D$ .

Therefore the preceding proportion becomes,

$$\text{rad.} : \cos. \text{ lat.} :: \text{dif. lon.} : \text{dep.}$$

$$\text{Or,} \quad \cos. \text{ lat.} : \text{rad.} :: \text{dep.} : \text{dif. lon.}$$

In words, *The cosine of the latitude is to the radius so is the departure to the difference of longitude.*

These words are indelibly engraved on the memory of every navigator, and they embrace all the rules that can be made for changing departure into longitude, or longitude into departure.

When a ship sails east or west, the distance sailed is called departure, and is reduced to longitude by the preceding proportion.

This is called parallel sailing.

#### EXAMPLES.

1. What difference of longitude corresponds to 47 miles departure in the latitude of  $37^\circ 23'$  ?

Ans. 59.15 miles.

Let  $x$  = the difference of longitude required.

$$\text{Then} \quad \cos. 37^\circ 53' : \text{rad.} :: 47 : x = \frac{47 \times \text{rad.}}{\cos. 37^\circ 23'}$$

|         |                     |   |   |   |   |           |
|---------|---------------------|---|---|---|---|-----------|
| By log. | 47 R                | - | - | - | - | 11.672098 |
|         | Cos. $37^\circ 23'$ | - | - | - | - | 9.900144  |
|         | Diff. Lon. 59.15    | - | - | - | - | 1.771954  |

2. How many miles, or how much departure corresponds to a degree in longitude on the parallel of  $42^\circ$  of latitude ?

Ans. 44.59 miles.

Here the longitude of one degree is given.

$$R. : \cos. 42^\circ :: 60 : x = \frac{60 \cos. 42^\circ}{R.}$$

|         |                 |   |   |   |   |          |
|---------|-----------------|---|---|---|---|----------|
| By log. | 60              | - | - | - | - | 1.778151 |
|         | cos. $42^\circ$ | - | - | - | - | 9.871073 |
|         | 44.59           | - | - | - | - | 1.649224 |

N. B.—In this manner the length of a degree in longitude corresponding to each degree of latitude has been found and put in a table.

3. A ship sails east from Cape Race, 212 miles; required her longitude. The latitude of the cape is  $46^{\circ} 40' N.$ , longitude  $53^{\circ} 3' 15''$  west.      Ans. lon.  $47^{\circ} 54'$  west.

4. Two places in lat.  $50^{\circ} 12'$  differ in longitude  $34^{\circ} 48'$ ; required their distance asunder in miles.      Ans. 1336.

5. How far must a ship sail *W.* from the Cape of Good Hope, that her course to Jamestown, St. Helena, may be due north?

Ans. 1193 miles.

|      |   |                          |           |   |                              |
|------|---|--------------------------|-----------|---|------------------------------|
| Cape | { | lat. $34^{\circ} 29' S.$ | Jamestown | { | lat. $15^{\circ} 55' S.$     |
|      |   | lon. $18^{\circ} 23' E.$ |           |   | lon. $5^{\circ} 43' 30'' W.$ |

6. How far must a ship sail *E.* from Cape Horn to reach the meridian of the Cape of Good Hope? The latitude of Cape Horn being  $55^{\circ} 58' 30'' S.$ , lon.  $67^{\circ} 21' W.$ , and the latitude and longitude of the Cape of Good Hope being as stated in the last example.

Ans. 2878 miles.

7. In what latitude will the difference of longitude be three times its corresponding departure? In other words in what latitude will *FG* be one-third of *AB*? (See last figure.)

Ans. lat.  $70^{\circ} 31' 44''$ .

8. Take the 2d example in plane sailing (page 181), the departure made, as stated in the answer, is 233.4 miles. What is the corresponding difference of longitude?      Ans.  $5^{\circ} 18' 24''$ .

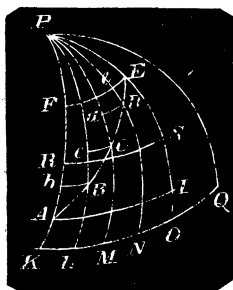
This inquiry now arises. To what latitude does this departure correspond? Is it to the latitude left,  $41^{\circ} 9'$ , or to the latitude arrived at,  $44^{\circ} 41'$ ? Or does it correspond to the mean latitude between the two?

If we suppose the departure corresponds to lat.  $41^{\circ} 9'$ , then the difference of longitude by the preceding rule must be  $5^{\circ} 10'$ , and if the departure corresponds to lat.  $44^{\circ} 41'$ , then the difference of longitude is  $5^{\circ} 28'$ ; the mean of these is  $5^{\circ} 19'$ , and if we take the departure to correspond with the mean latitude  $42^{\circ} 55'$ , then the difference of longitude would be  $5^{\circ} 18' 24''$ .

In the examples under Plane Sailing we have supposed the earth a plane, and the course a ship sails a straight line, but neither supposition is strictly true.

Meridians are *not parallel* with each other, and therefore when a ship sails by the compass, and cuts all the meridians at the same angle, the line that the ship sails will not be a right line : it will be a curve line peculiar to itself, called a *rhumb line*.

For the sake of illustration, let us suppose that in the annexed figure,  $P$  is the north pole,  $KQ$  the equator, or a great circle, every part of which is a quadrant distance from  $P$  ;  $PK, PL, PM$ , &c., great circles passing through  $P$ , and of course cutting the equator at right angles ;  $AI, bB, RS$ , &c., arcs of smaller circles parallel to the equator, and therefore cutting the meridians at right angles ;  $AE$  a curve cutting every meridian which it meets, as  $PK, PL, PM$ , &c., at the same angle. Then  $PK, PL$ , &c., produced till they meet at the opposite pole, are called meridians ;  $AI, bB, RS$ , &c., continued round the globe, are called parallels of latitude ;  $AE$  is called the rhumb line, passing through  $A$  and  $E$  ; the length of  $AE$  is called the nautical distance from  $A$  to  $E$  ; and the angle  $BAb$ , or any of its equals,  $cBC, dCD$ , &c., is called the course from  $A$  to  $E$ .



If a ship sail from  $A$  to  $E$ ,  $EF$  will be her meridian distance ; but if she sail from  $E$  to  $A$ ,  $AI$  will be her meridian distance.

If  $AB, BC, CD$ , &c., be conceived to be equal, and indefinitely small, and their number indefinitely great ; then the triangles  $ABb, BcC$ , &c., may be considered as indefinitely small right-angled plane triangles. And as the angles  $BAb, CBc$ , &c., are equal, and the right angles  $AbB, BcC$ , &c., are equal, the remaining angles  $ABb, BCc$ , &c., are equal ; and as the sides  $AB, BC$ , &c., are also equal, these elementary triangles  $ABb, BcC, CDd$ , &c., will be all identical triangles ; therefore  $AE$  will be the same multiple of  $AB$ , that the sum of  $Ab, Bc, Cd$ , &c., is of  $Ab$  ; and that the sum of  $Bb, Cc, Dd$ , &c., is of  $Bb$ .

It is obvious that

$$Ab + Bc + Cd + \&c. = AF$$

the whole difference of latitude. And,

$$Bb + Cc + Dd + Ee + \&c. = \text{the whole departure.}$$

But this departure is neither  $EF$  nor  $AI$ , it is greater than  $EF$  and less than  $AI$ ; because  $bB$  is less than its corresponding portion on  $AI$  and greater than its corresponding portion on  $FE$ . The same may be said of  $cC$ ,  $Dd$ , &c. Therefore the departure on any course corresponds to *neither of the extreme latitudes*, but to *some mean* between the two, and it is so near the arithmetical mean, that the arithmetical mean is taken as the true.

Therefore in all those examples in Plane Sailing, on page 181, we can take the departures and find the corresponding differences of longitude, provided we take the *middle latitude* and consider the departure run on that parallel.

This method of connecting the change in longitude with a ship's change of place is called

#### MIDDLE LATITUDE SAILING.

But in reality there is no such thing as middle latitude sailing; the cosine of the middle latitude is compared to the radius, as *the ratio* between the departure made and the corresponding difference of longitude, but the departure made may be made on one course or on several courses. When a ship sails on several courses before the run is summed up, the summing up and finding the result in one course and distance is called *working a traverse*, and sailing from one point to another by several courses is called

#### TRAVERSE SAILING.

With adverse winds or crooked channels, vessels are obliged to run a traverse. Going round a survey and keeping an account of our course and distance from the starting point is *working a traverse*, and the operation is the same on sea or land, except on land we aim at coming round to the same point again, but on sea we wish to make some other point.

With this explanation it is obvious that we must make a table as in a survey, and compute the course and distance from the starting point, and this is called the *course and distance made good*.

To work a *traverse* we use the *traverse table* of course; that table is made to every half degree, and the column in the table nearest to the course is sufficiently exact.

The following table gives the degree and parts of a degree corresponding to every point and quarter point of the compass.

| Points.        | Deg.                  | Points.        | Deg.                  |
|----------------|-----------------------|----------------|-----------------------|
| $\frac{1}{4}$  | $2^{\circ} 48' 45''$  | $4\frac{1}{4}$ | $47^{\circ} 48' 45''$ |
| $\frac{1}{2}$  | $5^{\circ} 37' 30''$  | $4\frac{1}{2}$ | $50^{\circ} 37' 30''$ |
| $\frac{3}{4}$  | $8^{\circ} 26' 15''$  | $4\frac{3}{4}$ | $53^{\circ} 26' 15''$ |
| 1              | $11^{\circ} 15'$      | 5              | $56^{\circ} 15'$      |
| $1\frac{1}{4}$ | $14^{\circ} 3' 45''$  | $5\frac{1}{4}$ | $59^{\circ} 3' 45''$  |
| $1\frac{1}{2}$ | $16^{\circ} 52' 30''$ | $5\frac{1}{2}$ | $61^{\circ} 52' 30''$ |
| $1\frac{3}{4}$ | $19^{\circ} 41' 15''$ | $5\frac{3}{4}$ | $64^{\circ} 41' 15''$ |
| 2              | $22^{\circ} 30'$      | 6              | $67^{\circ} 30'$      |
| $2\frac{1}{4}$ | $25^{\circ} 18' 45''$ | $6\frac{1}{4}$ | $70^{\circ} 18' 45''$ |
| $2\frac{1}{2}$ | $28^{\circ} 7' 30''$  | $6\frac{1}{2}$ | $73^{\circ} 7' 30''$  |
| $2\frac{3}{4}$ | $30^{\circ} 56' 15''$ | $6\frac{3}{4}$ | $75^{\circ} 56' 15''$ |
| 3              | $33^{\circ} 45'$      | 7              | $78^{\circ} 45'$      |
| $3\frac{1}{4}$ | $36^{\circ} 33' 45''$ | $7\frac{1}{4}$ | $81^{\circ} 33' 45''$ |
| $3\frac{1}{2}$ | $39^{\circ} 22' 30''$ | $7\frac{1}{2}$ | $84^{\circ} 22' 30''$ |
| $3\frac{3}{4}$ | $42^{\circ} 11' 15''$ | $7\frac{3}{4}$ | $87^{\circ} 11' 15''$ |
| 4              | $45^{\circ} 0'$       | 8              | $90^{\circ} 0'$       |

In works exclusively designed for practical navigation, the traverse table is adapted exactly to the points and quarter points of the compass, but the table in this work is sufficient for the purpose.

The use of this table is to find the degree corresponding to any given course, thus: *N. by E.*, *N. by W.*, *S. by E.*, *S. by W.*, each correspond to 1 point or  $11^{\circ} 15'$ . In using our traverse table for 1 point we should take a mean result between  $11^{\circ}$  and  $11^{\circ} 30'$ , which *mean* result can be taken by the eye.

Again *S.E. by E.  $\frac{1}{4}$  E.* is  $5\frac{1}{4}$  points, or  $S. 59^{\circ} E.$  nearly, and so on for any other course that may be named.

The student is now fully prepared to work the following examples in traverse sailing.

#### EXAMPLES.

1. A ship from Cape Clear, Ireland, in lat.  $51^{\circ} 25' N.$  and longitude  $9^{\circ} 29' W.$ , sails as follows :

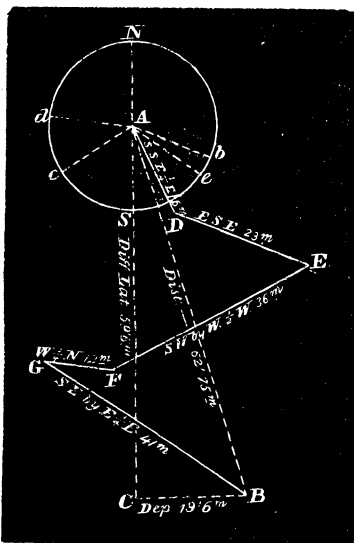
*S. S. E.  $\frac{1}{4}$  E.* 16 miles, *E. S. E.* 23 miles, *S. W. by W.  $\frac{1}{2}$  W.* 36 miles, *W.  $\frac{3}{4}$  N.* 12 miles, and *S. E. by E.  $\frac{1}{4}$  E.* 41 miles.



Required her course and distance made good, the departure, latitude and longitude of the ship.

*By Construction.*—Take *A* for the place sailed from, and draw the vertical line *NASC*, to represent the meridian. About *A*, as a

center, with the chord of  $60^\circ$ , describe a circle, cutting *NC* in *N* and *S*; then *N* and *S* will represent the north and south points of the compass. Take  $2\frac{1}{4}$  points from the line of chords,\* and apply it from *S* to *a*, join *Aa*, and on it take *AD*=16 from a line of equal parts. Then *D* will be the place of the ship at the end of the first course. From *S*, set off *Sb*=6 points from the line of chords; join *Ab*, and through *D* draw *DE*, parallel to *Ab*, and make it equal to 23 from the same scale of equal parts that *AD* was taken from. Then *E* will be the place of the ship,



at the end of the second course. Make *Sc*= $5\frac{1}{2}$  points, *Nd*  $7\frac{1}{4}$  points, and *Se*  $5\frac{1}{4}$  points, taken from the line of chords. Through *E* draw *EF*, parallel to *Ac*, and make it equal to 36, from the scale of equal parts; through *F* draw *FG* parallel to *Ad*, and make it equal to 12, from the scale of equal parts; through *G* draw *GB* parallel to *Ae*, and make it equal to 21, from the scale of equal parts. Demit *BC* a perpendicular, on the meridian *NC*, and join *AB*; then *B* will be the place of the ship, *AB* her distance from the place which she left, *AC* her difference of latitude, *BC* her departure, and *BAC* the course which she has made on the whole. Now, *AB*, *AC*, and *BC* being measured on the scale of equal parts from which the distances were taken, we have *AB*=62.7, *AC*=59.6, and *BC*=19.6 miles. And the arc included by *AC* and *BC*, if measured on the line of chords, gives about  $18^\circ$  for the measure of the course *BAC*.

\* Two and  $\frac{1}{4}$  points is  $25^\circ 18' 45''$ , that is, take the chord of  $25^\circ 18'$  in the dividers, and set it off from *s* to *a*, and so on, for other angles.

TRAVERSE TABLE.

| Courses.                                       | Points.        | Dis. | Diff. Lat. |      | Dep. |      |
|------------------------------------------------|----------------|------|------------|------|------|------|
|                                                |                |      | N.         | S.   | E.   | W.   |
| <i>S. S. E. <math>\frac{1}{4}</math> E.</i>    | $2\frac{1}{4}$ | 16   |            | 14.5 | 6.8  |      |
| <i>E. S. E.</i>                                | 6              | 23   |            | 8.8  | 21.3 |      |
| <i>S. W. by W. <math>\frac{1}{2}</math> W.</i> | $5\frac{1}{2}$ | 36   |            | 17.9 |      | 31.8 |
| <i>W. <math>\frac{3}{4}</math> N.</i>          | $7\frac{1}{4}$ | 12   | 1.8        |      |      | 11.9 |
| <i>S. E. by E. <math>\frac{1}{4}</math> E.</i> | $5\frac{1}{2}$ | 41   |            | 21.1 | 35.2 |      |
|                                                |                |      | 1.8        | 61.4 | 63.3 | 45.7 |
|                                                |                |      |            | 1.8  | 43.7 |      |
| Result                                         |                |      | 59.6       |      | 19.6 |      |

Lat. left - - -  $51^{\circ} 25' N.$

diff. lat. - - -  $1^{\circ} 00' S.$

Lat. in - - -  $50^{\circ} 25' N.$  Mid. lat.  $50^{\circ} 55'$

To find the course and distance, by trigonometry,

|                                 |           |
|---------------------------------|-----------|
| As diff. lat. 59.6 miles        | 1.775246  |
| : radius $90^{\circ}$           | 10.000000 |
| : : dep. 19.6 miles             | 1.292256  |
| : tan. course $18^{\circ} 12'$  | 9.517010  |
| As sin. course $18^{\circ} 12'$ | 9.484621  |
| : dep. 19.6 miles               | 1.292256  |
| : Radius                        | 10.000000 |
| : Dis. 62.75 miles              | 1.797635  |

To find the dif. of longitude.

|                          |                      |
|--------------------------|----------------------|
| As cos. $50^{\circ} 55'$ | 9.799651             |
| : Radius                 | 10.000000            |
| : : dep. 19.6            | 1.292256             |
| : diff. lon. 31.09 miles | 1.492605             |
| Longitude left           | $9^{\circ} 29'$ west |
| diff. lon.               | $31$ east            |
| Lon. in                  | $8^{\circ} 58'$ west |

Thus, we have found the course  $18^{\circ} 12'$ ; Distance 62.75 miles; diff. longitude  $31' E.$ ; lat. in  $50.25 N.$ ; lon.  $8^{\circ} 58' W.$

If these be the distances run in a day, from noon to noon again, then the preceding operation is called working a day's work; otherwise it is called working a traverse, as we have mentioned before.

But this is not the *seaman's* way of working a day's work, he does it all by inspection, in the traverse table. For example, taking the result of the traverse 59.6 south, and 19.6 east, which shows that the resulting course is between the south and the east, and with these numbers he enters the traverse table, and finds, as near as possible, 59.6 and 19.6, standing as latitude and departure; and they are found nearly under the angle of  $18^\circ$ , and opposite the distance 63. nearly.

Hence, he takes his course as *S.  $18^\circ$  E.*, and dis. 63. To find the difference of longitude, he takes the *middle* latitude as a course, and the departure as difference of latitude, *then the distance in the table is difference of longitude.*

In this instance, we take  $51^\circ$  as a course, and in the difference of latitude column we find 19.5, and the distance opposite to it is 31., which we take for difference of longitude.

The reason for this is as follows :

For the longitude we have,

$$\cos. \text{ mid. } L : R :: \text{ dep. } : \text{ diff. lon.} \quad (1)$$

In the construction of the traverse table, we have,

$$\cos. \text{ course } : R :: \text{ diff. lat. } : \text{ dist.} \quad (2)$$

Now, in proportion (2), if we take the middle latitude for a course, and the dep. for difference of latitude; it necessarily follows, that the last term of proportion (2) must be diff. of longitude; for proportion (2) would then be transformed into proportion (1).

2. A ship sails from Cape Clear, as follows; *S. by W. 23 miles; W. S. W. 40 miles; S. W.  $\frac{3}{4}$  W. 18 miles; W.  $\frac{1}{2}$  N. 28 miles; S. by E. 12 miles; S. S. E.  $\frac{3}{4}$  E. 16 miles.*

Required the course and distance made good, and the latitude and longitude arrived at.

Ans. Course *S.  $45^\circ 47'$  W.*; dis. 102.4 miles.

Latitude of ship  $50^\circ 14' N.$ ; Lon.  $11^\circ 25' W.$

3. A ship at noon, on a certain day, was in lat.  $41^\circ 12' N.$ , and longitude  $37^\circ 21' W.$ , she then sailed as follows :

*S. W. by W. 21 m.; S. W.  $\frac{1}{2}$  S. 31 m.; W. S. W.  $\frac{1}{2}$  S. 16 m.; S.  $\frac{3}{4}$  E. 18 m.; S. W.  $\frac{1}{4}$  W. 14, and W.  $\frac{1}{2}$  N. 30 miles.*

Required her course, distance, latitude, and longitude.

Ans. course *S.  $52^\circ 49'$  W.*; dis. 111.7; lat.  $40^\circ 5' N.$ ; lon.  $39^\circ 18' W.$

4. Last noon we were in latitude  $28^{\circ} 46'$  south, and longitude  $32^{\circ} 20'$  west; since then we have sailed by the log :

$S. W. \frac{3}{4} W. 62$  m. ;  $S.$  by  $W. 16$  m. ;  $W. \frac{1}{4} S. 40$  m. ;  $S. W. \frac{3}{4} W. 29$  m. ;  $S.$  by  $E. 30$  m. ; and  $S. \frac{3}{4} E. 14$  miles. Required the direct course and distance, and our present latitude and longitude.

Ans. Course  $S. 43^{\circ} 14' W.$  ; Dis. 158 m. ; lat.  $30^{\circ} 41' S.$  ;  
lon.  $34^{\circ} 24' W.$

5. A ship from Toulon, lat.  $43^{\circ} 7' N.$ , lon.  $5^{\circ} 56' E.$ , sailed

$S. S. W. 48$  m. ;  $S.$  by  $E. 34$  m. ;  $S. W. \frac{1}{4} W. 26$  m. ; and  $E. 17$  miles. Required her course and distance to Port Mahon, Lat.  $39^{\circ} 52' N.$ , and longitude  $4^{\circ} 18' 30''$  east.

Ans. Lat. of ship  $41^{\circ} 32' N.$  ; lon.  $5^{\circ} 37'$  east.  
Course to Port M.  $S. 31^{\circ} W.$  nearly, and distance 117.5 miles.

6. On leaving the Cape of Good Hope, for St. Helena, we took our departure from Cape Town, bearing  $S. E.$  by  $S. 12$  miles, after running  $N. W. 36$  m., and  $N. W.$  by  $W. 140$  miles. Required our latitude and longitude, and the course and distance made.

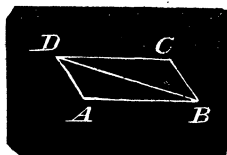
N. B. Lat. of Cape Town  $33^{\circ} 56' S.$  Lon.  $18^{\circ} 23' E.$

Lat. of St. Helena  $15^{\circ} 55' S.$  Lon.  $5^{\circ} 43' 30'' W.$

Ans. Lat.  $32^{\circ} 3' S.$  ; Lon.  $18^{\circ} 25' E.$  ; course  $N. 52^{\circ} 41' W.$  ;  
dis. 187 miles.

## SAILING IN CURRENTS.

If a ship at  $B$ , sailing in the direction  $BA$ , were in a current which would carry her from  $B$  to  $C$ , in the same time that in still water she would sail from  $B$  to  $A$ , then, by the joint action of the current and the wind, she would in the same time, describe the diagonal  $BD$  of the parallelogram  $ABCD$ . For her being carried by the current in a direction parallel to  $BC$ , would neither alter the force of the wind, nor the position of the ship, nor the sails, with respect to it ; the wind would therefore continue to propel the ship in a direction parallel to  $AB$ , in the manner as if the current had no existence. Hence, as she would be swept to the line  $CD$ , by the independent action of the current, in the same time that she reached the line  $AD$ , by the independent action of the wind on her sails, she would be found at  $D$ , the point of intersection of the lines  $AD$  and  $CD$ , having moved along the diagonal  $BD$ .



Now the log heaved from the ship in the ordinary way, can give no imitation of a current; for the line withdrawn from the reel is only the measure of what the ship sails *from the log*; and, consequently, as the log itself, as well as the ship, will move with the current, the distance shown by the log in a current, is merely what it would have been if the ship had been in still water.

If the ship sail in the direction of the current, the whole effect of the current will be to increase the distance; but if she sail against the current, the difference between the rate of sailing given by the log and drift of the current, will be the distance which the ship actually goes; and she will move forward, if her rate of sailing be greater than the drift of the current, but otherwise, her motion will be retrograde, or she will be carried backwards, in the direction of the current.

Problems relating to the oblique action of a current upon a ship, may be resolved by the solution of an oblique-angled plane triangle, such as  $ABD$ , in the preceding figure, where if  $AB$  represent the distance which a ship would sail in still water, and  $AD$  the drift of the current in the same time,  $BD$  will be the actual distance sailed, and  $ABD$  the change in the course produced by the current.

A great variety of problems might be proposed relative to currents, but the chief ones of any practical importance, are the following:

1. To determine a ship's actual course and distance in a current, when her course and distance by the compass and the log, and the setting and drift of the current, are given.

2. To find the course to be steered through a known current, the required course in still water, and the ship's rate of sailing, being known.

3. To find the setting and drift of a current, from a ship's actual place, compared with that deduced from the compass and the log.

The first of these cases may be conveniently resolved, by considering the ship as having performed a traverse, the setting and drift of the current being taken as a separate course and distance.

#### EXAMPLES.

1. If a ship sail  $W$ . 28 miles in a current, which in the same time carries her  $N. N. W$ . 8 miles, required her true course and distance.

N. B. Conceive the current to be one course and distance, and with the other courses find the course and distance made good.

Thus, by the traverse table :

| Course.         | Dis. | Diff. Lat |    | Dep. |       |
|-----------------|------|-----------|----|------|-------|
|                 |      | N.        | S. | E.   | W.    |
| <i>W.</i>       | 28   |           |    |      | 28    |
| <i>N. N. W.</i> | 8    |           |    |      |       |
|                 |      | 7.39      |    |      | 2.06  |
|                 |      | 7.39      |    |      | 31.06 |

As  $7.39 : \text{rad.} :: 31.06 : \tan. 76^{\circ} 36'$ , the course,  
 $\cos. 76^{\circ} 31' : R :: 31.06 : 31.93$ , the distance.

2. If a ship sail *E.* 7 miles an hour by the log, in a current setting *E. N. E.* 2.5 miles per hour ; required her true course, and hourly rate of sailing.

Ans. Course *N.*  $84^{\circ} 8' E.$ , and rate 9.358 per hour.

3. A ship has made by the reckoning *N.*  $\frac{1}{2}$  *W.* 20 miles, but by observation it is found, that, owing to a current, she has actually gone *N. N. E.* 28 miles. Required the setting and drift of the current in the time which the ship has been running.

Ans. Setting *N.*  $64^{\circ} 48' E.$ , and drift 14.1 miles.

4. A ship's course to her port is *W. N. W.*, and she is running by the log 8 miles an hour, but meeting with a current setting *W.*  $\frac{1}{2}$  *S.* 4 miles an hour, what course must she steer in the current that her true course may be *W. N. W.*?

Ans. Course *N.*  $44^{\circ} 39' W.$

5. In a tide running *N. W.* b *W.* 3 miles an hour, I wished to weather a point of land, which bore *N. E.* 14 miles. What course must I steer so as to clear the point, the ship sailing 7 miles an hour by the log, and what time shall I be in reaching the point ?

Ans. Course *N.*  $69^{\circ} 51' E.$ , and time 2 hours 25 minutes.

6. From a ship in a current, steering *W. S. W.* 6 miles an hour by the log, a rock was seen at 6 in the evening, bearing *S. W.*  $\frac{1}{2}$  *S.* 20 miles. The ship was lost on the rock at 11 P. M. Required the setting and drift of the current.

Ans. Setting *S.*  $75^{\circ} 10' E.$ , and drift 3.11 miles per. hour.

## CHAPTER III.

## MERCATOR'S CHART AND MERCATOR'S SAILING.

IN representing any small portion of the earth's surface, it is sufficiently accurate to represent the meridians as parallel; but if the portion of the earth is considerable, the representation will not be true unless the meridians are curved.

If we make a chart and draw all the meridians parallel with each other, the length of a degree of longitude in all places, *except on the equator*, will be greater on the chart than its true distance, but the true bearing of one place from another will be preserved, provided we increase the degrees of latitude in the *same ratio* as the degrees of longitude are increased.

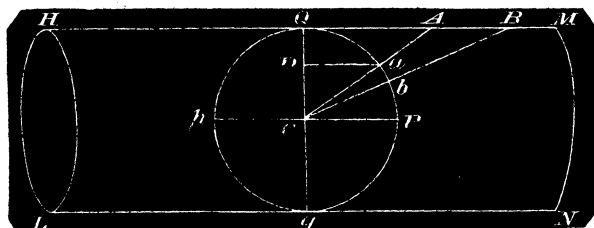
Gerrard Mercator, a Fleming, in 1556, published a chart which seemed to embrace this idea, but he did not show its construction, nor were his degrees in their true proportion; but from this came the name of Mercator's Chart.

A Mr. Wright, an Englishman, in 1599, it is said, published the true sea chart, constructed on the following principles.

1. *The distance between two meridians at the equator, is to their distance in any parallel of latitude, as the radius is to the cosine of that latitude.*
2. *Any part of a parallel of latitude, is to a like part of the meridian, as the radius is to the secant of that parallel.*

We shall make an effort to illustrate these principles by the following figure.

Conceive the equator to be extended both ways parallel to the earth's axis, thus forming a cylinder, whose circumference is just equal to the circumference of the earth.



Let  $Qq$  be the plane of the equator,  $Pp$  the earth's axis ; conceive a globe enclosed in the cylinder,  $HLNM$ .

Suppose there is an island on the earth at  $a$ , that island is projected on the cylinder at  $A$ . The surface of the earth at  $b$  is projected at  $B$ . Conceive this paper cylinder cut by a line at right angles to the equator and rolled out, it will then be a true representation of Mercator's chart.

The scale on the globe at  $a$  is, to the scale on the chart at  $A$ , as  $Ca$  to  $CA$ , that is, as radius to the secant of the latitude at  $a$ .

The scale on the chart at  $A$  is, to the scale on the chart at  $B$ , as  $CA$  is to  $CB$ , that is, the scale on the chart increases as the secants of the latitudes increase.

The poles of the earth, and places very near the poles, can never be represented on this chart.

The meridian distance of a degree on the globe, as at  $a$ , is 60 miles, on the chart at  $A$  it is 60, into  $AC$  the secant of the latitude, calling  $Ca$  unity.

If we commence at the equator at  $Q$ , and take one mile for unity. Then,

Mer. pts. of  $1' = \text{nat. sec. } 1$

Mer. pts. of  $2' = \text{nat. sec. } 1 + \text{nat. sec. } 2$

Mer. pts. of  $3' = \text{nat. sec. } 1' + \text{nat. sec. } 2' + \text{nat. sec. } 3$

Mer. pts. of  $4' = \text{nat. sec. } 1' + \text{nat. sec. } 2' + \text{nat. sec. } 3' + \text{nat. sec. } 4', \&c., \&c.$

In this manner the table of *meridional parts* was originally constructed. It is Table IV of this work.

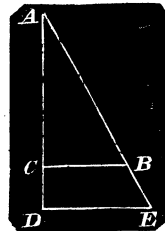
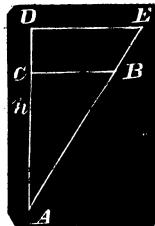
The following figures represent any problem than can arise in Mercator's sailing.

$AC$  represents the true difference of latitude.

$AD$  represents the meridional difference of latitude, which is always taken from the table.

$CB$  represents the departure.

$DE$  the difference of longitude.





$AB$  represents the distance.

$A$ , the angle at  $A$ , represents the course.

Three of these six quantities must be given to solve a problem.

Observe that the difference of longitude  $DE$  is always greater than the departure  $CB$ , as it ought to be.

#### EXAMPLES.

1. A ship from Cape Finisterre, in lat.  $42^{\circ} 56' N.$ , and longitude  $8^{\circ} 16' W.$ , sailed  $S. W. \frac{1}{4} W.$  till her difference of longitude is 134 miles; required the distance sailed and the latitude in.

*By logarithms.* As radius - - - - - 10.000000  
 : diff. lon. 134 miles - - - - - 2.127105  
 : : cot. course  $4\frac{1}{4}$  points - - - - - 9.957295  
 : mer. diff. lat. 121.5 miles - - - - - 2.084400

Lat. Cape Finisterre  $42^{\circ} 56' N.$  Mer. parts - 2858

Mer. diff. - 121

Lat.  $41^{\circ} 27' N.$ , corresponding to - - - 2737 in table

As cosine course - - - - - 9.827085  
 : proper diff. lat. 89 miles - - - 1.949390  
 : : radius - - - - - 10.000000  
 : dis. 132.5 miles - - - - - 2.122305

2. A ship from lat.  $40^{\circ} 41' N.$ , lon.  $16^{\circ} 37' W.$ , sails in the  $N. E.$  quarter till she arrives in lat.  $43^{\circ} 57' N.$ , and has made 248 miles departure; required her course, distance, and longitude in.

Ans. course  $N. 51^{\circ} 41' E.$ , dis. 316 miles, and lon. in  $11^{\circ} W.$

3. How far must a ship sail  $N. E. \frac{1}{2} E.$  from lat.  $44^{\circ} 12' N.$ , lon.  $23^{\circ} W.$ , to reach the parallel of  $47^{\circ} N.$ , and what from that point will be the bearing and distance of Ushant, which is in lat.  $48^{\circ} 28' N.$  and lon.  $5^{\circ} 3' W.$ ?

Ans. She must sail 262 miles, and her course and distance to Ushant will then be  $N. 80^{\circ} 32' E.$ , and dis. 535 miles.

4. A ship from the Cape of Good Hope steers  $E. \frac{1}{2} S.$  446 miles; required her place, and her course, and distance to Kerguelen's Land, in lat.  $48^{\circ} 41' S.$ , and lon.  $69^{\circ}$  east.

Ans. lat. in  $35^{\circ} 13' S.$ , lon. in  $27^{\circ} 21' E.$ , course  $S. 66^{\circ} 25' E.$ , and distance 2018 miles.

5. By observation, a ship was found to be in lat.  $41^{\circ} 50' S.$ , lon.  $68^{\circ} 14' E.$  She then sailed  $N. E.$  140m, and  $E \frac{1}{2} S.$  76m; required her place, and her course, and distance to the island of St. Paul, which is in lat.  $38^{\circ} 42' S.$ , and in lon.  $77^{\circ} 18' E.$

Ans. lat  $40^{\circ} 18' S.$ , lon.  $72^{\circ} 2'$ , course  $N. 68^{\circ} 35' E.$ , and dis. 263 miles, nearly.

# CELESTIAL OBSERVATIONS.

---

## CHAPTER I.

WE now come to the more scientific and essential parts of navigation, the determination of latitude and longitude by celestial observations.

We shall at present confine ourselves to latitude, first calling to mind the following necessary definitions and explanations :

1. **MERIDIAN.** — The meridian of any place is the north and south line passing through that place, and it may be conceived to run along the ground or pass in the same direction in the heavens, through the point vertically over the place. The line on the earth is the terrestrial meridian, the line in the heavens is called the celestial meridian ; they are both in one plane with the center of the earth.

2. **EQUATOR.** — The equator is that circle around the earth over which the sun seems to pass when the days and nights are equal all over the earth.

3. **LATITUDE.** — The latitude of any place is the meridian distance of that place from the equator, measured by degrees and parts of a degree of arc.

4. **LONGITUDE.** — The longitude of any place is the inclination of the plane of its meridian, with the plane of some other definite meridian from which reckoning is made. This inclination is measured on the equator by degrees, minutes, and seconds of *arc*, and it is either east or west.\*

---

\* The first meridian to reckon from may be arbitrarily chosen, and different nations have taken different meridians for the commencement of longitude, but custom and long association have pretty firmly fixed the meridian of Greenwich (England) as the first meridian for all who use the English language.

5. **DECLINATION.** — The declination of a heavenly body is its meridian distance from the equator north or south.

6. **POLAR DISTANCE.** — The polar distance of a body is its declination added to, or subtracted from  $90^{\circ}$ . If both added and subtracted, we shall have the meridian distances from each pole.

The distance from the north pole, is called north polar distance, and from the south pole, south polar distance. The two polar distances must of course make  $180^{\circ}$ .

7. **ZENITH.** — Zenith is the point in the heavens directly overhead.

8. **HORIZON.** — The horizon is either apparent or real, or as commonly expressed, *sensible* or *rational*.

The *sensible* horizon is a plane conceived to touch the earth at any point at which an observer is situated.

The *rational* horizon is a plane parallel to the sensible one, passing through the center of the earth.

The zenith is the pole to the horizon.

9. **GREAT CIRCLES.** — A great circle in the heavens is any circle whose plane passes through the center of the earth.

All great circles which pass through the zenith are perpendicular to the horizon, and such circles are called *vertical circles*, *azimuth circles*, or *circles of altitude*.

10. **AZIMUTH.** — The angle which the meridian makes with that vertical circle which passes through any object is said to be the azimuth of that object. Hence, azimuths may be reckoned from the north or south points of the horizon.

11. **ALTITUDE.** — The altitude of any object is its angular distance from the horizon, measured on a vertical circle.\*

Altitudes are very frequently measured at sea, several times in a day in fair weather; but altitudes observed from the surface of the earth, or above it, require several corrections before the true altitudes can be deduced from them.

---

\* We do not pretend to give all the definitions of the sphere, but we suppose the reader is already acquainted with them, from his knowledge of Geography and Astronomy.

These corrections are for *semi-diameter*, *dip*, *refraction*, and *parallax*. The correction for semi-diameter is obvious.

At sea, the visible horizon (from which all observed altitudes are taken) is where the sea and sky apparently meet, and when the eye of the observer is above the water, this visible horizon is below the *sensible* horizon, and the amount of the depression is called the *dip of the horizon*. Its correction is always subtractive, and its amount is to be found in Table VI.

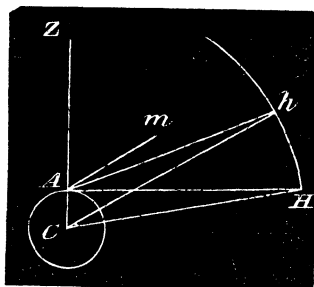
Refraction is to be found in Table V. It is always subtractive, and for the reason, see some treatise on natural philosophy.

Parallax is always additive. Conceive two lines drawn to a heavenly body; one from an observer at the circumference of the earth and the other from the center of the earth, the inclination of these two lines is parallax, and when the body is in the horizon its parallax is greatest, and it is then called horizontal parallax.

Parallax always tends to depress the object, but the parallax of any celestial object, except that of the moon, is so small, that we shall pay attention to lunar parallax only, but this is so important to navigation that we shall give it a full explanation.

The moon's horizontal parallax is given in the Nautical Almanac for every noon and midnight of Greenwich time, and from the horizontal parallax we must deduce the parallax corresponding to any other altitude.

Let  $AC$  be the radius of the earth,  $A$  the position of an observer,  $Z$  his zenith, and suppose  $H$  to be the moon in the horizon; then the angle  $AHC^*$  is the moon's horizontal parallax, and the angle  $AhC$  is the parallax corresponding to the apparent altitude  $hAH$ . Draw  $Am$  parallel to  $Ch$ , then  $mAH$  would be the *true altitude*.



\* From this figure we draw the following definition for horizontal parallax.

*The horizontal parallax of any body is the angle under which the semi-diameter of the earth would appear as seen from that body.* Of course then, when the body is at a great distance its horizontal parallax must be small, hence the sun and the remote planets have very little parallax, and the fixed stars none at all.

Let  $CH$  and  $Ch$  be each represented by  $R$ . Put  $p$  = the horizontal parallax, and  $x$  = the parallax in altitude, or the angle  $mAh$  or  $AhC$ .

Now in the triangle  $ACH$ , right-angled at  $A$ , we have

$$1 : \sin. p :: R : AC.$$

In the triangle  $ACh$  we have

$$\sin. CAh : \sin. x :: R : AC.$$

By comparing these two proportions, we perceive that

$$1 : \sin. p :: \sin. CAh : \sin. x$$

Whence,  $\sin. x = \sin. p. \sin. CAh$

But  $\sin. CAh = \cos. hAH$ , for the sine of any arc greater than  $90^\circ$  is equal to the cosine of the excess over  $90^\circ$ , hence,

$$\sin. x = \sin. p \cos. hAH$$

The lunar horizontal parallax is rarely over a degree, commonly less, and the sine of a degree does not materially differ from the arc itself, hence, the preceding equation becomes the following, without any essential error.

That is,  $x = p \cos. \text{altitude.}$

Or, in words, *the parallax in altitude is equal to the horizontal parallax multiplied into the cosine of the apparent altitude (radius being unity).*

#### EXAMPLES.

1. The apparent altitude of the moon's center after being corrected for dip and refraction was  $31^\circ 25'$ ; and its horizontal parallax at that time, taken from a nautical almanac, was  $57' 37''$ ; what was the correction for parallax, and what was the true altitude as seen from the center of the earth?

|                         |      |   |   |          |
|-------------------------|------|---|---|----------|
| $p = 57' 37'' = 3457''$ | log. | - | - | 3.538699 |
| $31^\circ 25' \cos.$    | -    | - | - | 9.931152 |
| $x = 49' 10'' = 2950$   | log. | - | - | 3.469851 |

Ans. Cor. for parallax  $49' 10''$

True altitude  $32^\circ 13' 10''$

2. The apparent altitude of the moon's center on a certain occasion was  $42^\circ 17'$ ; and its horizontal parallax at the same time was  $58' 12''$ ; what was the parallax in altitude, and what was the moon's true altitude?

Ans. Parallax in alt.  $43' 4''$

True alt.  $43^\circ 0' 4''$

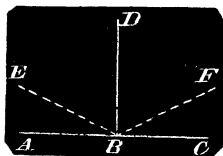
No other examples of this kind are necessary, as they will incidentally occur in several places further on.

It now remains to describe the instrument used for taking angles at sea. We, therefore, give the following illustrations on the

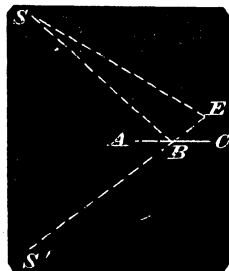
#### QUADRANT AND SEXTANT.

The quadrant and sextant are essentially the same instrument, and the following is an explanation of the principle on which they are constructed.

Let  $ABC$  be a section of a reflecting surface,  $FB$  a ray of light falling upon it, and reflected again in the direction  $BE$ , and  $BD$  a perpendicular at the point of impact; then it is a well known optical fact, that the angles  $FBC$  and  $EBA$  are equal, and that  $FB$ ,  $DB$ , and  $EB$  are in the same plane.



Again, if  $AC$  were a reflecting surface, and a ray of light,  $SB$ , from any celestial object  $S$ , were reflected to an eye at  $E$ , the image of the object would appear at  $S'$  on the other side of the plane, the angles  $SBA$  and  $ABS'$ , as well as  $EBC$ , being equal; and if  $EB$  bear no sensible proportion to the distance of  $S$ , the angles  $SES'$  and  $SBS'$  may be considered as equal; for their difference,  $BSE$ , will be of no sensible magnitude.



Before we proceed to the direct description of the sextant, it is necessary to give the following important

#### LEMMA.

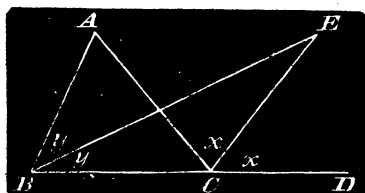
*If the exterior angle of a triangle be bisected, and also one of the interior opposite angles, and the bisecting lines produced until they meet, the angle so formed will be half the other interior opposite angle.*

Let  $ABC$  be the triangle, and bisect the exterior angle  $ACD$  by the line  $CE$ , and the angle  $B$  by the line  $BE$ .

*The angle  $E$  will be half the angle  $A$ .*

Let each of the angles  $ACE$ ,  $ECD$ , be designated by  $x$  (as rep-

resented in the figure ), and each of the equal parts of the angle  $B$  by  $y$ . Let  $A$  represent the angle  $A$ , and  $E$  the angle  $E$ .



Now as the sum of the three angles of every plane triangle is equal to  $180^\circ$ ; therefore, in the triangle  $ABC$ , we have

$$A + 2y + C = 180^\circ \quad (1)$$

Also, in the triangle  $EBC$ , we have

$$E + y + C + x = 180^\circ \quad (2)$$

Subtracting (2) from (1) gives us

$$A - E + y - x = 0 \quad (3)$$

$$\text{Whence,} \quad A = E + (x - y) \quad (4)$$

But because  $x$  is the exterior angle of the triangle  $ECB$

$$x = E + y \quad (\text{see Elementary Geometry.})$$

$$\text{Or,} \quad (x - y) = E$$

This value of  $(x - y)$  substituted in (4) gives

$$A = 2E, \text{ or } E = \frac{A}{2} \quad Q. E. D.$$

*Another Demonstration.*— The angle  $x$  being the half of  $ACD$  is equal to

$$\frac{A + 2y}{2}$$

The angle  $x$  is also equal to  $E + y$ , because it is the exterior angle to the triangle  $EBC$ .

Therefore, by equality,

$$E + y = \frac{A + 2y}{2}$$

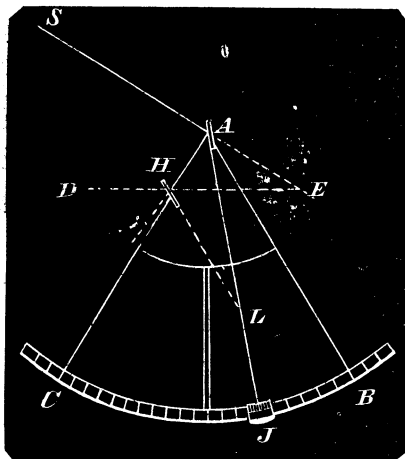
$$\text{Whence,} \quad E = \frac{1}{2}A \quad Q. E. D.$$



We are now prepared to show the construction of the sextant and quadrant.

The instrument represented by the annexed cut is a quadrant or a sextant, according as the arc contains  $90^\circ$  or  $120^\circ$ , but each actual degree of arc is graduated to  $2^\circ$ , and the space that covers  $90^\circ$  is really but  $45^\circ$ , and so on.

*The reason why a half degree is counted and marked as a whole one, we are about to explain.*



$ABC$  is a firm plane sector, commonly made of metal or ebony;  $AJ$  is a revolving index bar, turning on the center  $A$ , to which is attached a vernier scale, revolving over the graduated arc.

The graduation commences at  $B$ . At  $A$  is a small plane mirror, perpendicular to the plane of the sector, it is attached to the index bar and revolves with it. This is called the index mirror or index glass.

At  $H$  is another small mirror, half silvered and the other half transparent. This is called the *horizon glass*; it might be called the *image glass*.

The horizon glass must be perpendicular to the instrument, and parallel to  $AB$ .

Now conceive a ray of light coming from an object  $S$ , striking the mirror  $A$ , the index and mirror being turned so as to throw the reflecting ray into the mirror  $H$ , this mirror again reflects it toward  $E$ , and an eye anywhere in the line  $DH$  will see the image of the object behind the mirror  $H$ . Conceive the ray of light from  $S$  to pass right through the mirror at  $A$ , to meet the line  $HE$ ; then, it is obvious that the angle  $SED$  measures the angle between the object  $S$  and its image  $D$ .

Now, in the triangle  $AEH$ , by a little inspection, it will be found that  $HL$  bisects the exterior angle, and  $AJ$ , the index, bisects one of

the interior opposite angles ; therefore, by the *preceding lemma*, the angle  $HLA$  is half the angle at  $E$ , but as  $AB$  and the mirror  $H$  are parallel, the angle  $HLA$  is equal  $JAB$ . It is obvious that  $JAB$  is measured by the arc  $BJ$ , or it measures the angle at  $E$ , if half degrees on  $BC$  are counted as whole ones, which was to be shown.

A tube, and sometimes a small telescope, is attached to the bar  $AB$ , and placed in the direction of the line  $EH$ . This is called the *line of sight*.

#### THE ADJUSTMENT OF THE INSTRUMENT.

When this instrument is in adjustment, the two mirrors are perpendicular to the plane of the sector, and are parallel to each other when 0 on the vernier coincides with 0 on the arc. We therefore inquire : First

*Is the index mirror perpendicular to the plane of the instrument ?*

The following experiment decides the question.

Put the index on about the middle of the arch, and look into the index mirror, and you will see part of the arch reflected, and the same part direct ; and if the arch appears perfect, the mirror is in adjustment ; but if the arch appears broken, the mirror is not in adjustment, and must be put so by a screw behind it, adapted to this purpose. Second,

*Are the mirrors parallel when the index is at 0 ?*

Place the index at 0, and clamp it fast ; then look at some well-defined, distant object, like an even portion of the distant horizon, and see part of it in the mirror of the horizon glass, and the other part through the transparent part of the glass ; and, if the whole has a natural appearance, the same as without the instrument, the mirrors are parallel ; but, if the object appears broken and distorted, the mirrors are not parallel, and must be made so, by means of the lever and screws attached to the *horizon glass*. Third,

*Is the horizon glass perpendicular to the plane of the instrument ?*

The former adjustments being made, place the index at 0, and clamp it ; look at some smooth line of the distant horizon, while holding the instrument perpendicular ; a continued unbroken line will be seen in both parts of the horizon glass ; and if, on turning the instrument from the perpendicular, the horizontal line *continues*

*unbroken*, the horizon glass is in full adjustment; but, if a break in the line is observed, the glass is not perpendicular to the plane of the instrument, and must be made so, by the screw adapted to that purpose.

After an instrument has been examined according to these directions, it may be considered as in an approximate adjustment—a re-examination will render it more perfect—and, finally, we may find its *index error* as follows:—measure the sun's diameter both on and off the arch—that is, both ways from 0, and if it measures the same, there is *no index error*; but if there is a difference, half that difference will be the index error, *additive*, if the greater measure is off the arch, *subtractive*, if on the arch.

*To measure the altitude of the sun at sea.*

Turn down the proper screen or screens, to defend the eye. Put the index at 0, having it loose, look directly at the sun through the tube, and you will see its image in the silvered part of the horizon glass. Now move the index, and the image will drop; drop it to the horizon, and clamp the index.

Let the instrument slightly vibrate each side of the perpendicular, on the line of sight as a center, and the image of the sun will apparently sweep along the horizon in a circle. While thus sweeping, move the tangent screw,\* so that the lower limb of the sun will just touch the horizon, without going below it. The reading of the index will be the altitude corresponding to that instant, provided there be no index error.

*To measure the angular distance between two bodies as the sun and moon, or the moon and a star.*

The most brilliant of the two objects is always reflected to the other. Loosen the index, place it at 0, and direct the line of sight to the brighter object, and catch a view of its image in the silvered part of the horizon glass.

Turn the plane of the instrument into the plane between the two objects; now move the index, keeping the eye on the image, and

---

\* The screens, adjusting screws, clamp screw, and tangent screw, are not given in our description of the instrument, it is not necessary to describe them; should we attempt it, there is danger that the spirit and clearness of the description would be lost in the multitude of words.

bring it along to the other object; bring them as near as possible, then gently clamp the index.

Hold up the instrument again, in the plane between the two objects, and view one object through the transparent part of the horizon glass; and when the instrument is in the right position, the image of the other object will appear also in the same field of view, and then with the tangent screw, make the limb of the reflected object just touch the other, as it moves past it to and fro, by the gentle motion of the instrument. When the observer is satisfied that he has got the measure as near as he can, he cries out, *mark*, and his assistants mark the time by the watch, and the altitudes of the objects are also marked for the same time, if required, and observers are present to take them.

The first experiments in the use of this instrument, other than measuring a simple altitude, are generally failures, but a little practice will establish dexterity, skill, and confidence.

We are now prepared to give examples for finding latitude.

Let it be remembered, that latitude is the observer's zenith distance from the equator, and the nautical almanac gives the distance of all the heavenly bodies from the equator, under the name of Declination. We can therefore observe our zenith distance from any celestial object, and then apply its declination, and we shall have our zenith distance from the equator, *which is the latitude*.

#### EXAMPLES.

1. On a certain day, the meridian\* altitude of the sun's lower limb was observed to be  $31^{\circ} 44'$ , bearing south. At that time its declination was  $7^{\circ} 25' 8''$  south, semi-diameter  $16' 9''$ , index error  $+2' 12''$ , height of the eye 17 feet. What was the latitude?

Ans.  $50^{\circ} 48'$  north.

---

\*To obtain the meridian altitude of the sun, the observer commences observations before noon, while the sun is still rising; driving the index forward as fast as the image appears to rise, and there will come a time, a few minutes in succession, in which the image appears to rest on the horizon, neither rises nor falls, but at length the image will fall; then the observer knows that noon has passed, and the greatest apparent altitude will be shown by reading the index.

|               |         |        |                 |            |
|---------------|---------|--------|-----------------|------------|
| Semi-diameter | +16' 9" | N.A.   | Alt. ob.        | 31° 44'    |
| Index error   | + 2.12  |        | Correction      | 12' 46"    |
| Refraction    | — 1.31  | Table. | Alt. ☉'s center | 31 56 46   |
| Dip           | — 4.04  | Table. |                 | 90°        |
| Sum           | +12'46" |        | ☉'s zenith dis. | 58° 3' 14" |
|               |         |        | Dec. south      | 7° 25' 8"  |
|               |         |        | Latitude north  | 50° 48' 6" |

In this example, if the meridian altitude had been observed in the north, in place of the south, what then would have been the observer's latitude ?      Ans. 65° 28' 22" south.

We may note the following

**RULE.** — *Subtract the corrected altitude from 90°. Then if the observer and the object are both on the same side of the equator, add the declination, but if on different sides, subtract the declination, and the sum or difference will be the latitude of the observer.*

Find the latitude from each of the following meridian observations:

|   | Object.          | Alt. ob. | Direc. | S. D.   | Height. | Declination.   | Latitude.      |
|---|------------------|----------|--------|---------|---------|----------------|----------------|
| 1 | Sun <i>L. L.</i> | 45° 27'  | South  | 16' 15" | 20 feet | 17° 19' 31" S. | 27° 2' 51" N.  |
| 2 | " <i>L. L.</i>   | 81° 43'  | South  | 15' 47" | 14 "    | 22° 13' 7" N.  | 30° 18' 10" N. |
| 3 | Jupiter          | 73° 17'  | South  |         | 17 "    | 24° 10' 13" S. | 7° 22' 52" S   |
| 4 | Saturn           | 82° 12'  | North  |         | 17 "    | 12° 9' 6" N.   | 4° 16' 54" N.  |
| 5 | Sirius           | 75° 5'   | North  |         | 18 "    | 16° 31' S.     | 31° 30' 26" S. |
| 6 | Sun <i>U. L.</i> | 40° 42'  | North  | 16' 17" | 16 "    | 23° 22' S.     | 73° 1' 19" S.  |
| 7 | Sun <i>L. L.</i> | 87° 29'  | South  | 16' 17" | 16 "    | 22° 9' S.      | 19° 50' 12" S. |
| 8 | Sun <i>L. L.</i> | 15° 45'  | South  | 16' 0"  | 16 "    | 4° 43' N.      | 69° 23' 16" N  |

In this table *L. L.* indicates lower limb, *U. L.* upper limb, *S. D.* semi-diameter, *N.* north, *S.* south, *Direc.* direction. In these examples, the instrument is supposed to have no index error.

Night observations at sea are of little value, for it is very seldom that the horizon can be defined, unless it is in bright moon-light, in the tropical climates.

For this reason, very few navigators attempt to find the latitude, by observations on the planets and fixed stars.

Occasionally, however, when one of the bright planets, or a conspicuous fixed star, comes to the meridian in the morning or evening, twilight observations can be made on them, and the latitude deduced.

Some navigators apply a summary correction to the sun's lower limb, for semi-diameter, dip, and refraction, such as is comprised in the following table.

| Sun's Obs.<br>Altitude. | Correction to be added to the Observed Altitude of the<br>Sun's Lower Limb, to find the True Altitude. |               |                |                |               |                |      |      |      |      |      |      |      |      |      |  |
|-------------------------|--------------------------------------------------------------------------------------------------------|---------------|----------------|----------------|---------------|----------------|------|------|------|------|------|------|------|------|------|--|
|                         | Height of the Eye above the Sea in Feet.                                                               |               |                |                |               |                |      |      |      |      |      |      |      |      |      |  |
|                         | 6                                                                                                      | 8             | 10             | 12             | 14            | 16             | 18   | 20   | 22   | 24   | 26   | 28   | 30   | 32   | 34   |  |
| 0                       | '                                                                                                      | '             | '              | '              | '             | '              | '    | '    | '    | '    | '    | '    | '    | '    | '    |  |
| 5                       | 3.8                                                                                                    | 3.5           | 3.1            | 2.8            | 2.5           | 2.3            | 2.1  | 1.8  | 1.6  | 1.4  | 1.2  | 1.0  | 0.8  | 0.6  | 0.5  |  |
| 6                       | 5.3                                                                                                    | 4.9           | 4.6            | 4.3            | 4.0           | 3.7            | 3.5  | 3.3  | 3.0  | 2.8  | 2.6  | 2.4  | 2.2  | 2.1  | 1.9  |  |
| 7                       | 6.4                                                                                                    | 6.0           | 5.7            | 5.4            | 5.1           | 4.8            | 4.6  | 4.4  | 4.1  | 3.9  | 3.7  | 3.5  | 3.3  | 3.2  | 3.0  |  |
| 8                       | 7.2                                                                                                    | 6.8           | 6.5            | 6.2            | 5.9           | 5.7            | 5.4  | 5.3  | 5.0  | 4.8  | 4.6  | 4.4  | 4.2  | 4.0  | 3.9  |  |
| 9                       | 7.9                                                                                                    | 7.5           | 7.2            | 6.9            | 6.6           | 6.4            | 6.1  | 5.9  | 5.7  | 5.5  | 5.3  | 5.1  | 4.9  | 4.7  | 4.5  |  |
| 10                      | 8.5                                                                                                    | 8.1           | 7.8            | 7.5            | 7.2           | 6.9            | 6.7  | 6.5  | 6.2  | 6.0  | 5.8  | 5.6  | 5.4  | 5.3  | 5.1  |  |
| 11                      | 8.9                                                                                                    | 8.6           | 8.2            | 7.9            | 7.6           | 7.4            | 7.2  | 6.9  | 6.7  | 6.5  | 6.3  | 6.1  | 5.9  | 5.7  | 5.6  |  |
| 12                      | 9.3                                                                                                    | 9.0           | 8.7            | 8.3            | 8.0           | 7.8            | 7.6  | 7.3  | 7.1  | 6.9  | 6.7  | 6.5  | 6.3  | 6.2  | 6.0  |  |
| 14                      | 9.9                                                                                                    | 9.6           | 9.2            | 8.9            | 8.7           | 8.4            | 8.2  | 7.9  | 7.7  | 7.5  | 7.3  | 7.1  | 6.9  | 6.8  | 6.6  |  |
| 16                      | 10.4                                                                                                   | 10.1          | 9.7            | 9.4            | 9.1           | 8.9            | 8.7  | 8.4  | 8.2  | 8.0  | 7.8  | 7.0  | 7.4  | 7.2  | 7.1  |  |
| 18                      | 10.8                                                                                                   | 10.4          | 10.1           | 9.8            | 9.5           | 9.3            | 9.0  | 8.8  | 8.6  | 8.4  | 8.2  | 8.0  | 7.8  | 7.6  | 7.5  |  |
| 20                      | 11.1                                                                                                   | 10.7          | 10.4           | 10.1           | 9.8           | 9.6            | 9.3  | 9.1  | 8.9  | 8.7  | 8.5  | 8.2  | 8.1  | 7.9  | 7.7  |  |
| 22                      | 11.4                                                                                                   | 11.0          | 10.7           | 10.4           | 10.1          | 9.8            | 9.6  | 9.4  | 9.1  | 8.9  | 8.7  | 8.5  | 8.3  | 8.2  | 8.0  |  |
| 26                      | 11.7                                                                                                   | 11.4          | 11.0           | 10.7           | 10.5          | 10.2           | 10.0 | 9.7  | 9.5  | 9.3  | 9.1  | 8.9  | 8.7  | 8.6  | 8.4  |  |
| 30                      | 12.0                                                                                                   | 11.7          | 11.3           | 11.0           | 10.8          | 10.5           | 10.3 | 10.0 | 9.8  | 9.6  | 9.4  | 9.2  | 9.0  | 8.9  | 8.7  |  |
| 35                      | 12.3                                                                                                   | 11.9          | 11.6           | 11.3           | 11.0          | 10.7           | 10.6 | 10.3 | 10.1 | 9.9  | 9.7  | 9.4  | 9.2  | 9.2  | 9.0  |  |
| 40                      | 12.5                                                                                                   | 12.2          | 11.8           | 11.5           | 11.3          | 11.0           | 10.8 | 10.5 | 10.3 | 10.1 | 9.9  | 9.7  | 9.5  | 9.4  | 9.2  |  |
| 45                      | 12.7                                                                                                   | 12.4          | 12.0           | 11.7           | 11.5          | 11.2           | 11.0 | 10.7 | 10.5 | 10.2 | 10.1 | 9.8  | 9.7  | 9.6  | 9.4  |  |
| 50                      | 12.8                                                                                                   | 12.5          | 12.2           | 11.9           | 11.6          | 11.3           | 11.1 | 10.9 | 10.6 | 10.4 | 10.3 | 10.0 | 9.8  | 9.7  | 9.5  |  |
| 55                      | 13.0                                                                                                   | 12.6          | 12.3           | 12.0           | 11.7          | 11.5           | 11.2 | 11.0 | 10.7 | 10.5 | 10.3 | 10.1 | 9.9  | 9.8  | 9.6  |  |
| 60                      | 13.1                                                                                                   | 12.7          | 12.4           | 12.1           | 11.8          | 11.6           | 11.3 | 11.1 | 10.9 | 10.6 | 10.4 | 10.2 | 10.1 | 9.9  | 9.7  |  |
| 65                      | 13.2                                                                                                   | 12.8          | 12.5           | 12.2           | 11.9          | 11.7           | 11.4 | 11.2 | 11.0 | 10.7 | 10.5 | 10.3 | 10.1 | 10.0 | 9.8  |  |
| 70                      | 13.3                                                                                                   | 12.9          | 12.6           | 12.3           | 12.0          | 11.8           | 11.5 | 11.3 | 11.0 | 10.8 | 10.6 | 10.4 | 10.2 | 10.1 | 9.9  |  |
| 75                      | 13.4                                                                                                   | 13.1          | 12.7           | 12.4           | 12.1          | 11.9           | 11.7 | 11.4 | 11.2 | 11.0 | 10.8 | 10.6 | 10.4 | 10.2 | 10.1 |  |
| 80                      | 13.6                                                                                                   | 13.2          | 12.9           | 12.6           | 12.3          | 12.0           | 11.8 | 11.6 | 11.3 | 11.1 | 10.9 | 10.7 | 10.5 | 10.4 | 10.2 |  |
| Monthly<br>Correction   | Jan.<br>+0'.3                                                                                          | Feb.<br>+0'.2 | Mar.<br>+0'.1  | April,<br>0'.0 | May,<br>-0'.1 | June,<br>-0'.2 |      |      |      |      |      |      |      |      |      |  |
| for Sun's<br>Semi-diam. | July,<br>-0'.3                                                                                         | Aug.<br>-0'.2 | Sept.<br>-0'.1 | Oct.<br>+0'.1  | Nov.<br>+0'.2 | Dec.<br>+0'.3  |      |      |      |      |      |      |      |      |      |  |

The most practical method of obtaining the latitude by observation, other than the meridian altitude of the sun, is by the meridian altitude of the moon; but to correct the observed altitude for semi-diameter, parallax, refraction, and dip, and do it to the utmost accuracy, requires more computation and attention than the mere practical navigator is disposed to give. Moreover, such like accuracy is not required in practical navigation. To know the latitude within a mile is all the ship master requires; and this can be done in a very summary manner, by observing the moon's meridian altitude and using, the following tables, according as the lower\* or upper limb of the moon is observed.

\* The bright limb, is the one observed, whether it be the upper or lower.

These tables make but one correction for semi-diameter, parallax and refraction

TABLE I.  
CORRECTIONS to be added to the OBSERVED ALTITUDE of the Moon's lower limb.

| Part 1st. HORIZONTAL PARALLAX. |      |      |      |      |      |      |      |      |      |
|--------------------------------|------|------|------|------|------|------|------|------|------|
| Alt.                           | 53'  | 54'  | 55'  | 56'  | 57'  | 58'  | 59'  | 60'  | 61'  |
| 6                              | 0.59 | 1.0  | 1.1  | 1.3  | 1.4  | 1.5  | 1.6  | 1.8  | 1.9  |
| 8                              | 1.0  | 1.2  | 1.3  | 1.4  | 1.6  | 1.7  | 1.8  | 1.9  | 1.11 |
| 10                             | 1.1  | 1.3  | 1.4  | 1.5  | 1.7  | 1.8  | 1.9  | 1.10 | 1.12 |
| 15                             | 1.2  | 1.3  | 1.5  | 1.6  | 1.7  | 1.9  | 1.10 | 1.11 | 1.12 |
| 20                             | 1.2  | 1.3  | 1.4  | 1.5  | 1.6  | 1.8  | 1.9  | 1.10 | 1.11 |
| 25                             | 1.0  | 1.2  | 1.3  | 1.4  | 1.5  | 1.6  | 1.7  | 1.9  | 1.10 |
| 30                             | 0.59 | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  | 1.7  | 1.8  |
| 35                             | 0.57 | 0.59 | 0.59 | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  |
| 40                             | 0.55 | 0.55 | 0.56 | 0.57 | 0.58 | 0.59 | 1.0  | 1.1  | 1.2  |
| 45                             | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 0.56 | 0.57 | 0.58 | 0.59 |
| 50                             | 0.48 | 0.49 | 0.50 | 0.51 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 |
| 55                             | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 0.49 | 0.49 | 0.50 | 0.51 |
| 60                             | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.44 | 0.45 | 0.46 | 0.47 |
| 65                             | 0.36 | 0.37 | 0.38 | 0.39 | 0.39 | 0.40 | 0.40 | 0.41 | 0.42 |
| 70                             | 0.33 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 |
| 75                             | 0.28 | 0.28 | 0.29 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.32 |
| 80                             | 0.24 | 0.24 | 0.24 | 0.25 | 0.25 | 0.25 | 0.26 | 0.26 | 0.27 |
| 85                             | 0.19 | 0.19 | 0.19 | 0.20 | 0.20 | 0.21 | 0.21 | 0.21 | 0.22 |

TABLE II. CORRECTIONS to be applied to OBSERVED ALTITUDE of the Moon's upper limb

| Part 2nd. HORIZONTAL PARALLAX. |       |       |       |       |       |       |       |       |       |
|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Alt.                           | 53'   | 54'   | 55'   | 56'   | 57'   | 58'   | 59'   | 60'   | 61'   |
| 10                             | +0.33 | +0.33 | +0.34 | +0.34 | +0.34 | +0.36 | +0.37 | +0.37 | +0.38 |
| 15                             | 0.33  | 0.33  | 0.35  | 0.35  | 0.35  | 0.37  | 0.37  | 0.39  | 0.39  |
| 20                             | 5.32  | 0.33  | 0.34  | 0.35  | 0.35  | 0.36  | 0.37  | 0.37  | 0.38  |
| 26                             | 0.30  | 0.32  | 0.32  | 0.33  | 0.33  | 0.34  | 0.35  | 0.35  | 0.36  |
| 30                             | 0.29  | 0.30  | 0.31  | 0.31  | 0.32  | 0.32  | 0.33  | 0.34  | 0.34  |
| 36                             | 0.26  | 0.26  | 0.27  | 0.27  | 0.28  | 0.28  | 0.29  | 0.29  | 0.30  |
| 40                             | 0.24  | 0.25  | 0.26  | 0.26  | 0.27  | 0.27  | 0.28  | 0.29  | 0.29  |
| 46                             | 0.19  | 0.22  | 0.22  | 0.22  | 0.23  | 0.24  | 0.24  | 0.24  | 0.26  |
| 50                             | 0.17  | 0.19  | 0.19  | 0.20  | 0.20  | 0.21  | 0.21  | 0.21  | 0.21  |
| 55                             | 0.14  | 0.15  | 0.16  | 0.16  | 0.16  | 0.16  | 0.17  | 0.17  | 0.17  |
| 60                             | 0.10  | 0.11  | 0.12  | 0.12  | 0.12  | 0.12  | 0.13  | 0.13  | 0.13  |
| 65                             | 0.6   | 0.7   | 0.7   | 0.8   | 0.8   | 0.8   | 0.8   | 0.8   | 0.9   |
| 70                             | 0.3   | 0.3   | 0.3   | 0.3   | 0.3   | 0.3   | 0.3   | 0.3   | 0.4   |
| 75                             | -0.1  | -0.1  | -0.1  | -0.1  | -0.1  | -0.2  | -0.2  | -0.2  | -0.2  |
| 80                             | 0.6   | 0.6   | 0.6   | 0.6   | 0.6   | 0.6   | -0.6  | 0.7   | 0.7   |
| 85                             | 0.10  | 0.11  | 0.11  | 0.11  | 0.11  | 0.11  | 0.11  | 0.12  | 0.12  |
| Height of the eye,             | 4ft.  |       | 9ft.  |       | 16ft. |       | 25ft. |       | 36ft. |
| Dip of the Horizon,            | -2'   |       | -3'   |       | -4'   |       | -5'   |       | -6'   |

#### EXAMPLES.

1. In longitude about  $45^{\circ}$  west, on the 5th of January, 1852, at about 11h. in the evening, I observed the altitude of the moon's lower limb as she passed the meridian, and found it to be  $68^{\circ} 12'$  from the south, height of the eye 16 feet. What was my latitude?

On the 5th of Jan., at 11h. evening, lon. 45 west, corresponds to 2 hours after midnight at Greenwich.

From the Nautical Almanac, we find, that,

At midnight of the 5th, the moon's horizontal parallax was - - 57' 40"

At noon of the 6th, - - - - - 58' 1"

Therefore, by proportion, the horizontal parallax at the time of observation, must have been 57' 43".

Moon's declination at midnight of the 5th, (N. Almanac),  $21^{\circ} 47' 53''$  N.

" " noon of the 6th, - - - - -  $22^{\circ} 16' 55''$  N

Variation in 12 hours, - - - - -  $29' 2''$

Therefore, the variation for 2 hours, was not far from -  $4' 50''$

Hence the dec. at the time of observation was, - -  $22^{\circ} 52' 43''$  N.

We enter table 1, and under the parallax, and opposite to the altitude *as near as we can find them*, we perceive that 37' must be about the correction for the altitude.

|         |                            |                        |
|---------|----------------------------|------------------------|
| Whence, | Observed alt. <i>L. L.</i> | $68^{\circ} 12'$       |
|         | Correction,                | $+ 37$                 |
|         |                            | <hr/> 68 49            |
|         | Dip. always sub.           | $- 4$                  |
|         |                            | <hr/> 68 45            |
|         |                            | 90                     |
|         | Zenith dis. $\searrow$     | $21 15$                |
|         | $\searrow$ 's dec.         | $22 53$                |
|         |                            | <hr/>                  |
|         | Lat. in                    | $44^{\circ} 8'$ North. |

Find the true altitude of the moon's center, in each of the following examples. *L. L.* means lower limb ; *U. L.* upper limb.

|                            | Observed Alt.    | H. P.      | Height of the eye. | Ans. True Alt.           |
|----------------------------|------------------|------------|--------------------|--------------------------|
| 1. $\searrow$ <i>L. L.</i> | $53^{\circ} 23'$ | $58' 14''$ | 14 feet            | $54^{\circ} 10'$ nearly. |
| 2. $\searrow$ <i>L. L.</i> | $48^{\circ} 58'$ | $60' 27''$ | 19 "               | $49^{\circ} 48'$ "       |
| 3. $\searrow$ <i>U. L.</i> | $57^{\circ} 11'$ | $54' 30''$ | 20 "               | $57^{\circ} 19'$ "       |
| 4. $\searrow$ <i>L. L.</i> | $63^{\circ} 38'$ | $55' 29''$ | 12 "               | $64^{\circ} 14'$ "       |
| 5. $\searrow$ <i>U. L.</i> | $20^{\circ} 3'$  | $54' 14''$ | 16 "               | $20^{\circ} 32'$ "       |
| 6. $\searrow$ <i>L. L.</i> | $16^{\circ} 2'$  | $59' 38''$ | 23 "               | $17^{\circ} 12'$ "       |

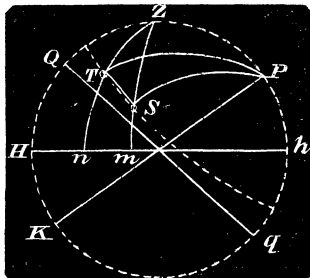
When the weather makes it doubtful whether meridian observations can be obtained, navigator's resort to double altitudes, or to the altitudes of two objects taken at the same time. We shall only show



the principle on which this method is founded ; it is the application of spherical trigonometry.

Let  $Pp$  be the earth's axis,  $Qq$  the equator. Suppose the sun to be the object, and let its position be  $S$  and  $T$  at two different times.

The elapsed time measures the angle  $SPT$ . In the triangle  $PTS$ , we have the two sides  $PT$ ,  $PS$ , and the included angle, from which we compute the side  $TS$ , and the angle  $TSP$ .



Subtracting the altitudes  $Sm$  and  $Tn$  from  $90^\circ$ , we have  $ZS$ , and  $ZT$ , then we have all the sides of the triangle  $ZTS$ , from which we compute the angle  $TSZ$ . Subtracting this angle from  $TSP$ , gives us the angle  $ZSP$ . Now, in the triangle  $ZSP$ , we have the two sides  $ZS$ ,  $SP$ , and their included angle, from which we compute  $PZ$  the complement of the latitude.

If the ship sails, during the interval between the observations, a correction will be required for the first altitude, and such corrections are found by the traverse table ; a nautical mile in the direction of the sun, corresponds to one minute of a degree, to be applied to the altitude. When the proper correction is made, the result is equivalent to having both altitudes taken at the last station, and the deduced latitude is the latitude of that station.

## CHAPTER II.

### LONGITUDE.

LONGITUDE, from celestial observations, is measured by time. A place  $15^\circ$  west of another, will have noon one hour of absolute time later ; if  $30^\circ$  west, the local time, noon will be two hours later, &c., &c.;  $15^\circ$  corresponding to an hour in time. Therefore, if we have any way of determining the times at two places, corresponding to the same absolute instant, the difference of such times will

correspond to the difference of longitude between the two places at the rate of  $15^{\circ}$  to an hour, or 4 minutes to a degree.

A *perfect* time piece will keep the time at any *particular meridian*, and by carrying that *perfect* time piece with us, by it we can see the time at that *particular meridian*; and then if we can find the time at the place where we are, the comparison of these two times will give the difference of longitude, that is, the difference between our longitude and that of the *particular meridian*, to which the time piece refers.

For instance, a gentleman leaves Boston; his watch is a perfect time piece, and it is set to Boston time, he travels west on the rail-roads, his watch all the while shows Boston time; when it is twelve o'clock by his watch it is really so in Boston, but not so at the place where he is. The sun has arrived at the meridian of Boston, but not yet at the meridian of Albany, or Buffalo, or Detroit; and when the gentleman arrives at any of these places, or any intermediate place, the local time, compared with the time in Boston, will give the longitude of that locality from Boston, counting *one* degree for every four minutes in the difference of time.

Unfortunately, however, there is no such thing as a *perfect time piece*, but some do approximate toward perfection. Such ones, made with the greatest care and solely for accuracy in rate of motion, are called *chronometers*; they are supposed to keep time within certain known limits, and in the place of perfect time keepers, they are used at sea for finding longitude.

Chronometers show the time at the distant place, it then remains to find the time at ship, and this is done most accurately by spherical trigonometry, as will soon appear.

The sun's altitude is greatest just at apparent noon, but by observations we cannot define just the moment when that takes place; hence meridian observations, valuable as they are for latitudes, are worth nothing for time, when time is to be settled to anything like accuracy.

The best position of the sun (or any other celestial object) for an observation to find local time, is when it is nearly east or west, and its altitude more than ten degrees.

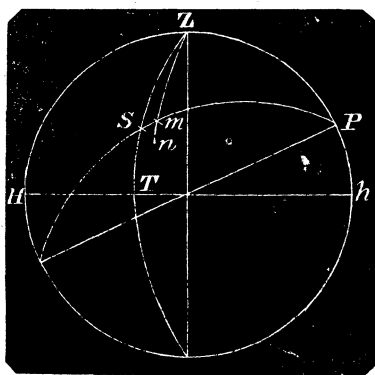
In such circumstances, an observer can find the local time

within 5 or 6 seconds, by taking an altitude of the sun, provided he at the same time knows his latitude and the sun's polar distance.

The operation is a beautiful application of spherical trigonometry, and it is illustrated by the following figure.

Let  $Z$  be the zenith of the observer,  $P$  the pole,  $S$  the position of the sun, and  $PS$  the sun's polar distance.\*

When  $S$  comes on to the meridian, it is then apparent noon; and the angle  $ZPS$  of the triangle  $ZPS$  measures the interval from apparent noon, at the rate of four minutes to one degree.



The side  $PS$  is the polar distance, the side  $ZS$  is the co-altitude, and the side  $PZ$  is the co-latitude.

Now, in every treatise on spherical trigonometry, it is demonstrated as a fundamental principle, that

*The cosine of any angle, of a spherical triangle, is equal to the cosine of its opposite side, diminished by the rectangle of the cosines of the adjacent sides, divided by the rectangle of the sines of the adjacent sides.*

$$\text{That is, } \cos. P = \frac{\cos. ZS - \cos. PZ \cos. PS}{\sin. PZ \sin. PS}$$

Now, in place of  $\cos. ZS$ , we take its equal,  $\sin. ST$ , or the sine of the altitude, and in place of  $\cos. PZ$ , we take its equal, the sine of the latitude.

In short, let  $A$  = the altitude,  $L$  = the latitude, and  $D$  = the polar distance.

$$\text{Then } \cos. P = \frac{\sin. A - \sin. L \cos. D}{\cos. L \sin. D}$$

---

\* When the observer is in the northern hemisphere, the polar distance is counted from the north pole; when in the southern hemisphere, from the south pole.

From a general equation, in plane trigonometry, we have

$$2 \sin^2 \frac{1}{2} P = 1 - \cos. P$$

Substituting the value of  $\cos. P$ , in this last equation, we have

$$\begin{aligned} 2 \sin^2 \frac{1}{2} P &= 1 - \frac{\sin. A - \sin. L \cos. D}{\cos. L \sin. D} \\ &= \frac{(\cos. L \sin. D + \sin. L \cos. D) - \sin. A}{\cos. L \sin. D} \end{aligned}$$

By comparing the quantity in parentheses with eq. (7), plane trigonometry, we perceive that

$$2 \sin^2 \frac{1}{2} P = \frac{\sin. (L+D) - \sin. A}{\cos. L \sin. D}$$

Considering  $(L+D)$  to be a single arc, and then applying equation (18), plane trigonometry, and dividing by 2, we shall have

$$\sin^2 \frac{1}{2} P = \frac{\cos. \left( \frac{L+D+A}{2} \right) \sin. \left( \frac{L+D-A}{2} \right)}{\cos. L \sin. D.}$$

But  $\frac{L+D-A}{2} = \frac{L+D+A}{2} - A$ , and now if we put

$$S = \frac{L+D+A}{2} \text{ we shall have}$$

$$\sin^2 \frac{1}{2} P = \frac{\cos. S \sin. (S-A)}{\cos. L \sin. D}$$

Or,  $\sin. \frac{1}{2} P = \sqrt{\frac{\cos. S \sin. (S-A)}{\cos. L \sin. D}}$

This is the final result when radius is unity, when it is  $R$  times greater, then the  $\sin. \frac{1}{2} P$  will be  $R$  times greater, and if  $R$  represents the radius of our tables, to correspond with these tables we must multiply the second member by  $R$ , and if we put it under the radical sign, we must multiply by  $R^2$ ; in short we shall have,

$$\sin. \frac{1}{2} P = \sqrt{\left( \frac{R}{\cos. L} \right) \left( \frac{R}{\sin. D} \right) \cos. S \sin. (S-A)}$$

The right hand member of this equation, shows *four* distinct logarithms; thus,  $\frac{R}{\cos. L}$  is the cosine of the latitude subtracted from 10, which we shall call cosine complement.

This equation furnishes the following rule for finding apparent local time, when the sun's altitude, its polar distance, and the latitude of the observer, are given.

The *altitude must be observed*, the latitude must be known, and the Nautical Almanac will furnish the polar distance.

**RULE.**—1. *Add together the altitude, latitude, and polar distance; take the half sum, and from the said half sum subtract the altitude, thus finding the remainder.*

2. *The logarithms. Find the cosine complement of the latitude, the sine complement of the polar distance, the cosine of the half sum, and the sine of the remainder.*

3. *Add these four logarithms together, and divide by 2, the logarithm thus found, is the sine of half the polar angle, or half the sun's meridian distance.*

4. *Take out the arc corresponding to this sine, and divide its double by 15 (as in compound division in arithmetic), and the quotient will be the hours, minutes, and seconds from apparent noon; and if the sun is east of the meridian, the hours, minutes, and seconds, must be subtracted from 12 hours, for the corresponding time of day.*

The time shown by a chronometer or a perfect clock, or rather graduation of clocks, is to mean and not to apparent time, and to convert apparent into mean time, the equation\* of time is given in the Nautical Almanac for the noon of every day at Greenwich. The amount of it, reduced or modified to correspond to the time of observation, can be applied to apparent time, and the mean time of taking the observation will be determined. The difference between this time and the mean time at Greenwich, as determined by the chronometer, will be the longitude. The longitude will be *west*, if the time at Greenwich is *latest* in the day, otherwise it will be *east*.

If the observer is on land, without a sea horizon, and uses a reflecting instrument, he must have an artificial horizon. A proper artificial horizon, is a small dish of mercury, with a glass roof to put over it, to keep the mercury from being agitated by the wind. In place of the mercury, a plate of molasses will answer. In still calm weather any clear pool of water is a good artificial horizon.

In either of these, the reflected image of the object appears as much below the horizon as it is above it, and to measure the altitude,

---

\* For the theory of equation of time, see works on astronomy.

the image reflected by the mirror of the instrument must be carried to the image in the artificial horizon; half of the angle shown by the index will be the apparent altitude. In using an artificial horizon there is no dip, other corrections are to be applied according to circumstances.

EXAMPLES UNDER THE PRECEDING RULE.

1. Being at sea, May 20th, 1823, in latitude  $43^{\circ} 30' N.$ , and in longitude about  $20^{\circ}$  west, I observed the altitude of the sun's lower limb, and found it to be  $32^{\circ} 4'$  rising, when an assistant marked the time per watch, at 7h. 43m. A. M.; height of the eye 16 feet. What was the true mean time?

Just before the observation, the watch was compared with the chronometer in the cabin\*, and found to be 1 hour, 21 minutes, and 12 seconds slow of chronometer.

On the 8th of May, the chronometer was 3m. 7s. fast of Greenwich time, and gaining 1s.6 daily. What was the longitude?

|                        |             |                                  |   |         |                 |
|------------------------|-------------|----------------------------------|---|---------|-----------------|
| ☉ <i>S. D.</i> , - - - | + 15' 49"   |                                  |   |         | <i>H. M. S.</i> |
| Dip., - - -            | - 3 56      | Watch, - - -                     | - | 7 43 0  |                 |
| Ref., - - -            | - 1 30      | Diff., - - -                     | - | 1 21 12 |                 |
| Correction, - - -      | + 10 23     | Face of chron. at ob., -         | - | 9 4 12  |                 |
| Observation, - - -     | 32 4        | Error 3m. 7s., increase of error |   |         |                 |
| Alt. ☉ center, - - -   | 32° 14' 23" | in 12 days 19s., whole error, -  | - | 3 26    |                 |
|                        |             | Greenwich time, - - -            | - | 9 0 46  |                 |

At noon on the 20th of May, 1823, the sun's declination, by the *N. A.*, was  $19^{\circ} 52' 18''$  north, increasing at the rate of  $30''.6$  per hour, and the time of taking the observation was 3 hours before noon at Greenwich; therefore, the declination must have been  $19^{\circ} 50' 47'' N.$

|                |             |             |          |
|----------------|-------------|-------------|----------|
| Altitude,      | 32° 14' 23" |             |          |
| Lat.,          | 43 30       | cos. com.   | .139435  |
| <i>P. D.</i> , | 70 9 13     | sin. com.   | .026603  |
|                | 2)145 53 36 |             |          |
| <i>S.</i>      | 72 56 48    | cosine      | 9.467253 |
|                | 32 14 23    |             |          |
| ( <i>S—A</i> ) | 40 42 25    | sine        | 9.814363 |
|                |             | 2)19.447657 |          |
|                | 31° 58' 8"  | sin.        | 9.723828 |

\* Chronometers should never be, and by careful persons, never are, taken out of their places during a voyage.

|                               |                            |               |
|-------------------------------|----------------------------|---------------|
|                               | 31° 58' 8"                 |               |
|                               | 2                          |               |
|                               | <hr/>                      |               |
|                               | 63° 56' 16" =              | 4h. 15m. 45s. |
|                               | 12                         |               |
| Apparent time, - - -          | 7 44 15                    | A. M.         |
| Equation of time N. A. - -    | — 3 51                     |               |
| Mean time at ship, - - -      | 7 40 24                    |               |
| Watch, - - - - -              | 7 43                       |               |
| Watch too fast, - - -         | 2m. 36s.                   |               |
| Time at Greenwich per chron., | 9h. 0m. 46s.               |               |
| Time at ship per observation, | 7 40 24                    |               |
| Diff., - - - - -              | 1 20 22 = 20° 5' west lon. |               |

2. August 10th, 1824, in latitude  $54^{\circ} 12'$  north, at 2h 33m per watch, height of the eye 18 feet, I observed the altitude of the sun's upper limb  $16^{\circ} 50'$  falling. My chronometer was 2h 20m 37s fast of the watch; and on the 7th, the same month, the chronometer was 40m 29.4s fast of Greenwich time, gaining  $7\frac{5}{8}$  seconds daily. What was the error of the watch, and the longitude per chronometer?

*Preparation.*

|                           |              |       |
|---------------------------|--------------|-------|
| Time per watch -          | 5h. 33m. 0s. | P. M. |
| Diff. per chron. - -      | 2 20 37      |       |
| Face of chronometer -     | 7 53 37      | P. M. |
| Chron. fast (whole error) | —40 52       |       |
| Greenwich, mean time,     | 7 12 45      | P. M. |

On the 10 of August, 1824, the sun's declination at noon, Greenwich time, was  $15^{\circ} 32' 14''$  north, decreasing at the rate of  $45''$  per hour, as given in the Nautical Almanac. The decrease for  $7\frac{1}{2}$  hours must be  $5^{\circ} 24''$ ; whence, the declination at the time of observation,  $15^{\circ} 26' 50''$  N., and the polar distance  $74^{\circ} 33' 10''$ .

|                        |                       |                                          |
|------------------------|-----------------------|------------------------------------------|
| Observed altitude -    | $16^{\circ} 50' 00''$ | Equation of time, per N. A., Aug.        |
| Semi-diameter, N. A. - | —15 48                | 10, 1824, was - - - $+5m. 2s.$           |
| Dip and Ref. - - -     | —7 20                 | Hourly decrease $1\frac{3}{8}s.$ - - - 2 |
| True alt. center - -   | $16^{\circ} 37' 52''$ | Equation at ob. - - - $5m. 0s.$          |

We now leave the problem to be worked through by the pupil, giving only the answer.

Ans. Watch slow of local mean time, 3m 27s.

Longitude by chronometer,  $24^{\circ} 4' 30''$  west.

3. When it was 6h 0m 21s, P. M., mean time, at Greenwich, by my chronometer, I observed the altitude of the sun's lower limb to be  $30^{\circ} 17'$ , in the afternoon of January 12th, 1852. At noon our

latitude, by a meridian observation, was  $21^{\circ} 47'$  north, and since that time we have made 11 miles of southing, by the log. The dip was  $4'$ , and semi-diameter  $16' 17''$ . What was the longitude by chronometer?

|                                                      |   |                              |
|------------------------------------------------------|---|------------------------------|
| Sun's Declination Jan. 12, '52, at noon, G. T.       | - | $21^{\circ} 44' 10''$ south. |
| Hourly decrease, per N. A., $25''$ , giving          | - | $-2' 30''$                   |
| Declination at the time of observation               | - | $21^{\circ} 41' 40''$ south  |
| Equation of time at noon, Greenwich                  | - | $+8m. 25s.$                  |
| Hourly increase $\frac{9}{10}\%$ of a second, making | - | $6s.$ nearly                 |
| Equation at time of observation (to add)             | - | $+8m. 31s.$                  |

Were we sure that pupils would have access to nautical almanacs, we would give neither declination nor equation of time.

Ans. Lon.  $45^{\circ} 39'$  west.

4. On the 16th of January, 1852, when my chronometer showed 11h 27m 41s, A. M., for the mean time at Greenwich, I observed the altitude of the sun's lower limb and found it  $32^{\circ} 21'$  rising, height of the eye 16 feet, latitude  $0^{\circ} 41'$  south. What was the longitude by chronometer?

By the N. A., the sun's declination at that time was  $21^{\circ} 2' 36''$  south, and the equation of time 9m. 53s. additive.

Ans. Lon.  $46^{\circ} 39'$  west.

N. B.—Time at any place, is but the difference between the right ascension of the meridian and the right ascension of the sun; and to find the time from these two elements, we always subtract the right ascension of the sun from the right ascension of the meridian, increasing the latter by 24 hours, to render subtraction possible, when necessary.

The right ascensions of the stars are given, and the right ascension of the sun is given, in the Nautical Almanac, for the noon of every day in the year, Greenwich time. Now, if we can find the meridian distance of any known star, by observation, we can establish the right ascension of the meridian, and, consequently, the local time. Hence, we can find longitude by comparing the chronometer with the altitudes of the stars, as the following example will illustrate.

5. If on the 8th of March, 1852, when my chronometer showed the Greenwich time to be 7h 22m 3s, P. M., I found by observation,



that the true altitude of Sirius was  $37^{\circ} 52'$  west of the meridian. My latitude was  $32^{\circ} 28'$  south. What was the time at ship, and what was my longitude; the elements for computation being as follows?

1. Right ascension of the star - - 6h. 38m. 38s.
2. Declination of the star  $16^{\circ} 31'$  south
3. Right ascension of the sun - - 23h. 17m. 25s.

By means of the triangle we find,

|                                                        |   |    |                         |
|--------------------------------------------------------|---|----|-------------------------|
| The meridian distance of the star                      | - | -  | 3h. 40m. 58s.           |
| To which add $\star$ 's R. A., because $\star$ is west | 6 | 38 | 38                      |
| Right ascension of the meridian                        | - | -  | 10 19 36                |
| Add                                                    | - | -  | 24                      |
|                                                        |   |    | <hr/>                   |
|                                                        |   |    | 34 19 36                |
| Subtract the R. A. of the sun                          | - | -  | 23 17 25                |
| Diff. is apparent time at ship                         | - | -  | 11 2 11 P. M.           |
| Equation of time, add                                  | - | -  | 10 48                   |
|                                                        |   |    | <hr/>                   |
| Mean time at ship                                      | - | -  | 11 12 59                |
| Time at Greenwich                                      | - | -  | 7 22 3                  |
|                                                        |   |    | <hr/>                   |
| Longitude in time                                      | - | -  | 3 50 56                 |
|                                                        |   |    | = $57^{\circ} 44'$ east |

N. B.—When the chronometer remains in the same place for a week or more, its rate can be determined by comparing it with the observed altitudes of the sun, taken from day to day. In different climates the same chronometer will have different rates, and on returning to its original station it will frequently resume its original rate.

For azimuths, and variations of the compass, see page 106.

## CHAPTER III.

### LUNAR OBSERVATIONS.

A good and well-tried chronometer is a valuable and reliable instrument for finding the longitude at sea, during short runs; but still it is but an instrument, and is not one of the reliable works of

nature. Near the end of a long voyage, the best of chronometers very frequently give false longitude, and in such cases, good navigators always resort to lunar observations, which from the hands of a good observer, can be relied upon to within 10 or 12 minutes of a degree, and they usually come within 5 or 6 miles, and sometimes even more exact, but that is accidental and unfrequent.

To comprehend the theory of lunars, we must call to mind the fact that the moon moves through the heavens, apparently among the stars, at the rate of more than  $13^{\circ}$  in a day, and any angular distance it may have from the sun or any star corresponds to some moment of Greenwich time.

About three days before and after the change of the moon, she is too near the sun to be visible, but at all other times, her distance from the sun, some of the larger planets, and certain bright fixed stars, called lunar stars,\* which lie near her path, are computed and put down in the nautical almanac, for every third hour of mean Greenwich time commencing at noon. For any particular day, the distances are given to such objects only, east and west of her, as will be convenient to measure with the common instruments.

The distances put down in the nautical almanac, are such as would be seen if viewed from the center of the earth; but observers are always on the surface of the earth, and the distances thence observed, must always be reduced to equivalent distances seen from the center, and this reduction is called *working a lunar*, which is generally the highest scientific ambition of the young navigator. †

The true distance between the sun and moon, or between a star and the moon, can be deduced from the apparent distance by the application of spherical trigonometry.

The moon is never seen by an observer in its *true place*, unless the observer is in a line between the center of the earth and the moon, that is, unless the moon is in the zenith of the observer; in all other

---

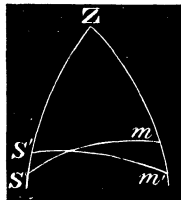
\* There are nine lunar stars, Arietis, Aldebaran, Pollax, Regulus, Spica, Antares, Aquilæ, Fomalhaut, and Pegasi.

† Many navigators, both old and young, direct all their efforts to knowing how to do, without attempting to comprehend the reasons for so doing; and this the world calls *practical*, — a complete perversion of the term. On the other hand, some men of the schools spend their energies in metaphysical nothings, splitting hairs in logic, and calling it scientific; this is equally a perversion.

positions, the moon is depressed by parallax, and appears nearer to those stars that are below her, and further from those stars that are above her, than would appear from the center of the earth. Therefore, the apparent altitudes of the two objects, must be taken at the same time that their distance asunder is measured. The altitudes must be corrected for parallax and refraction, thus obtaining the true altitudes.

The annexed figure is a general representation of the triangles pertaining to a lunar observation.

Let  $Z$  be the zenith of an observer,  $S'$  the apparent place of the sun or star, and  $S$  its true place. Also, let  $m'$  be the apparent place of the moon, and  $m$  its true place as seen from the center of the earth.



Here are two distinct triangles,  $ZS'm'$ , and  $ZSm$ . The apparent altitudes subtracted from  $90^\circ$ , give  $ZS'$  and  $Zm'$ , and  $S'm'$  is the *apparent* distance; with these three sides, the angle  $Z$  can be found. Correcting the altitudes, and subtracting them from  $90^\circ$ , will give the sides  $ZS$  and  $Zm$ ; these two sides, and their included angle at  $Z$ , will give the side  $Sm$ , which is the *true distance*.

The definite true distance must have a definite Greenwich time, which can be readily found; and this, compared with the local time deduced from an altitude of the sun, will of course give the *longitude*.

We shall now make a formula to clear the distance.

Let  $S'$  = the apparent altitude of the sun or star,  
 and  $S$  = the true altitude. Also,  
 Let  $m'$  = the apparent altitude of the moon,  
 and  $m$  = the true altitude.

Observe that the letters with the accent, indicate *apparent*, and without the accent, the *true* altitudes.

Put  $d$  to represent the apparent distance, and  $x$  to represent the true distance.

Bear in mind, that the sine of an altitude is the same as the cosine of its zenith distance, and conversely, the sine of a zenith distance is the same thing as the cosine of the corresponding altitude.

Now, by the fundamental equation of spherical trigonometry noted in the last chapter, we have

$$\cos. Z = \frac{\cos. d - \sin. S' \sin. m'}{\cos. S' \cos. m'}. \text{ Also } \cos. Z = \frac{\cos. x - \sin. S \sin. m}{\cos. S \cos. m}.$$

$$\text{Whence, } \frac{\cos. d - \sin. S' \sin. m'}{\cos. S' \cos. m'} = \frac{\cos. x - \sin. S \sin. m}{\cos. S \cos. m}.$$

By adding unity to each member we have

$$1 + \frac{\cos. d - \sin. S' \sin. m'}{\cos. S' \cos. m'} = 1 + \frac{\cos. x - \sin. S \sin. m}{\cos. S \cos. m}.$$

$$\frac{(\cos. S' \cos. m' - \sin. S' \sin. m') + \cos. d}{\cos. S' \cos. m'} = \frac{(\cos. S \cos. m - \sin. S \sin. m) + \cos. x}{\cos. S \cos. m}.$$

By observing equation 9, plane trigonometry, we perceive that the preceding equation reduces to

$$\frac{\cos. (S' + m') + \cos. d}{\cos. S' \cos. m'} = \frac{\cos. (S + m) + \cos. x}{\cos. S \cos. m},$$

$$\text{Whence } \cos. x = (\cos. (S' + m') + \cos. d) \frac{\cos. S \cos. m}{\cos. S' \cos. m'} - \cos. (S + m).$$

It is here important to notice that the moon's horizontal parallax given in the Nautical Almanac, is the equatorial horizontal parallax; that is, it corresponds to the greatest radius of the earth. The diameter of the earth through any other latitude is less, and of course the corresponding parallax is less.

We therefore give the following table for the reduction of the equatorial horizontal parallax, to the horizontal parallax of any other latitude; it is computed on the supposition that the equatorial diameter is to its polar as 230 to 229. For example if the horizontal parallax in the Nautical Almanac is 55', in the latitude of 40° the reduction would be 6'', and the parallax reduced would be 54' 54'', and if the parallax from the Nautical Almanac were 60' the reduction would be 6'' 6, and reduced would be 59' 53'' 4.

The semi-diameter of the moon given in the Nautical Almanac is her horizontal semi-diameter, but when she is in the zenith she is nearer to us by the whole radius of the earth, about one-sixtieth part of her whole distance, consequently she must appear under a larger and larger angle as she rises from the horizon, and this is called the augmentation of the semi-diameter.

We give the reduction for the parallax; and the augmentation for the semi-diameter in the following tables:

| Red. of ☉'s Eq. hor. parallax. |              |              |
|--------------------------------|--------------|--------------|
| Lat. itude.                    | Eq. par. 55' | Eq. par. 60' |
| 20°                            | 0".9         | 1"           |
| 25                             | 2.8          | 3            |
| 30                             | 3.7          | 4            |
| 35                             | 4.6          | 5            |
| 40                             | 6.0          | 6.6          |
| 45                             | 7.3          | 8            |
| 50                             | 8.6          | 9.4          |
| 55                             | 10.1         | 11           |
| 60                             | 11           | 12           |
| 65                             | 11.8         | 13           |
| 70                             | 12.8         | 14           |
| 75                             | 13.9         | 15           |
| 80                             | 14.6         | 16           |

| Augmentation of the Moon's semi-diam. |      |
|---------------------------------------|------|
| Ap. Alt.                              | Aug. |
| 6°                                    | 2"   |
| 12                                    | 3    |
| 18                                    | 5    |
| 24                                    | 6    |
| 30                                    | 8    |
| 36                                    | 9    |
| 42                                    | 11   |
| 48                                    | 12   |
| 54                                    | 13   |
| 60                                    | 14   |
| 66                                    | 15   |
| 72                                    | 16   |
| 90                                    | 16   |

We now give an example showing all the details of finding the longitude by a lunar observation.

## EXAMPLE.

Suppose that on the 25th of January, 1852, between three and four o'clock in the afternoon, local time, the observed distance between the nearest limbs of the sun and moon was  $50^{\circ} 3' 20''$ , the altitude of the sun's lower limb was  $20^{\circ} 1'$ , and of the moon's lower limb  $48^{\circ} 57'$ , height of the eye 16 feet. The latitude corrected for the run from noon was  $34^{\circ} 12' N.$ , and the supposed longitude about  $65^{\circ}$  west. What was the longitude? (the Nautical Almanac being at hand.)

*Preparation.*

|                             |            |
|-----------------------------|------------|
| Supposed time at ship,      | H. M.      |
| Supposed longitude 65       | 3 15 P. M. |
|                             | 4 20       |
| Supposed time at Greenwich, | 7 25 P. M. |

On the 25th at noon the N. A. gives the ☉'s *S. D.* at  $14' 47''$ , and at midnight at  $14' 45''$ ; therefore at the time of observation we take it at  $14' 46''$ , by simple inspection. In the same summary manner we take the Equatorial horizontal parallax at  $54' 12''$ .

|                             |                           |                         |                          |
|-----------------------------|---------------------------|-------------------------|--------------------------|
| ☉'s semi-diameter, - - -    | 14' 46''                  | ☉'s Eq. hor. par. - - - | 54' 12''                 |
| Aug for Alt. - - -          | 12                        | Red. for lat. - - -     | 4                        |
| ☉'s true <i>S. D.</i> - - - | 14' 58''                  | Reduced hor. par. - - - | 54' 8''                  |
| Observed distance, -        | $50^{\circ} 3' 20''$      | Alt. ☉'s <i>L L</i>     | $48^{\circ} 57'$         |
| Sun's <i>S. D.</i> - - -    | 16 16                     | ☉'s <i>S. D.</i>        | 14' 58''                 |
| Moon's <i>S. D.</i> - - -   | 14 58                     | Dip                     | — 3 56                   |
| Apparent central dis.       | $50^{\circ} 34' 34'' = d$ | ☉'s app. alt.           | $49^{\circ} 8' 2'' = m.$ |

|                    |                    |                                      |               |
|--------------------|--------------------|--------------------------------------|---------------|
| Alt. ☉'s lower $L$ | 20° 1'             | N. B. To find the moon's parallax in |               |
| Semi-diameter,     | +16' 16"           | altitude see problems on page 201.   |               |
| Dip,               | — 3' 56"           | ☉'s app. Alt. 49° 8'                 | ocs. 9.815778 |
| ☉'s app. alt.      | 20° 13' 12" = $S'$ | 54' 8" = 3488'                       | log. 3.542576 |
| Refraction,        | — 2' 34"           | 36' 2" = 2282                        | 3.358354      |
| ☉'s true alt.      | 20° 10' 38" $S$ .  |                                      |               |
|                    | ☉'s app. alt.      | 49° 8' 2"                            |               |
|                    | Parallax in alt.   | 36' 2                                |               |
|                    | Refraction,        | —49                                  |               |
|                    | True alt.          | 49° 43' 15" = $m$                    |               |

$$(S' + m') = 69^\circ 21' 14'' \quad (S + m) = 69^\circ 53' 53''.$$

We are now prepared to apply the equation to compute the true distance. The equation requires the use of natural sines and cosines.

$$\cos. x = (\cos. (S' + m') + \cos. d) \frac{\cos. S' \cos. m}{\cos. S' \cos. m'} - \cos. (S + m)$$

$$(S' + m') = 69^\circ 21' 14'' \text{ N. cos. } .35259$$

$$d = 50^\circ 34' 34'' \text{ N. cos. } * .63449$$

$$.98708 \log. -1.994350$$

$$S = 20^\circ 10' 38'' \log. \cos. 9.972496$$

$$m = 49^\circ 43' 15'' \log. \cos. 9.810578$$

$$S' = 20^\circ 13' 12'' \cos. \text{com. } 0.017626$$

$$m' = 49^\circ 8' 2'' \cos. \text{com. } 0.184228$$

$$\text{Num. } .97561 \log. -1.989278$$

$$= \text{sum less } 20. \dagger$$

$$\text{N. cos. } (S + m) = 69^\circ 53' 53'' - .34369$$

$$\text{True distance, } 50^\circ 48' 29'' \cos. 63192$$

In the Nautical Almanac, we find that at 6 P. M. mean Greenwich time, on said day, the true distance between the sun and moon was  $49^\circ 59' 26''$ , and at 9 P. M., the distance was  $51^\circ 20' 49''$ , showing a change of  $1^\circ 21' 23''$  in three hours of time. But the change

\* When  $d$  is greater than  $90^\circ$  its cosine becomes *minus*, and its numerical value is then the natural sine of the excess over  $90^\circ$ . Thus if  $d$  were  $105^\circ$ , its cosine would be numerically equal to the sine of  $15^\circ$ , and must then be subtracted from the cosine of the sum of apparent altitudes. The result ( $\cos. x$ ) would then be the sine of the excess over  $90^\circ$ .

† Less 20 because the table of natural sines is to radius unity, and we used  $\cos. S$  and  $\cos. m$  to the radius of 10, making two tens to take away.

from  $49^{\circ} 59' 26''$  to  $50^{\circ} 48' 29''$  is  $49' 3''$ ; and now on the supposition that the change is in proportion to the time (and it is very nearly), we have the following analogy

$$1^{\circ} 21' 23'' : 49' 3'' :: 3h. : t$$

Or,  $4883 : 2943 :: 3 : 1h. 48m. 29s.$

That is, the time that this observation was taken 1h 48m 29s after 6 at Greenwich. Or, 7h 48m 29s mean Greenwich time.

With the true altitude of the sun  $20^{\circ} 10' 38''$ , the latitude  $34^{\circ} 12'$ , and the polar distance  $109^{\circ} 0' 48''$ , we find the apparent time at ship 3h 10m 5s, to which we would add the equation of time, 12m 34, making the mean time 3h 22m 39s.

|                         |     |      |      |                             |
|-------------------------|-----|------|------|-----------------------------|
| From the Greenwich time | 7h. | 48m. | 29s. |                             |
| Sub. time at ship       | -   | 3    | 22   | 39                          |
|                         |     |      |      |                             |
| Giving lon. in time     | -   | 4    | 25   | $50=66^{\circ} 27' 38'' W.$ |

West, because the time at Greenwich was later in the day.

If a lunar is taken with a star, or with the sun, when the sun is not in a proper position to depend upon its altitude for local time, the time must be noted by a watch, and the difference between the watch and true time made known, by a previous or subsequent observation on the sun, or some star which is nearly east or west of the observer.

The most material part of working a lunar is that of clearing the distance. We, therefore, give the following examples, without the little incidental details.

We show the working of one in which the distance is greater than  $90^{\circ}$ .

The apparent distance between the center of the sun and moon on a certain occasion, was  $98^{\circ} 12'$ ; the apparent altitude of the sun's center was  $74^{\circ} 10'$ , and of the moon's  $20^{\circ} 37'$ ; the moon's horizontal parallax at the same time was  $57' 12''$ . What was the true distance?

Ans.  $97^{\circ} 34' 27''$

|                                        |      |          |  |
|----------------------------------------|------|----------|--|
| Horizontal par. $57' 12''=3432$        | log. | 3.532547 |  |
| D Alt. $20^{\circ} 37'$                | cos. | 9.971256 |  |
| Parallax in alt. $53^{\circ} 31'=3211$ | log. | 3.586803 |  |

$\odot$ 's app. alt.  $20^{\circ} 37'$        $\odot$  app. alt.  $74^{\circ} 10' 0''$   
 Refraction  $-2 31$       Refraction  $16$   
 Parallax  $+53 31$        $\odot$ 's true alt.  $74^{\circ} 9' 44''$   
 $\odot$ 's True alt.  $21^{\circ} 28'$

$$(S' + m') = 94^{\circ} 47' \quad (S + m) = 95^{\circ} 37' 44''$$

$$94^{\circ} 47' \text{ Nat. cos. } -0.08339$$

$$d = 98^{\circ} 12' \text{ Nat. cos. } -0.14263$$

$$-0.22602 \text{ log. } -1.356150^*$$

$$S = 74^{\circ} 9' 44'' \text{ cos. } - \quad - \quad 9.436021$$

$$m = 21^{\circ} 28' 00'' \text{ cos. } - \quad - \quad 9.968777$$

$$S' = 74^{\circ} 10' 00'' \text{ cos. complement } 0.364092$$

$$m' = 20^{\circ} 37' 00'' \text{ cos. complement } 0.028744$$

$$-0.23007 \text{ log. } -1.361784$$

$$\text{cos. } (S + m) = 95^{\circ} 37' 66'' + 0.09826\dagger$$

$$\text{True dis. } 97^{\circ} 34' 27'' \text{ N. cos. } -0.13181$$

## EXAMPLES FOR PRACTICE.

| No. | Ap. alt. of sun<br>or a fixed star. | Moon's ap.<br>altitude. | Apparent cen-<br>tral distance. | Moon's<br>hor. par. | True<br>distances.    |
|-----|-------------------------------------|-------------------------|---------------------------------|---------------------|-----------------------|
| 1   | $\odot$ $86^{\circ} 3'$             | $39^{\circ} 18'$        | $46^{\circ} 45' 0''$            | $53' 51''$          | $46^{\circ} 4' 25''$  |
| 2   | $\star$ $29^{\circ} 47'$            | $57^{\circ} 22'$        | $27^{\circ} 35' 0''$            | $60' 3''$           | $28^{\circ} 8' 24''$  |
| 3   | $\odot$ $31^{\circ} 14'$            | $28^{\circ} 7'$         | $14^{\circ} 21' 30''$           | $53' 29''$          | $14^{\circ} 9' 24''$  |
| 4   | $\odot$ $60^{\circ} 5'$             | $63^{\circ} 12'$        | $51^{\circ} 3' 21''$            | $58' 30''$          | $50^{\circ} 41' 15''$ |
| 5   | $\star$ $34^{\circ} 28'$            | $10^{\circ} 42'$        | $49^{\circ} 18' 38''$           | $61' 11''$          | $48^{\circ} 45' 39''$ |
| 6   | $\odot$ $8^{\circ} 26'$             | $19^{\circ} 24'$        | $120^{\circ} 18' 46''$          | $57' 14''$          | $120^{\circ} 1' 46''$ |
| 7   | $\star$ $43^{\circ} 27'$            | $40^{\circ} 9'$         | $18^{\circ} 21' 35''$           | $60' 20''$          | $18^{\circ} 8' 12''$  |
| 8   | $\star$ $53^{\circ} 13'$            | $57^{\circ} 32'$        | $60^{\circ} 13' 49''$           | $60' 52''$          | $59^{\circ} 48' 12''$ |
| 9   | $\odot$ $72^{\circ} 26'$            | $18^{\circ} 30'$        | $81^{\circ} 2' 28''$            | $60' 58''$          | $80^{\circ} 9' 33''$  |
| 10  | $\odot$ $60^{\circ} 33'$            | $9^{\circ} 26'$         | $70^{\circ} 36' 16''$           | $59' 57''$          | $69^{\circ} 49' 12''$ |

\* The factor 0.22602 being minus renders the product minus.

† The cos.  $(S + m)$  in the equation is minus, but where the value of  $(S + m)$  is more than  $90^{\circ}$ , the *minus* necessarily becomes *plus*.



## APPENDIX.

---

It is comparatively an easy matter, to conduct a survey, or navigate a vessel, when there are no important difficulties to be overcome ; but the true test of knowledge or skill in any pursuit, is to be found only in real adversity. The mariner who successfully manages his ship, when every thing is provided, when all is in order, and the weather favorable, is deserving of little credit ; but let the ship become disabled, and the storm terrific, and then there is scope for the exercise of every necessary acquirement, and its kindred talent.

So it is with the man of science ; when every instrument is at hand, and all in order, it requires little skill, and but common knowledge, to make observations and experiments ; but when we reverse the case ; the tact, knowledge, and ingenuity of the man, may oft times more or less overcome the difficulties.

For instance, suppose it were necessary to find the altitude of the sun, for the purpose of finding the latitude of the place (on shore), or for the purpose of finding the time ; and we had no sextant or quadrant, and in fact, no instrument to measure angles. It could be done approximately as follows :

Let a plumb line be suspended in water ; have a knot in the line, and let the knot be at a known distance above the water. The knot will cast a shadow on the water ; measure the distance of this shadow from the plumb line. The knot and its shadow, with the plumb line and water, will form a right angled triangle, and the angle at the base, computed by plane trigonometry, will be the altitude of the sun's *upper limb*, and this altitude may be used for any purpose, the same as if it were measured by a sextant, but the accuracy is not to be depended upon, for the want of delicacy in the instrument.

A person on shore having a good watch, and knowing his latitude, can regulate his watch, or at least determine its rate and error for a short period of time. Then, if he have a nautical almanac, the common tables of logarithms, and a knowledge of spherical trigonometry, and a

corresponding knowledge of astronomy, he can find the longitude by a lunar observation, *without* a sextant, as follows:

By the means of his watch and a plumb line, he will be able to range off an *approximate* meridian line. He will then observe the transits of stars, and of the moon across that meridian, taking those stars which are at that time near the moon's meridian, some to the east and some to the west of the moon, and some more north, and others more south than the moon.

He will note the difference in time, between the transit of each star and the moon, across his approximate meridian, and by a combination, or rather comparison of these observations, he will be able to determine the moon's right ascension very nearly. By the moon's right ascension, and the aid of the nautical almanac, he can find the Greenwich time.

The Greenwich time, compared with the local time, will give the longitude.

When we can find the moon in a vertical plane with any two fixed stars, and it be at the time the moon changes her declination very slowly, so that we can depend upon a declination taken from the nautical almanac for the supposed time, we can then determine the moon's right ascension, and from thence the longitude as before, whether we are on land or sea.

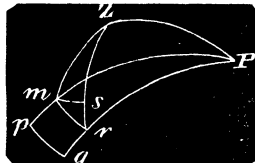
Ship-wrecked mariners, and travelers similarly situated, have frequently resorted to these artifices to obtain their approximate localities.

We have frequently remarked in the course of this work, that the best position of a celestial object, at the time of taking its altitude, for the purpose of more exactly defining the time, is when the object is nearly east or west; we now propose to show this conclusively, and therefore give the following

#### INVESTIGATION.

*To find under what circumstances, in a given latitude, a small mistake in observing or correcting the altitude of a celestial object, will produce the smallest error in the time computed from it.*

Let  $Z$  be the zenith,  $P$  the pole,  $r$  the supposed place, and  $m$  the true place of the object. Let  $ms$  be a parallel of altitude, join the points  $m$  and  $r$ , and let  $pq$  be the arc of the equator contained between the meridians  $Pm$  and  $Pr$ .



Then as  $Pm$  and  $Pr$  are equal,  $mr$  may be considered as a small portion of a parallel of declination  $rs$  will be the error in

altitude, and  $pq$  the measure of the required error in time. And as the sides of the triangle  $msr$  will necessarily be small, that triangle may be considered as a rectilinear one, right angled at  $s$ ; and because the angle  $Prm$  is also a right angle, the angles  $smr$  and  $PrZ$ , being each the complement of  $mrs$ , are equal to each other

We now have,

$$rs : mr :: \sin. smr (ZrP) : \text{rad.} \quad (1)$$

$$\text{Also, } mr : pq :: \cos. qr : \text{rad.} \quad (2)$$

Multiplying these two proportions together, omitting the common factor  $mr$ , gives,

$$rs : pq :: \cos. qr \sin. (ZrP) : (\text{rad.})^2 \quad (3)$$

$$\text{But, } \sin. rP \text{ or } \cos. qr : \sin. rZP : \sin. ZP : \sin. (ZrP) \quad (4)$$

$$\text{Whence, } \cos. qr \sin. ZrP = \sin. rZP \sin. ZP \quad (5)$$

The first member of equation (5), is the same as the third term in proportion (3); therefore, proportion (3) may be changed to the following,

$$rs : pq :: \sin. rZP \sin. ZP : (\text{rad.})^2$$

$$\text{Whence, } pq = \left( \frac{rs (\text{rad.})^2}{\sin. ZP} \right) \frac{1}{\sin. rZP}$$

Now, as the quantities in parentheses are supposed to be constant the value of  $pq$ , the error in time must vary as  $\frac{1}{\sin. rZP}$  varies; and it is obvious that  $pq$  will be *least*, when  $\sin. rZP$  is *greatest*, that is, when  $rZP=90^\circ$ , or the object due east or west.



# LOGARITHMS OF NUMBERS

FROM

1 TO 10000.

| N. | Log.     | N. | Log.     | N. | Log.     | N.  | Log.     |
|----|----------|----|----------|----|----------|-----|----------|
| 1  | 0 000000 | 26 | 1 414973 | 51 | 1 707570 | 76  | 1 880814 |
| 2  | 0 301030 | 27 | 1 431364 | 52 | 1 716003 | 77  | 1 886491 |
| 3  | 0 477121 | 28 | 1 447158 | 53 | 1 724276 | 78  | 1 892095 |
| 4  | 0 602060 | 29 | 1 462398 | 54 | 1 732394 | 79  | 1 897627 |
| 5  | 0 698970 | 30 | 1 477121 | 55 | 1 740363 | 80  | 1 903090 |
| 6  | 0 778151 | 31 | 1 491362 | 56 | 1 748188 | 81  | 1 908485 |
| 7  | 0 845098 | 32 | 1 505150 | 57 | 1 755875 | 82  | 1 913814 |
| 8  | 0 903090 | 33 | 1 518514 | 58 | 1 763428 | 83  | 1 919078 |
| 9  | 0 954243 | 34 | 1 531479 | 59 | 1 770852 | 84  | 1 924279 |
| 10 | 1 000000 | 35 | 1 544068 | 60 | 1 778151 | 85  | 1 929419 |
| 11 | 1 041393 | 36 | 1 556303 | 61 | 1 785330 | 86  | 1 934498 |
| 12 | 1 079181 | 37 | 1 568202 | 62 | 1 792392 | 87  | 1 939519 |
| 13 | 1 113943 | 38 | 1 579784 | 63 | 1 799341 | 88  | 1 944483 |
| 14 | 1 146128 | 39 | 1 591065 | 64 | 1 806180 | 89  | 1 949390 |
| 15 | 1 176091 | 40 | 1 602060 | 65 | 1 812913 | 90  | 1 954243 |
| 16 | 1 204120 | 41 | 1 612784 | 66 | 1 819544 | 91  | 1 959041 |
| 17 | 1 230449 | 42 | 1 623249 | 67 | 1 826075 | 92  | 1 963788 |
| 18 | 1 255273 | 43 | 1 633468 | 68 | 1 832509 | 93  | 1 968483 |
| 19 | 1 278754 | 44 | 1 643453 | 69 | 1 838849 | 94  | 1 973128 |
| 20 | 1 301030 | 45 | 1 653213 | 70 | 1 845098 | 95  | 1 977724 |
| 21 | 1 322219 | 46 | 1 662578 | 71 | 1 851258 | 96  | 1 982271 |
| 22 | 1 342423 | 47 | 1 672098 | 72 | 1 857333 | 97  | 1 986772 |
| 23 | 1 361728 | 48 | 1 681241 | 73 | 1 863323 | 98  | 1 991226 |
| 24 | 1 380211 | 49 | 1 690196 | 74 | 1 869232 | 99  | 1 995635 |
| 25 | 1 397940 | 50 | 1 698970 | 75 | 1 875061 | 100 | 2 000000 |

N. B. In the following table, in the last nine columns of each page, where the first or leading figures change from 9's to 0's, points or dots are now introduced instead of the 0's through the rest of the line, to catch the eye, and to indicate that from thence the corresponding natural numbers in the first column stands in the *next lower line*, and its annexed first two figures of the Logarithms in the second column.

# LOGARITHMS OF NUMBERS.

3

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 100 | 000000 | 0434 | 0868 | 1301 | 1734 | 2166 | 2598 | 3029 | 3461 | 3891 |
| 101 | 4321   | 4750 | 5181 | 5609 | 6038 | 6466 | 6894 | 7321 | 7748 | 8174 |
| 102 | 8600   | 9026 | 9451 | 9876 | .300 | .724 | 1147 | 1570 | 1993 | 2415 |
| 103 | 012837 | 3259 | 3680 | 4100 | 4521 | 4940 | 5360 | 5779 | 6197 | 6616 |
| 104 | 7033   | 7451 | 7868 | 8284 | 8700 | 9116 | 9532 | 9947 | .361 | .775 |
| 105 | 021189 | 1603 | 2016 | 2428 | 2841 | 3252 | 3664 | 4075 | 4486 | 4896 |
| 106 | 5306   | 5715 | 6125 | 6533 | 6942 | 7350 | 7757 | 8164 | 8571 | 8978 |
| 107 | 9384   | 9789 | .195 | .600 | 1004 | 1408 | 1812 | 2216 | 2619 | 3021 |
| 108 | 033424 | 3826 | 4227 | 4628 | 5029 | 5430 | 5830 | 6230 | 6629 | 7028 |
| 109 | 7426   | 7825 | 8223 | 8620 | 9017 | 9414 | 9811 | .207 | .602 | .998 |
| 110 | 041393 | 1787 | 2182 | 2576 | 2969 | 3362 | 3755 | 4148 | 4540 | 4932 |
| 111 | 5323   | 5714 | 6105 | 6495 | 6885 | 7275 | 7664 | 8053 | 8442 | 8830 |
| 112 | 9218   | 9606 | 9993 | .380 | .766 | 1153 | 1538 | 1924 | 2309 | 2694 |
| 113 | 053078 | 3463 | 3846 | 4230 | 4613 | 4996 | 5378 | 5760 | 6142 | 6524 |
| 114 | 6905   | 7286 | 7666 | 8046 | 8426 | 8805 | 9185 | 9563 | 9942 | .320 |
| 115 | 060698 | 1075 | 1452 | 1829 | 2206 | 2582 | 2958 | 3333 | 3709 | 4083 |
| 116 | 4458   | 4832 | 5206 | 5580 | 5953 | 6326 | 6699 | 7071 | 7443 | 7815 |
| 117 | 8186   | 8557 | 8928 | 9298 | 9668 | .38  | .407 | .776 | 1145 | 1514 |
| 118 | 071882 | 2250 | 2617 | 2985 | 3352 | 3718 | 4085 | 4451 | 4816 | 5182 |
| 119 | 5547   | 5912 | 6276 | 6640 | 7004 | 7368 | 7731 | 8094 | 8457 | 8819 |
| 120 | 9181   | 9543 | 9904 | .266 | .626 | .987 | 1347 | 1707 | 2067 | 2426 |
| 121 | 082785 | 3144 | 3503 | 3861 | 4219 | 4576 | 4934 | 5291 | 5647 | 6004 |
| 122 | 6360   | 6716 | 7071 | 7426 | 7781 | 8136 | 8490 | 8845 | 9198 | 9552 |
| 123 | 9905   | .258 | .611 | .963 | 1315 | 1667 | 2018 | 2370 | 2721 | 3071 |
| 124 | 093422 | 3772 | 4122 | 4471 | 4820 | 5169 | 5518 | 5866 | 6215 | 6562 |
| 125 | 6910   | 7257 | 7604 | 7951 | 8298 | 8644 | 8990 | 9335 | 9681 | 1026 |
| 126 | 100371 | 0715 | 1059 | 1403 | 1747 | 2091 | 2434 | 2777 | 3119 | 3462 |
| 127 | 3804   | 4146 | 4487 | 4828 | 5169 | 5510 | 5851 | 6191 | 6531 | 6871 |
| 128 | 7210   | 7549 | 7888 | 8227 | 8565 | 8903 | 9241 | 9579 | 9916 | .253 |
| 129 | 110590 | 0926 | 1263 | 1599 | 1934 | 2270 | 2605 | 2940 | 3275 | 3609 |
| 130 | 3943   | 4277 | 4611 | 4944 | 5278 | 5611 | 5943 | 6276 | 6608 | 6940 |
| 131 | 7271   | 7603 | 7934 | 8265 | 8595 | 8926 | 9256 | 9586 | 9915 | 0245 |
| 132 | 120574 | 0903 | 1231 | 1560 | 1888 | 2216 | 2544 | 2871 | 3198 | 3525 |
| 133 | 3852   | 4178 | 4504 | 4830 | 5156 | 5481 | 5806 | 6131 | 6456 | 6781 |
| 134 | 7105   | 7429 | 7753 | 8076 | 8399 | 8722 | 9045 | 9368 | 9690 | .112 |
| 135 | 130334 | 0655 | 0977 | 1298 | 1619 | 1939 | 2260 | 2580 | 2900 | 3219 |
| 136 | 3539   | 3858 | 4177 | 4496 | 4814 | 5133 | 5451 | 5769 | 6086 | 6403 |
| 137 | 6721   | 7037 | 7354 | 7671 | 7987 | 8303 | 8618 | 8934 | 9249 | 9564 |
| 138 | 9879   | .194 | .508 | .822 | 1136 | 1450 | 1763 | 2076 | 2389 | 2702 |
| 139 | 143015 | 3327 | 3630 | 3951 | 4263 | 4574 | 4885 | 5196 | 5507 | 5818 |
| 140 | 6128   | 6438 | 6748 | 7058 | 7367 | 7676 | 7985 | 8294 | 8603 | 8911 |
| 141 | 9219   | 9527 | 9835 | .142 | .449 | .756 | 1063 | 1370 | 1676 | 1982 |
| 142 | 152288 | 2594 | 2900 | 3205 | 3510 | 3815 | 4120 | 4424 | 4728 | 5032 |
| 143 | 5336   | 5640 | 5943 | 6246 | 6549 | 6852 | 7154 | 7457 | 7759 | 8061 |
| 144 | 8362   | 8664 | 8965 | 9266 | 9567 | 9868 | .168 | .469 | .769 | 1068 |
| 145 | 161368 | 1667 | 1967 | 2266 | 2564 | 2863 | 3161 | 3460 | 3758 | 4055 |
| 146 | 4353   | 4650 | 4947 | 5244 | 5541 | 5838 | 6134 | 6430 | 6726 | 7022 |
| 147 | 7317   | 7613 | 7908 | 8203 | 8497 | 8792 | 9086 | 9380 | 9674 | 9968 |
| 148 | 170262 | 0555 | 0848 | 1141 | 1434 | 1726 | 2019 | 2311 | 2603 | 2895 |
| 149 | 3186   | 3478 | 3769 | 4060 | 4351 | 4641 | 4932 | 5222 | 5512 | 5802 |

| N.  | 0      | 1     | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|-------|------|------|------|------|------|------|------|------|
| 150 | 176091 | 6381  | 6670 | 6959 | 7248 | 7536 | 7825 | 8113 | 8401 | 8689 |
| 151 | 8977   | 9264  | 9552 | 9839 | .126 | .413 | .699 | .985 | 1272 | 1558 |
| 152 | 181844 | 2129  | 2415 | 2700 | 2985 | 3270 | 3555 | 3839 | 4123 | 4407 |
| 153 | 4691   | 4975  | 5259 | 5542 | 5825 | 6108 | 6391 | 6674 | 6956 | 7239 |
| 154 | 7521   | 7803  | 8084 | 8366 | 8647 | 8928 | 9209 | 9490 | 9771 | .51  |
| 155 | 190332 | 0612  | 0892 | 1171 | 1451 | 1730 | 2010 | 2289 | 2567 | 2846 |
| 156 | 3125   | 3403  | 3681 | 3959 | 4237 | 4514 | 4792 | 5069 | 5346 | 5623 |
| 157 | 5899   | 6176  | 6453 | 6729 | 7005 | 7281 | 7556 | 7832 | 8107 | 8382 |
| 158 | 8657   | .8932 | 9206 | 9481 | 9755 | .29  | .303 | .577 | .850 | 1124 |
| 159 | 201397 | 1670  | 1943 | 2216 | 2488 | 2761 | 3033 | 3305 | 3577 | 3848 |
| 160 | 4120   | 4391  | 4663 | 4934 | 5204 | 5475 | 5746 | 6016 | 6286 | 6556 |
| 161 | 6826   | 7096  | 7365 | 7634 | 7904 | 8173 | 8441 | 8710 | 8979 | 9247 |
| 162 | 9515   | 9783  | .51  | .319 | .586 | .853 | 1121 | 1388 | 1654 | 1921 |
| 163 | 212188 | 2454  | 2720 | 2986 | 3252 | 3518 | 3783 | 4049 | 4314 | 4579 |
| 164 | 4844   | 5109  | 5373 | 5638 | 5902 | 6166 | 6430 | 6694 | 6957 | 7221 |
| 165 | 7484   | 7747  | 8010 | 8273 | 8536 | 8798 | 9060 | 9323 | 9585 | 9846 |
| 166 | 220108 | 0370  | 0631 | 0892 | 1153 | 1414 | 1675 | 1936 | 2196 | 2456 |
| 167 | 2716   | 2976  | 3236 | 3496 | 3755 | 4015 | 4274 | 4533 | 4792 | 5051 |
| 168 | 5309   | 5568  | 5826 | 6084 | 6342 | 6600 | 6858 | 7115 | 7372 | 7630 |
| 169 | 7887   | 8144  | 8400 | 8657 | 8913 | 9170 | 9426 | 9682 | 9938 | .193 |
| 170 | 230449 | 0704  | 0960 | 1215 | 1470 | 1724 | 1979 | 2234 | 2488 | 2742 |
| 171 | 2996   | 3250  | 3504 | 3757 | 4011 | 4264 | 4517 | 4770 | 5023 | 5276 |
| 172 | 5528   | 5781  | 6033 | 6285 | 6537 | 6789 | 7041 | 7292 | 7544 | 7795 |
| 173 | 8046   | 8297  | 8548 | 8799 | 9049 | 9299 | 9550 | 9800 | .50  | .300 |
| 174 | 240549 | 0799  | 1048 | 1297 | 1546 | 1795 | 2044 | 2293 | 2541 | 2790 |
| 175 | 3038   | 3286  | 3534 | 3782 | 4030 | 4277 | 4525 | 4772 | 5019 | 5266 |
| 176 | 5513   | 5759  | 6006 | 6252 | 6499 | 6745 | 6991 | 7237 | 7482 | 7728 |
| 177 | 7973   | 8219  | 8464 | 8709 | 8954 | 9198 | 9443 | 9687 | 9932 | .176 |
| 178 | 250420 | 0664  | 0908 | 1151 | 1395 | 1638 | 1881 | 2125 | 2368 | 2610 |
| 179 | 2853   | 3096  | 3338 | 3580 | 3822 | 4064 | 4306 | 4548 | 4790 | 5031 |
| 180 | 5273   | 5514  | 5755 | 5996 | 6237 | 6477 | 6718 | 6958 | 7198 | 7439 |
| 181 | 7679   | 7918  | 8158 | 8398 | 8637 | 8877 | 9116 | 9355 | 9594 | 9833 |
| 182 | 260071 | 0310  | 0548 | 0787 | 1025 | 1263 | 1501 | 1739 | 1976 | 2214 |
| 183 | 2451   | 2688  | 2925 | 3162 | 3399 | 3636 | 3873 | 4109 | 4346 | 4582 |
| 184 | 4818   | 5054  | 5290 | 5525 | 5761 | 5996 | 6232 | 6467 | 6702 | 6937 |
| 185 | 7172   | 7406  | 7641 | 7875 | 8110 | 8344 | 8578 | 8812 | 9046 | 9279 |
| 186 | 9513   | 9746  | 9980 | .213 | .446 | .679 | .912 | 1144 | 1377 | 1609 |
| 187 | 271842 | 2074  | 2306 | 2538 | 2770 | 3001 | 3233 | 3464 | 3696 | 3927 |
| 188 | 4158   | 4389  | 4620 | 4850 | 5081 | 5311 | 5542 | 5772 | 6002 | 6232 |
| 189 | 6462   | 6692  | 6921 | 7151 | 7380 | 7609 | 7838 | 8067 | 8296 | 8525 |
| 190 | 8754   | 8982  | 9211 | 9439 | 9667 | 9895 | .123 | .351 | .578 | .806 |
| 191 | 281033 | 1261  | 1488 | 1715 | 1942 | 2169 | 2396 | 2622 | 2849 | 3075 |
| 192 | 3301   | 3527  | 3753 | 3979 | 4205 | 4431 | 4656 | 4882 | 5107 | 5332 |
| 193 | 5557   | 5782  | 6007 | 6232 | 6456 | 6681 | 6905 | 7130 | 7354 | 7578 |
| 194 | 7802   | 8026  | 8249 | 8473 | 8696 | 8920 | 9143 | 9366 | 9589 | 9812 |
| 195 | 290035 | 0257  | 0480 | 0702 | 0925 | 1147 | 1369 | 1591 | 1813 | 2034 |
| 196 | 2256   | 2478  | 2699 | 2920 | 3141 | 3363 | 3584 | 3804 | 4025 | 4246 |
| 197 | 4466   | 4687  | 4907 | 5127 | 5347 | 5567 | 5787 | 6007 | 6226 | 6446 |
| 198 | 6665   | 6884  | 7104 | 7323 | 7542 | 7761 | 7979 | 8198 | 8416 | 8635 |
| 199 | 8853   | 9071  | 9289 | 9507 | 9725 | 9943 | .161 | .378 | .595 | .813 |

OF NUMBERS

5

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 200 | 301030 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 2980 |
| 201 | 3196   | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 |
| 202 | 5351   | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 |
| 203 | 7496   | 7710 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 |
| 204 | 9630   | 9843 | .56  | .268 | .481 | .693 | .906 | 1118 | 1330 | 1542 |
| 205 | 311754 | 1966 | 2177 | 2389 | 2600 | 2812 | 3023 | 3234 | 3445 | 3656 |
| 206 | 3867   | 4078 | 4289 | 4499 | 4710 | 4920 | 5130 | 5340 | 5551 | 5760 |
| 207 | 5970   | 6180 | 6390 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 |
| 208 | 8063   | 8272 | 8481 | 8689 | 8898 | 9106 | 9314 | 9522 | 9730 | 9938 |
| 209 | 320146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 |
| 210 | 2219   | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 |
| 211 | 4282   | 4488 | 4694 | 4899 | 5105 | 5310 | 5516 | 5721 | 5926 | 6131 |
| 212 | 6336   | 6541 | 6745 | 6950 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 |
| 213 | 8380   | 8583 | 8787 | 8991 | 9194 | 9398 | 9601 | 9805 | .8   | .211 |
| 214 | 330414 | 0617 | 0819 | 1022 | 1225 | 1427 | 1630 | 1832 | 2034 | 2236 |
| 215 | 2438   | 2640 | 2842 | 3044 | 3246 | 3447 | 3649 | 3850 | 4051 | 4253 |
| 216 | 4454   | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 6260 |
| 217 | 6460   | 6660 | 6860 | 7060 | 7260 | 7459 | 7659 | 7858 | 8058 | 8257 |
| 218 | 8456   | 8656 | 8855 | 9054 | 9253 | 9451 | 9650 | 9849 | .47  | .246 |
| 219 | 340444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 1830 | 2028 | 2225 |
| 220 | 2423   | 2620 | 2817 | 3014 | 3212 | 3409 | 3606 | 3802 | 3999 | 4196 |
| 221 | 4392   | 4589 | 4785 | 4981 | 5178 | 5374 | 5570 | 5766 | 5962 | 6157 |
| 222 | 6353   | 6549 | 6744 | 6939 | 7135 | 7330 | 7525 | 7720 | 7915 | 8110 |
| 223 | 8305   | 8500 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 9860 | .54  |
| 224 | 350248 | 0442 | 0636 | 0829 | 1023 | 1216 | 1410 | 1603 | 1796 | 1989 |
| 225 | 2183   | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 |
| 226 | 4108   | 4301 | 4493 | 4685 | 4876 | 5068 | 5260 | 5452 | 5643 | 5834 |
| 227 | 6026   | 6217 | 6408 | 6599 | 6790 | 6981 | 7172 | 7363 | 7554 | 7744 |
| 228 | 7935   | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 |
| 229 | 9835   | .25  | .215 | .404 | .593 | .783 | .972 | 1161 | 1350 | 1539 |
| 230 | 361728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 |
| 231 | 3612   | 3800 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 |
| 232 | 5488   | 5675 | 5862 | 6049 | 6236 | 6423 | 6610 | 6796 | 6983 | 7169 |
| 233 | 7356   | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 9030 |
| 234 | 9216   | 9401 | 9587 | 9772 | 9958 | .143 | .328 | .513 | .698 | .883 |
| 235 | 371068 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 2360 | 2544 | 2728 |
| 236 | 2912   | 3096 | 3280 | 3464 | 3647 | 3831 | 4015 | 4198 | 4382 | 4565 |
| 237 | 4748   | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 6029 | 6212 | 6394 |
| 238 | 6577   | 6759 | 6942 | 7124 | 7306 | 7488 | 7670 | 7852 | 8034 | 8216 |
| 239 | 8398   | 8580 | 8761 | 8943 | 9124 | 9306 | 9487 | 9668 | 9849 | .30  |
| 240 | 380211 | 0392 | 0573 | 0754 | 0934 | 1115 | 1296 | 1476 | 1656 | 1837 |
| 241 | 2017   | 2197 | 2377 | 2557 | 2737 | 2917 | 3097 | 3277 | 3456 | 3636 |
| 242 | 3815   | 3995 | 4174 | 4353 | 4533 | 4712 | 4891 | 5070 | 5249 | 5428 |
| 243 | 5606   | 5785 | 5964 | 6142 | 6321 | 6499 | 6677 | 6856 | 7034 | 7212 |
| 244 | 7390   | 7568 | 7746 | 7923 | 8101 | 8279 | 8456 | 8634 | 8811 | 8989 |
| 245 | 9166   | 9343 | 9520 | 9698 | 9875 | .51  | .228 | .405 | .582 | .759 |
| 246 | 390935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 |
| 247 | 2697   | 2873 | 3048 | 3224 | 3400 | 3575 | 3751 | 3926 | 4101 | 4277 |
| 248 | 4452   | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 5850 | 6025 |
| 249 | 6199   | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 250 | 397940 | 8114 | 8287 | 8461 | 8634 | 8808 | 8981 | 9154 | 9328 | 9501 |
| 251 | 9674   | 9847 | .20  | .192 | .365 | .538 | .711 | .883 | 1056 | 1228 |
| 252 | 401401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 |
| 253 | 3121   | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 4320 | 4492 | 4663 |
| 254 | 4834   | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 6370 |
| 255 | 6540   | 6710 | 6881 | 7051 | 7221 | 7391 | 7561 | 7731 | 7901 | 8070 |
| 256 | 8240   | 8410 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 |
| 257 | 9933   | .102 | .271 | .440 | .609 | .777 | .946 | 1114 | 1283 | 1451 |
| 258 | 411620 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 |
| 259 | 3300   | 3467 | 3635 | 3803 | 3970 | 4137 | 4305 | 4472 | 4639 | 4806 |
| 260 | 4973   | 5140 | 5307 | 5474 | 5641 | 5808 | 5974 | 6141 | 6308 | 6474 |
| 261 | 6641   | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 7970 | 8135 |
| 262 | 8301   | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 9460 | 9625 | 9791 |
| 263 | 9956   | .121 | .286 | .451 | .616 | .781 | .945 | 1110 | 1275 | 1439 |
| 264 | 421604 | 1788 | 1933 | 2097 | 2261 | 2426 | 2590 | 2754 | 2918 | 3082 |
| 265 | 3246   | 3410 | 3574 | 3737 | 3901 | 4065 | 4228 | 4392 | 4555 | 4718 |
| 266 | 4882   | 5045 | 5208 | 5371 | 5534 | 5697 | 5860 | 6023 | 6186 | 6349 |
| 267 | 6511   | 6674 | 6836 | 6999 | 7161 | 7324 | 7486 | 7648 | 7811 | 7973 |
| 268 | 8135   | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 |
| 269 | 9752   | 9914 | .75  | .236 | .398 | .559 | .720 | .881 | 1042 | 1203 |
| 270 | 431364 | 1525 | 1685 | 1846 | 2007 | 2167 | 2328 | 2488 | 2649 | 2809 |
| 271 | 2969   | 3130 | 3290 | 3450 | 3610 | 3770 | 3930 | 4090 | 4249 | 4409 |
| 272 | 4569   | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 |
| 273 | 6163   | 6322 | 6481 | 6640 | 6800 | 6957 | 7116 | 7275 | 7433 | 7592 |
| 274 | 7751   | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 |
| 275 | 9333   | 9491 | 9648 | 9806 | 9964 | .122 | .279 | .437 | .594 | .752 |
| 276 | 440909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 |
| 277 | 2480   | 2637 | 2793 | 2950 | 3106 | 3263 | 3419 | 3576 | 3732 | 3889 |
| 278 | 4045   | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | 5137 | 5293 | 5449 |
| 279 | 5604   | 5760 | 5915 | 6071 | 6226 | 6382 | 6537 | 6692 | 6848 | 7003 |
| 280 | 7158   | 7313 | 7468 | 7623 | 7778 | 7933 | 8088 | 8242 | 8397 | 8552 |
| 281 | 8706   | 8861 | 9015 | 9170 | 9324 | 9478 | 9633 | 9787 | 9941 | .95  |
| 282 | 450249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 1326 | 1479 | 1633 |
| 283 | 1786   | 1940 | 2093 | 2247 | 2400 | 2553 | 2706 | 2859 | 3012 | 3165 |
| 284 | 3318   | 3471 | 3624 | 3777 | 3930 | 4082 | 4235 | 4387 | 4540 | 4692 |
| 285 | 4845   | 4997 | 5150 | 5302 | 5454 | 5606 | 5758 | 5910 | 6062 | 6214 |
| 286 | 6366   | 6518 | 6670 | 6821 | 6973 | 7125 | 7276 | 7428 | 7579 | 7731 |
| 287 | 7882   | 8033 | 8184 | 8336 | 8487 | 8638 | 8789 | 8940 | 9091 | 9242 |
| 288 | 9392   | 9543 | 9694 | 9845 | 9995 | .146 | .296 | .447 | .597 | .748 |
| 289 | 460898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 2098 | 2248 |
| 290 | 2398   | 2548 | 2697 | 2847 | 2997 | 3146 | 3296 | 3445 | 3594 | 3744 |
| 291 | 3893   | 4042 | 4191 | 4340 | 4490 | 4639 | 4788 | 4936 | 5085 | 5234 |
| 292 | 5383   | 5532 | 5680 | 5829 | 5977 | 6126 | 6274 | 6423 | 6571 | 6719 |
| 293 | 6868   | 7016 | 7164 | 7312 | 7460 | 7608 | 7756 | 7904 | 8052 | 8200 |
| 294 | 8347   | 8495 | 8643 | 8790 | 8938 | 9085 | 9233 | 9380 | 9527 | 9675 |
| 295 | 9822   | 9969 | .116 | .263 | .410 | .557 | .704 | .851 | .998 | 1145 |
| 296 | 471292 | 1438 | 1585 | 1732 | 1878 | 2025 | 2171 | 2318 | 2464 | 2610 |
| 297 | 2756   | 2903 | 3049 | 3195 | 3341 | 3487 | 3633 | 3779 | 3925 | 4071 |
| 298 | 4216   | 4362 | 4508 | 4653 | 4799 | 4944 | 5090 | 5235 | 5381 | 5526 |
| 299 | 5671   | 5816 | 5962 | 6107 | 6252 | 6397 | 6542 | 6687 | 6832 | 6976 |

## OF NUMBERS.

7

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9     |
|-----|--------|------|------|------|------|------|------|------|------|-------|
| 300 | 477121 | 7266 | 7411 | 7555 | 7700 | 7844 | 7989 | 8133 | 8278 | 8422  |
| 301 | 8566   | 8711 | 8855 | 8999 | 9143 | 9287 | 9481 | 9675 | 9719 | 9863  |
| 302 | 480007 | 0151 | 0294 | 0438 | 0582 | 0725 | 0869 | 1012 | 1156 | 1299  |
| 303 | 1443   | 1586 | 1729 | 1872 | 2016 | 2159 | 2302 | 2445 | 2588 | 2731  |
| 304 | 2874   | 3016 | 3159 | 3302 | 3445 | 3587 | 3730 | 3872 | 4015 | 4157  |
| 305 | 4300   | 4442 | 4585 | 4727 | 4869 | 5011 | 5153 | 5295 | 5437 | 5579  |
| 306 | 5721   | 5863 | 6005 | 6147 | 6289 | 6430 | 6572 | 6714 | 6855 | 6997  |
| 307 | 7138   | 7280 | 7421 | 7563 | 7704 | 7845 | 7986 | 8127 | 8269 | 8410  |
| 308 | 8551   | 8692 | 8833 | 8974 | 9114 | 9255 | 9396 | 9537 | 9678 | 9818  |
| 309 | 9959   | .99  | .239 | .380 | .520 | .661 | .801 | .941 | 1081 | 1222  |
| 310 | 491362 | 1502 | 1642 | 1782 | 1922 | 2062 | 2201 | 2341 | 2481 | 2621  |
| 311 | 2760   | 2900 | 3040 | 3179 | 3319 | 3458 | 3597 | 3737 | 3876 | 4015  |
| 312 | 4155   | 4294 | 4433 | 4572 | 4711 | 4850 | 4989 | 5128 | 5267 | 5406  |
| 313 | 5544   | 5683 | 5822 | 5960 | 6099 | 6238 | 6376 | 6515 | 6653 | 6791  |
| 314 | 6930   | 7068 | 7206 | 7344 | 7483 | 7621 | 7759 | 7897 | 8035 | 8173  |
| 315 | 8311   | 8448 | 8586 | 8724 | 8862 | 8999 | 9137 | 9275 | 9412 | 9550  |
| 316 | 9687   | 9824 | 9962 | .99  | .236 | .374 | .511 | .648 | .785 | .922  |
| 317 | 501059 | 1196 | 1333 | 1470 | 1607 | 1744 | 1880 | 2017 | 2154 | 2291  |
| 318 | 2427   | 2564 | 2700 | 2837 | 2973 | 3109 | 3246 | 3382 | 3518 | 3655  |
| 319 | 3791   | 3927 | 4063 | 4199 | 4335 | 4471 | 4607 | 4743 | 1878 | 5014  |
| 320 | 5150   | 5286 | 5421 | 5557 | 5693 | 5828 | 5964 | 6099 | 6234 | 6370  |
| 321 | 6505   | 6640 | 6776 | 6911 | 7046 | 7181 | 7316 | 7451 | 7586 | 7721  |
| 322 | 7856   | 7991 | 8126 | 8260 | 8395 | 8530 | 8664 | 8799 | 8934 | 9068  |
| 323 | 9203   | 9337 | 9471 | 9606 | 9740 | 9874 | ...9 | .143 | .277 | .411  |
| 324 | 510545 | 0679 | 0813 | 0947 | 1081 | 1215 | 1349 | 1482 | 1616 | 1750  |
| 325 | 1883   | 2017 | 2151 | 2284 | 2418 | 2551 | 2684 | 2818 | 2951 | 3084  |
| 326 | 3218   | 3351 | 3484 | 3617 | 3750 | 3883 | 4016 | 4149 | 4282 | 4414  |
| 327 | 4548   | 4681 | 4813 | 4946 | 5079 | 5211 | 5344 | 5476 | 5609 | 5741  |
| 328 | 5874   | 6006 | 6139 | 6271 | 6403 | 6535 | 6668 | 6800 | 6932 | 7064  |
| 329 | 7196   | 7328 | 7460 | 7592 | 7724 | 7855 | 7987 | 8119 | 8251 | 8382  |
| 330 | 8514   | 8646 | 8777 | 8909 | 9040 | 9171 | 9303 | 9434 | 9566 | 9697  |
| 331 | 9828   | 9959 | .90  | .221 | .353 | .484 | .615 | .745 | .876 | 1007  |
| 332 | 521138 | 1269 | 1400 | 1530 | 1661 | 1792 | 1922 | 2053 | 2183 | 2314  |
| 333 | 2444   | 2575 | 2705 | 2835 | 2966 | 3096 | 3226 | 3356 | 3486 | 3616  |
| 334 | 3746   | 3876 | 4006 | 4136 | 4266 | 4396 | 4526 | 4656 | 4785 | 4915  |
| 335 | 5045   | 5174 | 5304 | 5434 | 5563 | 5693 | 5822 | 5951 | 6081 | 6210  |
| 336 | 6339   | 6469 | 6598 | 6727 | 6856 | 6985 | 7114 | 7243 | 7372 | 7501  |
| 337 | 7630   | 7759 | 7888 | 8016 | 8145 | 8274 | 8402 | 8531 | 8660 | 8788  |
| 338 | 8917   | 9045 | 9174 | 9302 | 9430 | 9559 | 9687 | 9815 | 9943 | ...72 |
| 339 | 530200 | 0328 | 0456 | 0584 | 0712 | 0840 | 0968 | 1096 | 1223 | 1351  |
| 340 | 1479   | 1607 | 1734 | 1862 | 1960 | 2117 | 2245 | 2372 | 2500 | 2627  |
| 341 | 2754   | 2882 | 3009 | 3136 | 3264 | 3391 | 3518 | 3645 | 3772 | 3899  |
| 342 | 4026   | 4153 | 4280 | 4407 | 4534 | 4661 | 4787 | 4914 | 5041 | 5167  |
| 343 | 5294   | 5421 | 5547 | 5674 | 5800 | 5927 | 6053 | 6180 | 6306 | 6432  |
| 344 | 6558   | 6685 | 6811 | 6937 | 7060 | 7189 | 7315 | 7441 | 7567 | 7693  |
| 345 | 7819   | 7945 | 8071 | 8197 | 8322 | 8448 | 8574 | 8699 | 8825 | 8951  |
| 346 | 9076   | 9202 | 9327 | 9452 | 9578 | 9703 | 9829 | 9954 | .79  | .204  |
| 347 | 540329 | 0455 | 0580 | 0705 | 0830 | 0955 | 1080 | 1205 | 1330 | 1454  |
| 348 | 1579   | 1704 | 1829 | 1953 | 2078 | 2203 | 2327 | 2452 | 2576 | 2701  |
| 349 | 2825   | 2950 | 3074 | 3199 | 3323 | 3447 | 3571 | 3696 | 3820 | 3944  |

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 350 | 544068 | 4192 | 4316 | 4440 | 4564 | 4688 | 4812 | 4936 | 5060 | 5183 |
| 351 | 5307   | 5431 | 5555 | 5678 | 5805 | 5925 | 6049 | 6172 | 6296 | 6419 |
| 352 | 6543   | 6666 | 6789 | 6913 | 7036 | 7159 | 7282 | 7405 | 7529 | 7652 |
| 353 | 7775   | 7898 | 8021 | 8144 | 8267 | 8389 | 8512 | 8635 | 8758 | 8881 |
| 354 | 9003   | 9126 | 9249 | 9371 | 9494 | 9616 | 9739 | 9861 | 9984 | .196 |
| 355 | 550228 | 0351 | 0473 | 0595 | 0717 | 0840 | 0962 | 1084 | 1206 | 1328 |
| 356 | 1450   | 1572 | 1694 | 1816 | 1938 | 2060 | 2181 | 2303 | 2425 | 2547 |
| 357 | 2668   | 2790 | 2911 | 3033 | 3155 | 3276 | 3393 | 3519 | 3640 | 3762 |
| 358 | 3883   | 4004 | 4126 | 4247 | 4368 | 4489 | 4610 | 4731 | 4852 | 4973 |
| 359 | 5094   | 5215 | 5346 | 5457 | 5578 | 5699 | 5820 | 5940 | 6061 | 6182 |
| 360 | 6303   | 6423 | 6544 | 6664 | 6785 | 6905 | 7026 | 7146 | 7267 | 7387 |
| 361 | 7507   | 7627 | 7748 | 7868 | 7988 | 8108 | 8228 | 8349 | 8469 | 8589 |
| 362 | 8709   | 8829 | 8948 | 9068 | 9188 | 9308 | 9428 | 9548 | 9667 | 9787 |
| 363 | 9907   | .26  | .146 | .265 | .385 | .504 | .624 | .743 | .863 | .982 |
| 364 | 561101 | 1221 | 1340 | 1459 | 1578 | 1698 | 1817 | 1936 | 2055 | 2173 |
| 365 | 2293   | 2412 | 2531 | 2650 | 2769 | 2887 | 3006 | 3125 | 3244 | 3362 |
| 366 | 3481   | 3600 | 3718 | 3837 | 3955 | 4074 | 4192 | 4311 | 4429 | 4548 |
| 367 | 4666   | 4784 | 4903 | 5021 | 5139 | 5257 | 5376 | 5494 | 5612 | 5730 |
| 368 | 5848   | 5966 | 6084 | 6202 | 6320 | 6437 | 6555 | 6673 | 6791 | 6909 |
| 369 | 7026   | 7144 | 7262 | 7379 | 7497 | 7614 | 7732 | 7849 | 7967 | 8084 |
| 370 | 8202   | 8319 | 8436 | 8554 | 8671 | 8788 | 8905 | 9023 | 9140 | 9257 |
| 371 | 9374   | 9491 | 9608 | 9725 | 9842 | 9959 | .76  | .193 | .309 | .426 |
| 372 | 570543 | 0660 | 0776 | 0893 | 1010 | 1126 | 1243 | 1359 | 1476 | 1592 |
| 373 | 1709   | 1825 | 1942 | 2058 | 2174 | 2291 | 2407 | 2523 | 2639 | 2755 |
| 374 | 2872   | 2988 | 3104 | 3220 | 3336 | 3452 | 3568 | 3684 | 3800 | 3915 |
| 375 | 4031   | 4147 | 4263 | 4379 | 4494 | 4610 | 4726 | 4841 | 4957 | 5072 |
| 376 | 5188   | 5303 | 5419 | 5534 | 5650 | 5765 | 5880 | 5996 | 6111 | 6226 |
| 377 | 6341   | 6457 | 6572 | 6687 | 6802 | 6917 | 7032 | 7147 | 7262 | 7377 |
| 378 | 7492   | 7607 | 7722 | 7836 | 7951 | 8066 | 8181 | 8295 | 8410 | 8525 |
| 379 | 8639   | 8754 | 8868 | 8983 | 9097 | 9212 | 9326 | 9441 | 9555 | 9669 |
| 380 | 9784   | 9898 | .12  | .126 | .241 | .355 | .469 | .583 | .697 | .811 |
| 381 | 580925 | 1039 | 1153 | 1267 | 1381 | 1495 | 1608 | 1722 | 1836 | 1950 |
| 382 | 2063   | 2177 | 2291 | 2404 | 2518 | 2631 | 2745 | 2858 | 2972 | 3085 |
| 383 | 3199   | 3312 | 3426 | 3539 | 3652 | 3765 | 3879 | 3992 | 4105 | 4218 |
| 384 | 4331   | 4444 | 4557 | 4670 | 4783 | 4896 | 5009 | 5122 | 5235 | 5348 |
| 385 | 5461   | 5574 | 5686 | 5799 | 5912 | 6024 | 6137 | 6250 | 6362 | 6475 |
| 386 | 6587   | 6700 | 6812 | 6925 | 7037 | 7149 | 7262 | 7374 | 7486 | 7599 |
| 387 | 7711   | 7823 | 7935 | 8047 | 8160 | 8272 | 8384 | 8496 | 8608 | 8720 |
| 388 | 8832   | 8944 | 9056 | 9167 | 9279 | 9391 | 9503 | 9615 | 9726 | 9834 |
| 389 | 9950   | .61  | .173 | .284 | .396 | .507 | .619 | .730 | .842 | .953 |
| 390 | 591065 | 1176 | 1287 | 1399 | 1510 | 1621 | 1732 | 1843 | 1955 | 2066 |
| 391 | 2177   | 2288 | 2399 | 2510 | 2621 | 2732 | 2843 | 2954 | 3064 | 3175 |
| 392 | 3286   | 3397 | 3508 | 3618 | 3729 | 3840 | 3950 | 4061 | 4171 | 4282 |
| 393 | 4393   | 4503 | 4614 | 4724 | 4834 | 4945 | 5055 | 5165 | 5276 | 5386 |
| 394 | 5496   | 5606 | 5717 | 5827 | 5937 | 6047 | 6157 | 6267 | 6377 | 6487 |
| 395 | 6597   | 6707 | 6817 | 6927 | 7037 | 7146 | 7256 | 7366 | 7476 | 7586 |
| 396 | 7695   | 7805 | 7914 | 8024 | 8134 | 8243 | 8353 | 8462 | 8572 | 8681 |
| 397 | 8791   | 8900 | 9009 | 9119 | 9228 | 9337 | 9446 | 9556 | 9666 | 9774 |
| 398 | 9883   | 9992 | .101 | .210 | .319 | .428 | .537 | .646 | .755 | .864 |
| 399 | 600973 | 1082 | 1191 | 1299 | 1408 | 1517 | 1625 | 1734 | 1843 | 1951 |

OF NUMBERS.

9

| N.  | 0      | 1    | 2    | 3    | 4    | 5     | 6     | 7     | 8     | 9     |
|-----|--------|------|------|------|------|-------|-------|-------|-------|-------|
| 400 | 602060 | 2169 | 2277 | 2386 | 2494 | 2603  | 2711  | 2819  | 2928  | 3036  |
| 401 | 3144   | 3253 | 3361 | 3469 | 3573 | 3686  | 3794  | 3902  | 4010  | 4118  |
| 402 | 4226   | 4334 | 4442 | 4550 | 4658 | 4766  | 4874  | 4982  | 5089  | 5197  |
| 403 | 5305   | 5413 | 5521 | 5628 | 5736 | 5844  | 5951  | 6059  | 6166  | 6274  |
| 404 | 6381   | 6489 | 6596 | 6704 | 6811 | 6919  | 7026  | 7133  | 7241  | 7348  |
| 405 | 7455   | 7562 | 7669 | 7777 | 7884 | 7991  | 8098  | 8205  | 8312  | 8419  |
| 406 | 8526   | 8633 | 8740 | 8847 | 8954 | 9061  | 9167  | 9274  | 9381  | 9488  |
| 407 | 9594   | 9701 | 9808 | 9914 | . 21 | . 128 | . 234 | . 341 | . 447 | . 554 |
| 408 | 610660 | 0767 | 0873 | 0979 | 1086 | 1192  | 1298  | 1405  | 1511  | 1617  |
| 409 | 1723   | 1829 | 1936 | 2042 | 2148 | 2254  | 2360  | 2466  | 2572  | 2678  |
| 410 | 2784   | 2890 | 2996 | 3102 | 3207 | 3313  | 3419  | 3525  | 3630  | 3736  |
| 411 | 3842   | 3947 | 4053 | 4159 | 4264 | 4370  | 4475  | 4581  | 4686  | 4792  |
| 412 | 4897   | 5003 | 5108 | 5213 | 5319 | 5424  | 5529  | 5634  | 5740  | 5845  |
| 413 | 5950   | 6055 | 6160 | 6265 | 6370 | 6476  | 6581  | 6686  | 6790  | 6895  |
| 414 | 7000   | 7105 | 7210 | 7315 | 7420 | 7525  | 7629  | 7734  | 7839  | 7943  |
| 415 | 8048   | 8153 | 8257 | 8362 | 8466 | 8571  | 8676  | 8780  | 8884  | 8989  |
| 416 | 9293   | 9198 | 9302 | 9406 | 9511 | 9615  | 9719  | 9824  | 9928  | . 32  |
| 417 | 620136 | 0140 | 0344 | 0448 | 0552 | 0656  | 0760  | 0864  | 0968  | 1072  |
| 418 | 1176   | 1280 | 1384 | 1488 | 1592 | 1695  | 1799  | 1903  | 2007  | 2110  |
| 419 | 2214   | 2318 | 2421 | 2525 | 2628 | 2732  | 2835  | 2939  | 3042  | 3146  |
| 420 | 3249   | 3353 | 3456 | 3559 | 3663 | 3766  | 3869  | 3973  | 4076  | 4179  |
| 421 | 4282   | 4385 | 4488 | 4591 | 4695 | 4798  | 4901  | 5004  | 5107  | 5210  |
| 422 | 5312   | 5415 | 5518 | 5621 | 5724 | 5827  | 5929  | 6032  | 6135  | 6238  |
| 423 | 6340   | 6443 | 6546 | 6648 | 6751 | 6853  | 6956  | 7058  | 7161  | 7263  |
| 424 | 7366   | 7468 | 7571 | 7673 | 7775 | 7878  | 7980  | 8082  | 8185  | 8287  |
| 425 | 8389   | 8491 | 8593 | 8695 | 8797 | 8900  | 9002  | 9104  | 9206  | 9308  |
| 426 | 9410   | 9512 | 9613 | 9715 | 9817 | 9919  | . 21  | . 123 | . 224 | . 326 |
| 427 | 630428 | 0530 | 0631 | 0733 | 0835 | 0936  | 1038  | 1139  | 1241  | 1342  |
| 428 | 1444   | 1545 | 1647 | 1748 | 1849 | 1951  | 2052  | 2153  | 2255  | 2356  |
| 429 | 2457   | 2559 | 2660 | 2761 | 2862 | 2963  | 3064  | 3165  | 3266  | 3367  |
| 430 | 3468   | 3569 | 3670 | 3771 | 3872 | 3973  | 4074  | 4175  | 4276  | 4376  |
| 431 | 4477   | 4578 | 4679 | 4779 | 4880 | 4981  | 5081  | 5182  | 5283  | 5383  |
| 432 | 5484   | 5584 | 5685 | 5785 | 5886 | 5986  | 6087  | 6187  | 6287  | 6388  |
| 433 | 6488   | 6588 | 6688 | 6789 | 6889 | 6989  | 7089  | 7189  | 7290  | 7390  |
| 434 | 7490   | 7590 | 7690 | 7790 | 7890 | 7990  | 8090  | 8190  | 8290  | 8389  |
| 435 | 8489   | 8589 | 8689 | 8789 | 8888 | 8988  | 9088  | 9188  | 9287  | 9387  |
| 436 | 9486   | 9586 | 9686 | 9785 | 9885 | 9984  | . 84  | . 183 | . 283 | . 382 |
| 437 | 640481 | 0581 | 0680 | 0779 | 0879 | 0978  | 1077  | 1177  | 1276  | 1375  |
| 438 | 1474   | 1573 | 1672 | 1771 | 1871 | 1970  | 2069  | 2168  | 2267  | 2366  |
| 439 | 2465   | 2563 | 2662 | 2761 | 2860 | 2959  | 3058  | 3156  | 3255  | 3354  |
| 440 | 3453   | 3551 | 3650 | 3749 | 3847 | 3946  | 4044  | 4143  | 4242  | 4340  |
| 441 | 4439   | 4537 | 4636 | 4734 | 4832 | 4931  | 5029  | 5127  | 5226  | 5324  |
| 442 | 5422   | 5521 | 5619 | 5717 | 5815 | 5913  | 6011  | 6110  | 6208  | 6306  |
| 443 | 6404   | 6502 | 6600 | 6698 | 6796 | 6894  | 6992  | 7089  | 7187  | 7285  |
| 444 | 7383   | 7481 | 7579 | 7676 | 7774 | 7872  | 7969  | 8067  | 8165  | 8262  |
| 445 | 8360   | 8458 | 8555 | 8653 | 8750 | 8848  | 8945  | 9043  | 9140  | 9237  |
| 446 | 9335   | 9432 | 9530 | 9627 | 9724 | 9821  | 9919  | . 16  | . 113 | . 210 |
| 447 | 650308 | 0405 | 0502 | 0599 | 0696 | 0793  | 0890  | 0987  | 1084  | 1181  |
| 448 | 1278   | 1375 | 1472 | 1569 | 1666 | 1762  | 1859  | 1956  | 2053  | 2150  |
| 449 | 2246   | 2343 | 2440 | 2530 | 2633 | 2730  | 2826  | 2923  | 3019  | 3116  |

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 450 | 653213 | 3309 | 3405 | 3502 | 3598 | 3695 | 3791 | 3888 | 3984 | 4080 |
| 451 | 4177   | 4273 | 4369 | 4465 | 4562 | 4658 | 4754 | 4850 | 4946 | 5042 |
| 452 | 5138   | 5235 | 5331 | 5427 | 5526 | 5619 | 5715 | 5810 | 5906 | 6002 |
| 453 | 6098   | 6194 | 6290 | 6386 | 6482 | 6577 | 6673 | 6769 | 6864 | 6960 |
| 454 | 7056   | 7152 | 7247 | 7343 | 7438 | 7534 | 7629 | 7725 | 7820 | 7916 |
| 455 | 8011   | 8107 | 8202 | 8298 | 8393 | 8488 | 8584 | 8679 | 8774 | 8870 |
| 456 | 8965   | 9060 | 9155 | 9250 | 9346 | 9441 | 9536 | 9631 | 9726 | 9821 |
| 457 | 9916   | .11  | .106 | .201 | .296 | .391 | .486 | .581 | .676 | .771 |
| 458 | 660865 | 0960 | 1055 | 1150 | 1245 | 1339 | 1434 | 1529 | 1623 | 1718 |
| 459 | 1813   | 1907 | 2002 | 2096 | 2191 | 2286 | 2380 | 2475 | 2569 | 2663 |
| 460 | 2758   | 2852 | 2947 | 3041 | 3135 | 3230 | 3324 | 3418 | 3512 | 3607 |
| 461 | 3701   | 3795 | 3889 | 3983 | 4078 | 4172 | 4266 | 4360 | 4454 | 4548 |
| 462 | 4642   | 4736 | 4830 | 4924 | 5018 | 5112 | 5206 | 5299 | 5393 | 5487 |
| 463 | 5581   | 5675 | 5769 | 5862 | 5956 | 6050 | 6143 | 6237 | 6331 | 6424 |
| 464 | 6518   | 6612 | 6705 | 6799 | 6892 | 6986 | 7079 | 7173 | 7266 | 7360 |
| 465 | 7453   | 7546 | 7640 | 7733 | 7826 | 7920 | 8013 | 8106 | 8199 | 8293 |
| 466 | 8386   | 8479 | 8572 | 8665 | 8759 | 8852 | 8945 | 9038 | 9131 | 9224 |
| 467 | 9317   | 9410 | 9503 | 9596 | 9689 | 9782 | 9875 | 9967 | .60  | .153 |
| 468 | 670241 | 0339 | 0431 | 0524 | 0617 | 0710 | 0802 | 0895 | 0988 | 1080 |
| 469 | 1173   | 1265 | 1358 | 1451 | 1543 | 1636 | 1728 | 1821 | 1913 | 2005 |
| 470 | 2098   | 2190 | 2283 | 2375 | 2467 | 2560 | 2652 | 2744 | 2836 | 2929 |
| 471 | 3021   | 3113 | 3205 | 3297 | 3390 | 3482 | 3574 | 3666 | 3758 | 3850 |
| 472 | 3942   | 4034 | 4126 | 4218 | 4310 | 4402 | 4494 | 4586 | 4677 | 4769 |
| 473 | 4861   | 4953 | 5045 | 5137 | 5228 | 5320 | 5412 | 5503 | 5595 | 5687 |
| 474 | 5778   | 5870 | 5962 | 6053 | 6145 | 6236 | 6328 | 6419 | 6511 | 6602 |
| 475 | 6694   | 6785 | 6876 | 6968 | 7059 | 7151 | 7242 | 7333 | 7424 | 7516 |
| 476 | 7607   | 7698 | 7789 | 7881 | 7972 | 8063 | 8154 | 8245 | 8336 | 8427 |
| 477 | 8518   | 8609 | 8700 | 8791 | 8882 | 8972 | 9064 | 9155 | 9246 | 9337 |
| 478 | 9428   | 9519 | 9610 | 9700 | 9791 | 9882 | 9973 | .63  | .154 | .245 |
| 479 | 680336 | 0426 | 0517 | 0607 | 0698 | 0789 | 0879 | 0970 | 1060 | 1151 |
| 480 | 1241   | 1332 | 1422 | 1513 | 1603 | 1693 | 1784 | 1874 | 1964 | 2055 |
| 481 | 2145   | 2235 | 2326 | 2416 | 2506 | 2596 | 2686 | 2777 | 2867 | 2957 |
| 482 | 3047   | 3137 | 3227 | 3317 | 3407 | 3497 | 3587 | 3677 | 3767 | 3857 |
| 483 | 3947   | 4037 | 4127 | 4217 | 4307 | 4396 | 4486 | 4576 | 4666 | 4756 |
| 484 | 4854   | 4935 | 5025 | 5114 | 5204 | 5294 | 5383 | 5473 | 5563 | 5652 |
| 485 | 5742   | 5831 | 5921 | 6010 | 6100 | 6189 | 6279 | 6368 | 6458 | 6547 |
| 486 | 6636   | 6726 | 6815 | 6904 | 6994 | 7083 | 7172 | 7261 | 7351 | 7440 |
| 487 | 7529   | 7618 | 7707 | 7796 | 7886 | 7975 | 8064 | 8153 | 8242 | 8331 |
| 488 | 8420   | 8509 | 8598 | 8687 | 8776 | 8865 | 8953 | 9042 | 9131 | 9220 |
| 489 | 9309   | 9398 | 9486 | 9575 | 9664 | 9753 | 9841 | 9930 | .19  | .107 |
| 490 | 690196 | 0285 | 0373 | 0462 | 0550 | 0639 | 0728 | 0816 | 0905 | 0993 |
| 491 | 1081   | 1170 | 1258 | 1347 | 1435 | 1524 | 1612 | 1700 | 1789 | 1877 |
| 492 | 1965   | 2053 | 2142 | 2230 | 2318 | 2406 | 2494 | 2583 | 2671 | 2759 |
| 493 | 2847   | 2935 | 3023 | 3111 | 3199 | 3287 | 3375 | 3463 | 3551 | 3639 |
| 494 | 3727   | 3815 | 3903 | 3991 | 4078 | 4166 | 4254 | 4342 | 4430 | 4517 |
| 495 | 4605   | 4693 | 4781 | 4868 | 4956 | 5044 | 5131 | 5210 | 5307 | 5394 |
| 496 | 5482   | 5569 | 5657 | 5744 | 5832 | 5919 | 6007 | 6094 | 6182 | 6269 |
| 497 | 6356   | 6444 | 6531 | 6618 | 6706 | 6793 | 6880 | 6968 | 7055 | 7142 |
| 498 | 7229   | 7317 | 7404 | 7491 | 7578 | 7665 | 7752 | 7839 | 7926 | 8014 |
| 499 | 8101   | 8188 | 8275 | 8362 | 8449 | 8535 | 8622 | 8709 | 8796 | 8883 |

OF NUMBERS.

11

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 500 | 698970 | 9057 | 9144 | 9231 | 9317 | 9404 | 9491 | 9578 | 9664 | 9751 |
| 501 | 9838   | 9924 | .11  | .98  | .184 | .271 | .358 | .444 | .531 | .617 |
| 502 | 700704 | 0790 | 0877 | 0963 | 1050 | 1136 | 1222 | 1309 | 1395 | 1482 |
| 503 | 1568   | 1654 | 1741 | 1827 | 1913 | 1999 | 2086 | 2172 | 2258 | 2344 |
| 504 | 2431   | 2517 | 2603 | 2689 | 2775 | 2861 | 2947 | 3033 | 3119 | 3205 |
| 505 | 3291   | 3377 | 3463 | 3549 | 3635 | 3721 | 3807 | 3895 | 3979 | 4065 |
| 506 | 4151   | 4236 | 4322 | 4408 | 4494 | 4579 | 4665 | 4751 | 4837 | 4922 |
| 507 | 5008   | 5094 | 5179 | 5265 | 5350 | 5436 | 5522 | 5607 | 5693 | 5778 |
| 508 | 5864   | 5949 | 6035 | 6120 | 6206 | 6291 | 6376 | 6462 | 6547 | 6632 |
| 509 | 6718   | 6803 | 6888 | 6974 | 7059 | 7144 | 7229 | 7315 | 7400 | 7485 |
| 510 | 7570   | 7655 | 7740 | 7826 | 7910 | 7996 | 8081 | 8166 | 8251 | 8336 |
| 511 | 8421   | 8506 | 8591 | 8676 | 8761 | 8846 | 8931 | 9015 | 9100 | 9185 |
| 512 | 9270   | 9355 | 9440 | 9524 | 9609 | 9694 | 9779 | 9863 | 9948 | .33  |
| 513 | 710117 | 0202 | 0287 | 0371 | 0456 | 0540 | 0625 | 0710 | 0794 | 0879 |
| 514 | 0963   | 1048 | 1132 | 1217 | 1301 | 1385 | 1470 | 1554 | 1639 | 1723 |
| 515 | 1807   | 1892 | 1976 | 2060 | 2144 | 2229 | 2313 | 2397 | 2481 | 2566 |
| 516 | 2650   | 2734 | 2818 | 2902 | 2986 | 3070 | 3154 | 3238 | 3326 | 3407 |
| 517 | 3491   | 3575 | 3659 | 3742 | 3826 | 3910 | 3994 | 4078 | 4162 | 4246 |
| 518 | 4330   | 4414 | 4497 | 4581 | 4665 | 4749 | 4833 | 4916 | 5000 | 5084 |
| 519 | 5167   | 5251 | 5335 | 5418 | 5502 | 5586 | 5669 | 5753 | 5836 | 5920 |
| 520 | 6003   | 6087 | 6170 | 6254 | 6337 | 6421 | 6504 | 6588 | 6671 | 6754 |
| 521 | 6838   | 6921 | 7004 | 7088 | 7171 | 7254 | 7338 | 7421 | 7504 | 7587 |
| 522 | 7671   | 7754 | 7837 | 7920 | 8003 | 8086 | 8169 | 8253 | 8336 | 8419 |
| 523 | 8502   | 8585 | 8668 | 8751 | 8834 | 8917 | 9000 | 9083 | 9165 | 9248 |
| 524 | 9331   | 9414 | 9497 | 9580 | 9663 | 9745 | 9828 | 9911 | 9994 | .77  |
| 525 | 720159 | 0242 | 0325 | 0407 | 0490 | 0573 | 0655 | 0738 | 0821 | 0903 |
| 526 | 0986   | 1068 | 1151 | 1233 | 1316 | 1398 | 1481 | 1563 | 1646 | 1728 |
| 527 | 1811   | 1893 | .975 | 2058 | 2140 | 2222 | 2305 | 2387 | 2469 | 2552 |
| 528 | 2634   | 2716 | 2798 | 2881 | 2963 | 3045 | 3127 | 3209 | 3291 | 3374 |
| 529 | 3456   | 3538 | 3620 | 3702 | 3784 | 3866 | 3948 | 4030 | 4112 | 4194 |
| 530 | 4276   | 4358 | 4440 | 4522 | 4604 | 4685 | 4767 | 4849 | 4931 | 5013 |
| 531 | 5095   | 5176 | 5258 | 5340 | 5422 | 5503 | 5585 | 5667 | 5748 | 5830 |
| 532 | 5912   | 5993 | 6075 | 6156 | 6238 | 6320 | 6401 | 6483 | 6564 | 6646 |
| 533 | 6727   | 6809 | 6890 | 6972 | 7053 | 7134 | 7216 | 7297 | 7379 | 7460 |
| 534 | 7541   | 7623 | 7704 | 7785 | 7866 | 7948 | 8029 | 8110 | 8191 | 8273 |
| 535 | 8354   | 8435 | 8516 | 8597 | 8678 | 8759 | 8841 | 8922 | 9003 | 9084 |
| 536 | 9165   | 9246 | 9327 | 9408 | 9489 | 9570 | 9651 | 9732 | 9813 | 9893 |
| 537 | 9974   | .55  | .136 | .217 | .298 | .378 | .459 | .440 | .621 | .702 |
| 538 | 730782 | 0863 | 0944 | 1024 | 1105 | 1186 | 1266 | 1347 | 1428 | 1508 |
| 539 | 1589   | 1669 | 1750 | 1830 | 1911 | 1991 | 2072 | 2152 | 2233 | 2313 |
| 540 | 2394   | 2474 | 2555 | 2635 | 2715 | 2796 | 2876 | 2956 | 3037 | 3117 |
| 541 | 3197   | 3278 | 3358 | 3438 | 3518 | 3598 | 3679 | 3759 | 3839 | 3919 |
| 542 | 3999   | 4079 | 4160 | 4240 | 4320 | 4400 | 4480 | 4560 | 4640 | 4720 |
| 543 | 4800   | 4880 | 4960 | 5040 | 5120 | 5200 | 5279 | 5359 | 5439 | 5519 |
| 544 | 5599   | 5679 | 5759 | 5838 | 5918 | 5998 | 6078 | 6157 | 6237 | 6317 |
| 545 | 6397   | 6476 | 6556 | 6636 | 6715 | 6795 | 6874 | 6954 | 7034 | 7113 |
| 546 | 7193   | 7272 | 7352 | 7431 | 7511 | 7590 | 7670 | 7749 | 7829 | 7908 |
| 547 | 7987   | 8067 | 8146 | 8225 | 8305 | 8384 | 8463 | 8543 | 8622 | 8701 |
| 548 | 8781   | 8860 | 8939 | 9018 | 9097 | 9177 | 9256 | 9335 | 9414 | 9493 |
| 549 | 9572   | 9651 | 9731 | 9810 | 9889 | 9968 | .47  | .126 | .205 | .284 |

| N.  | 0      | 1    | 2    | 3    | 4     | 5     | 6    | 7    | 8    | 9     |
|-----|--------|------|------|------|-------|-------|------|------|------|-------|
| 550 | 740863 | 0442 | 0621 | 0560 | 0678  | 0757  | 0836 | 0915 | 0994 | 1073  |
| 551 | 1152   | 1230 | 1309 | 1388 | 1467  | 1546  | 1624 | 1703 | 1782 | 1860  |
| 552 | 1939   | 2018 | 2096 | 2175 | 2254  | 2332  | 2411 | 2489 | 2568 | 2646  |
| 553 | 2725   | 2804 | 2882 | 2961 | 3039  | 3118  | 3196 | 3275 | 3353 | 3431  |
| 554 | 3510   | 3588 | 3667 | 3745 | 3823  | 3902  | 3980 | 4058 | 4136 | 4215  |
| 555 | 4293   | 4371 | 4449 | 4528 | 4606  | 4684  | 4762 | 4840 | 4919 | 4997  |
| 556 | 5075   | 5153 | 5231 | 5309 | 5387  | 5465  | 5543 | 5621 | 5699 | 5777  |
| 557 | 5855   | 5933 | 6011 | 6089 | 6167  | 6245  | 6323 | 6401 | 6479 | 6556  |
| 558 | 6634   | 6712 | 6790 | 6868 | 6945  | 7023  | 7101 | 7179 | 7256 | 7334  |
| 559 | 7412   | 7489 | 7567 | 7645 | 7722  | 7800  | 7878 | 7955 | 8033 | 8110  |
| 560 | 8188   | 8266 | 8343 | 8421 | 8498  | 8576  | 8653 | 8731 | 8808 | 8885  |
| 561 | 8963   | 9040 | 9118 | 9195 | 9272  | 9350  | 9427 | 9504 | 9582 | 9659  |
| 562 | 9736   | 9814 | 9891 | 9968 | . .45 | .123  | .200 | .277 | .354 | .431  |
| 563 | 760608 | 0686 | 0663 | 0740 | 0817  | 0894  | 0971 | 1048 | 1125 | 1202  |
| 564 | 1279   | 1356 | 1433 | 1510 | 1587  | 1664  | 1741 | 1818 | 1895 | 1972  |
| 565 | 2048   | 2125 | 2202 | 2279 | 2356  | 2433  | 2509 | 2586 | 2663 | 2740  |
| 566 | 2816   | 2893 | 2970 | 3047 | 3123  | 3200  | 3277 | 3353 | 3430 | 3506  |
| 567 | 3582   | 3660 | 3736 | 3813 | 3889  | 3966  | 4042 | 4119 | 4195 | 4272  |
| 568 | 4348   | 4425 | 4501 | 4578 | 4654  | 4730  | 4807 | 4883 | 4960 | 5036  |
| 569 | 5112   | 5189 | 5265 | 5341 | 5417  | 5494  | 5570 | 5646 | 5722 | 5799  |
| 570 | 5875   | 5951 | 6027 | 6103 | 6180  | 6256  | 6332 | 6408 | 6484 | 6560  |
| 571 | 6636   | 6712 | 6788 | 6864 | 6940  | 7016  | 7092 | 7168 | 7244 | 7320  |
| 572 | 7396   | 7472 | 7548 | 7624 | 7700  | 7775  | 7851 | 7927 | 8003 | 8079  |
| 573 | 8155   | 8230 | 8306 | 8382 | 8458  | 8533  | 8609 | 8685 | 8761 | 8836  |
| 574 | 8912   | 8988 | 9063 | 9139 | 9214  | 9290  | 9366 | 9441 | 9517 | 9592  |
| 575 | 9668   | 9743 | 9819 | 9894 | 9970  | . .45 | .121 | .196 | .272 | .347  |
| 576 | 760422 | 0498 | 0573 | 0649 | 0724  | 0799  | 0875 | 0950 | 1025 | 1101  |
| 577 | 1176   | 1251 | 1326 | 1402 | 1477  | 1552  | 1627 | 1702 | 1778 | 1853  |
| 578 | 1928   | 2003 | 2078 | 2153 | 2228  | 2303  | 2378 | 2453 | 2529 | 2604  |
| 579 | 2679   | 2754 | 2829 | 2904 | 2978  | 3053  | 3128 | 3203 | 3278 | 3353  |
| 580 | 3428   | 3503 | 3578 | 3653 | 3727  | 3802  | 3877 | 3952 | 4027 | 4101  |
| 581 | 4176   | 4251 | 4326 | 4400 | 4475  | 4550  | 4624 | 4699 | 4774 | 4848  |
| 582 | 4923   | 4998 | 5072 | 5147 | 5221  | 5296  | 5370 | 5445 | 5520 | 5594  |
| 583 | 5669   | 5743 | 5818 | 5892 | 5966  | 6041  | 6115 | 6190 | 6264 | 6338  |
| 584 | 6443   | 6487 | 6562 | 6636 | 6710  | 6785  | 6859 | 6933 | 7007 | 7082  |
| 585 | 7156   | 7230 | 7304 | 7379 | 7453  | 7527  | 7601 | 7675 | 7749 | 7823  |
| 586 | 7898   | 7972 | 8046 | 8120 | 8194  | 8268  | 8342 | 8416 | 8490 | 8564  |
| 587 | 8638   | 8712 | 8786 | 8860 | 8934  | 9008  | 9082 | 9156 | 9230 | 9303  |
| 588 | 9377   | 9451 | 9525 | 9599 | 9673  | 9746  | 9820 | 9894 | 9968 | . .42 |
| 589 | 770115 | 0189 | 0263 | 0336 | 0410  | 0484  | 0557 | 0631 | 0705 | 0778  |
| 590 | 0852   | 0926 | 0999 | 1073 | 1146  | 1220  | 1293 | 1367 | 1440 | 1514  |
| 591 | 1587   | 1661 | 1734 | 1808 | 1881  | 1955  | 2028 | 2102 | 2175 | 2248  |
| 592 | 2322   | 2395 | 2468 | 2542 | 2615  | 2688  | 2762 | 2835 | 2908 | 2981  |
| 593 | 3055   | 3128 | 3201 | 3274 | 3348  | 3421  | 3494 | 3567 | 3640 | 3713  |
| 594 | 3786   | 3860 | 3933 | 4006 | 4079  | 4152  | 4225 | 4298 | 4371 | 4444  |
| 595 | 4517   | 4590 | 4663 | 4736 | 4809  | 4882  | 4955 | 5028 | 5100 | 5173  |
| 596 | 5246   | 5319 | 5392 | 5465 | 5538  | 5610  | 5683 | 5756 | 5829 | 5902  |
| 597 | 5974   | 6047 | 6120 | 6193 | 6265  | 6338  | 6411 | 6483 | 6556 | 6629  |
| 598 | 6701   | 6774 | 6846 | 6919 | 6992  | 7064  | 7137 | 7209 | 7282 | 7354  |
| 599 | 7427   | 7499 | 7572 | 7644 | 7717  | 7789  | 7862 | 7934 | 8006 | 8079  |

## OF NUMBERS.

13

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 600 | 778151 | 8224 | 8296 | 8368 | 8441 | 8513 | 8585 | 8658 | 8730 | 8802 |
| 601 | 8874   | 8947 | 9019 | 9091 | 9163 | 9236 | 9308 | 9380 | 9452 | 9524 |
| 602 | 9596   | 9669 | 9741 | 9813 | 9885 | 9957 | .29  | .101 | .173 | .245 |
| 603 | 780317 | 0389 | 0461 | 0533 | 0605 | 0677 | 0749 | 0821 | 0893 | 0965 |
| 604 | 1037   | 1109 | 1181 | 1253 | 1324 | 1396 | 1468 | 1540 | 1612 | 1684 |
| 605 | 1755   | 1827 | 1899 | 1971 | 2042 | 2114 | 2186 | 2258 | 2329 | 2401 |
| 606 | 2473   | 2544 | 2616 | 2688 | 2759 | 2831 | 2902 | 2974 | 3046 | 3117 |
| 607 | 3189   | 3260 | 3332 | 3403 | 3475 | 3546 | 3618 | 3689 | 3761 | 3832 |
| 608 | 3904   | 3975 | 4046 | 4118 | 4189 | 4261 | 4332 | 4403 | 4475 | 4546 |
| 609 | 4617   | 4689 | 4760 | 4831 | 4902 | 4974 | 5045 | 5116 | 5187 | 5259 |
| 610 | 5330   | 5401 | 5472 | 5543 | 5615 | 5686 | 5757 | 5828 | 5899 | 5970 |
| 611 | 6041   | 6112 | 6183 | 6254 | 6325 | 6396 | 6467 | 6538 | 6609 | 6680 |
| 612 | 6751   | 6822 | 6893 | 6964 | 7035 | 7106 | 7177 | 7248 | 7319 | 7390 |
| 613 | 7460   | 7531 | 7602 | 7673 | 7744 | 7815 | 7885 | 7956 | 8027 | 8098 |
| 614 | 8168   | 8239 | 8310 | 8381 | 8451 | 8522 | 8593 | 8663 | 8734 | 8804 |
| 615 | 8875   | 8946 | 9016 | 9087 | 9157 | 9228 | 9299 | 9369 | 9440 | 9510 |
| 616 | 9581   | 9651 | 9722 | 9792 | 9863 | 9933 | .4   | .74  | .144 | .215 |
| 617 | 790285 | 0356 | 0426 | 0496 | 0567 | 0637 | 0707 | 0778 | 0848 | 0918 |
| 618 | 0988   | 1059 | 1129 | 1199 | 1269 | 1340 | 1410 | 1480 | 1550 | 1620 |
| 619 | 1691   | 1761 | 1831 | 1901 | 1971 | 2041 | 2111 | 2181 | 2252 | 2322 |
| 620 | 2392   | 2462 | 2532 | 2602 | 2672 | 2742 | 2812 | 2882 | 2952 | 3022 |
| 621 | 3092   | 3162 | 3231 | 3301 | 3371 | 3441 | 3511 | 3581 | 3651 | 3721 |
| 622 | 3790   | 3860 | 3930 | 4000 | 4070 | 4139 | 4209 | 4279 | 4349 | 4418 |
| 623 | 4488   | 4558 | 4627 | 4697 | 4767 | 4836 | 4906 | 4976 | 5045 | 5115 |
| 624 | 5185   | 5254 | 5324 | 5393 | 5463 | 5532 | 5602 | 5672 | 5741 | 5811 |
| 625 | 5880   | 5949 | 6019 | 6088 | 6158 | 6227 | 6297 | 6366 | 6436 | 6505 |
| 626 | 6574   | 6644 | 6713 | 6782 | 6852 | 6921 | 6990 | 7060 | 7129 | 7198 |
| 627 | 7268   | 7337 | 7406 | 7475 | 7545 | 7614 | 7683 | 7752 | 7821 | 7890 |
| 628 | 7960   | 8029 | 8098 | 8167 | 8236 | 8305 | 8374 | 8443 | 8513 | 8582 |
| 629 | 8651   | 8720 | 8789 | 8858 | 8927 | 8996 | 9065 | 9134 | 9203 | 9272 |
| 630 | 9341   | 9409 | 9478 | 9547 | 9610 | 9685 | 9754 | 9823 | 9892 | 9961 |
| 631 | 800026 | 0098 | 0167 | 0236 | 0305 | 0373 | 0442 | 0511 | 0580 | 0648 |
| 632 | 0717   | 0786 | 0854 | 0923 | 0992 | 1061 | 1129 | 1198 | 1266 | 1335 |
| 633 | 1404   | 1472 | 1541 | 1609 | 1678 | 1747 | 1815 | 1884 | 1952 | 2021 |
| 634 | 2089   | 2158 | 2226 | 2295 | 2363 | 2432 | 2500 | 2568 | 2637 | 2705 |
| 635 | 2774   | 2842 | 2910 | 2979 | 3047 | 3116 | 3184 | 3252 | 3321 | 3389 |
| 636 | 3457   | 3525 | 3594 | 3662 | 3730 | 3798 | 3867 | 3935 | 4003 | 4071 |
| 637 | 4139   | 4208 | 4276 | 4344 | 4412 | 4480 | 4548 | 4616 | 4685 | 4753 |
| 638 | 4821   | 4889 | 4957 | 5025 | 5093 | 5161 | 5229 | 5297 | 5365 | 5433 |
| 639 | 5501   | 5569 | 5637 | 5705 | 5773 | 5841 | 5908 | 5976 | 6044 | 6112 |
| 640 | 6180   | 6248 | 6316 | 6384 | 6451 | 6519 | 6587 | 6655 | 6723 | 6790 |
| 641 | 6858   | 6926 | 6994 | 7061 | 7129 | 7197 | 7264 | 7332 | 7400 | 7467 |
| 642 | 7535   | 7603 | 7670 | 7738 | 7806 | 7873 | 7941 | 8008 | 8076 | 8143 |
| 643 | 8211   | 8279 | 8346 | 8414 | 8481 | 8549 | 8616 | 8684 | 8751 | 8818 |
| 644 | 8886   | 8953 | 9021 | 9088 | 9156 | 9223 | 9290 | 9358 | 9425 | 9492 |
| 645 | 9560   | 9627 | 9694 | 9762 | 9829 | 9896 | 9964 | .31  | .98  | .165 |
| 646 | 810233 | 0300 | 0367 | 0434 | 0501 | 0566 | 0636 | 0703 | 0770 | 0837 |
| 647 | 0904   | 0971 | 1039 | 1106 | 1173 | 1240 | 1307 | 1374 | 1441 | 1508 |
| 648 | 1575   | 1642 | 1709 | 1776 | 1843 | 1910 | 1977 | 2044 | 2111 | 2178 |
| 649 | 2245   | 2312 | 2379 | 2445 | 2512 | 2579 | 2646 | 2713 | 2780 | 2847 |



| N.  | 0      | 1    | 2    | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|--------|------|------|-------|-------|-------|-------|-------|-------|-------|
| 650 | 812913 | 2980 | 3047 | 3114  | 3181  | 3247  | 3314  | 3381  | 3448  | 3514  |
| 651 | 3581   | 3648 | 3714 | 3781  | 3848  | 3914  | 3981  | 4048  | 4114  | 4181  |
| 652 | 4248   | 4314 | 4381 | 4447  | 4514  | 4581  | 4647  | 4714  | 4780  | 4847  |
| 653 | 4913   | 4980 | 5046 | 5113  | 5179  | 5246  | 5312  | 5378  | 5445  | 5511  |
| 654 | 5578   | 5644 | 5711 | 5777  | 5843  | 5910  | 5976  | 6042  | 6109  | 6175  |
| 655 | 6241   | 6308 | 6374 | 6440  | 6506  | 6573  | 6639  | 6705  | 6771  | 6838  |
| 656 | 6904   | 6970 | 7036 | 7102  | 7169  | 7233  | 7301  | 7367  | 7433  | 7499  |
| 657 | 7565   | 7631 | 7698 | 7764  | 7830  | 7896  | 7962  | 8028  | 8094  | 8160  |
| 658 | 8226   | 8292 | 8358 | 8424  | 8490  | 8556  | 8622  | 8688  | 8754  | 8820  |
| 659 | 8885   | 8951 | 9017 | 9083  | 9149  | 9215  | 9281  | 9346  | 9412  | 9478  |
| 660 | 9544   | 9610 | 9676 | 9741  | 9807  | 9873  | 9939  | ...   | .4    | .70   |
| 661 | 820201 | 0267 | 0333 | 0399  | 0464  | 0530  | 0595  | 0661  | 0727  | 0792  |
| 662 | 0858   | 0924 | 0989 | 1055  | 1120  | 1186  | 1251  | 1317  | 1382  | 1448  |
| 663 | 1514   | 1579 | 1645 | 1710  | 1775  | 1841  | 1906  | 1972  | 2037  | 2103  |
| 664 | 2168   | 2233 | 2299 | 2364  | 2430  | 2495  | 2560  | 2626  | 2691  | 2756  |
| 665 | 2822   | 2887 | 2952 | 3018  | 3083  | 3148  | 3213  | 3279  | 3344  | 3409  |
| 666 | 3474   | 3539 | 3605 | 3670  | 3735  | 3800  | 3865  | 3930  | 3996  | 4061  |
| 667 | 4126   | 4191 | 4256 | 4321  | 4386  | 4451  | 4516  | 4581  | 4646  | 4711  |
| 668 | 4776   | 4841 | 4906 | 4971  | 5036  | 5101  | 5166  | 5231  | 5296  | 5361  |
| 669 | 5426   | 5491 | 5556 | 5621  | 5686  | 5751  | 5815  | 5880  | 5945  | 6010  |
| 670 | 6075   | 6140 | 6204 | 6269  | 6334  | 6399  | 6464  | 6528  | 6593  | 6658  |
| 671 | 6723   | 6787 | 6852 | 6917  | 6981  | 7046  | 7111  | 7175  | 7240  | 7305  |
| 672 | 7369   | 7434 | 7499 | 7563  | 7628  | 7692  | 7757  | 7821  | 7886  | 7951  |
| 673 | 8015   | 8080 | 8144 | 8209  | 8273  | 8338  | 8402  | 8467  | 8531  | 8595  |
| 674 | 8660   | 8724 | 8789 | 8853  | 8918  | 8982  | 9046  | 9111  | 9175  | 9239  |
| 675 | 9304   | 9368 | 9432 | 9497  | 9561  | 9625  | 9690  | 9754  | 9818  | 9882  |
| 676 | 9947   | ..11 | ..75 | ..139 | ..204 | ..268 | ..332 | ..396 | ..460 | ..525 |
| 677 | 830589 | 0653 | 0717 | 0781  | 0845  | 0909  | 0973  | 1037  | 1102  | 1166  |
| 678 | 1230   | 1294 | 1358 | 1422  | 1486  | 1550  | 1614  | 1678  | 1742  | 1806  |
| 679 | 1870   | 1934 | 1998 | 2062  | 2126  | 2189  | 2253  | 2317  | 2381  | 2445  |
| 680 | 2509   | 2573 | 2637 | 2700  | 2764  | 2828  | 2892  | 2956  | 3020  | 3083  |
| 681 | 3147   | 3211 | 3275 | 3338  | 3402  | 3466  | 3530  | 3593  | 3657  | 3721  |
| 682 | 3784   | 3848 | 3912 | 3975  | 4039  | 4103  | 4166  | 4230  | 4294  | 4357  |
| 683 | 4421   | 4484 | 4548 | 4611  | 4675  | 4739  | 4802  | 4866  | 4929  | 4993  |
| 684 | 5056   | 5120 | 5183 | 5247  | 5310  | 5373  | 5437  | 5500  | 5564  | 5627  |
| 685 | 5691   | 5754 | 5817 | 5881  | 5944  | 6007  | 6071  | 6134  | 6197  | 6261  |
| 686 | 6324   | 6387 | 6451 | 6514  | 6577  | 6641  | 6704  | 6767  | 6830  | 6894  |
| 687 | 6957   | 7020 | 7083 | 7146  | 7210  | 7273  | 7336  | 7399  | 7462  | 7525  |
| 688 | 7588   | 7652 | 7715 | 7778  | 7841  | 7904  | 7967  | 8030  | 8093  | 8156  |
| 689 | 8219   | 8282 | 8345 | 8408  | 8471  | 8534  | 8597  | 8660  | 8723  | 8786  |
| 690 | 8849   | 8912 | 8975 | 9038  | 9101  | 9164  | 9227  | 9289  | 9352  | 9415  |
| 691 | 9478   | 9541 | 9604 | 9667  | 9729  | 9792  | 9855  | 9918  | 9981  | ..43  |
| 692 | 840106 | 0169 | 0232 | 0294  | 0357  | 0420  | 0482  | 0545  | 0608  | 0671  |
| 693 | 0733   | 0796 | 0859 | 0921  | 0984  | 1046  | 1109  | 1172  | 1234  | 1297  |
| 694 | 1359   | 1422 | 1485 | 1547  | 1610  | 1672  | 1735  | 1797  | 1860  | 1922  |
| 695 | 1985   | 2047 | 2110 | 2172  | 2235  | 2297  | 2360  | 2422  | 2484  | 2547  |
| 696 | 2609   | 2672 | 2734 | 2796  | 2859  | 2921  | 2983  | 3046  | 3108  | 3170  |
| 697 | 3233   | 3295 | 3357 | 3420  | 3482  | 3544  | 3606  | 3669  | 3731  | 3793  |
| 698 | 3855   | 3918 | 3980 | 4042  | 4104  | 4166  | 4229  | 4291  | 4353  | 4415  |
| 699 | 4477   | 4539 | 4601 | 4664  | 4726  | 4788  | 4850  | 4912  | 4974  | 5036  |

## OF NUMBERS.

15

| N.  | 0      | 1    | 2    | 3    | 4    | 5     | 6     | 7     | 8     | 9     |
|-----|--------|------|------|------|------|-------|-------|-------|-------|-------|
| 700 | 845098 | 5160 | 5222 | 5284 | 5346 | 5408  | 5470  | 5532  | 5594  | 5656  |
| 701 | 5718   | 5780 | 5842 | 5904 | 5966 | 6028  | 6090  | 6151  | 6213  | 6275  |
| 702 | 6337   | 6399 | 6461 | 6523 | 6585 | 6646  | 6708  | 6770  | 6832  | 6894  |
| 703 | 6955   | 7017 | 7079 | 7141 | 7202 | 7264  | 7326  | 7388  | 7449  | 7511  |
| 704 | 7573   | 7634 | 7676 | 7758 | 7819 | 7831  | 7943  | 8004  | 8066  | 8128  |
| 705 | 8189   | 8251 | 8312 | 8374 | 8435 | 8497  | 8559  | 8620  | 8682  | 8743  |
| 706 | 8805   | 8866 | 8928 | 8989 | 9051 | 9112  | 9174  | 9235  | 9297  | 9358  |
| 707 | 9419   | 9481 | 9542 | 9604 | 9665 | 9726  | 9788  | 9849  | 9911  | 9972  |
| 708 | 850033 | 0095 | 0156 | 0217 | 0279 | 0340  | 0401  | 0462  | 0524  | 0585  |
| 709 | 0646   | 0707 | 0769 | 0830 | 0891 | 0952  | 1014  | 1075  | 1136  | 1197  |
| 710 | 1258   | 1320 | 1381 | 1442 | 1503 | 1564  | 1625  | 1686  | 1747  | 1809  |
| 711 | 1870   | 1931 | 1992 | 2053 | 2114 | 2175  | 2236  | 2297  | 2358  | 2419  |
| 712 | 2480   | 2541 | 2602 | 2663 | 2724 | 2785  | 2846  | 2907  | 2968  | 3029  |
| 713 | 3090   | 3150 | 3211 | 3272 | 3333 | 3394  | 3455  | 3516  | 3577  | 3637  |
| 714 | 3698   | 3759 | 3820 | 3881 | 3941 | 4002  | 4063  | 4124  | 4185  | 4245  |
| 715 | 4306   | 4367 | 4428 | 4488 | 4549 | 4610  | 4670  | 4731  | 4792  | 4852  |
| 716 | 4913   | 4974 | 5034 | 5095 | 5156 | 5216  | 5277  | 5337  | 5398  | 5459  |
| 717 | 5519   | 5580 | 5640 | 5701 | 5761 | 5822  | 5882  | 5943  | 6003  | 6064  |
| 718 | 6124   | 6185 | 6245 | 6306 | 6366 | 6427  | 6487  | 6548  | 6608  | 6668  |
| 719 | 6729   | 6789 | 6850 | 6910 | 6970 | 7031  | 7091  | 7152  | 7212  | 7272  |
| 720 | 7332   | 7393 | 7453 | 7513 | 7574 | 7634  | 7694  | 7755  | 7815  | 7875  |
| 721 | 7935   | 7995 | 8056 | 8116 | 8176 | 8236  | 8297  | 8357  | 8417  | 8477  |
| 722 | 8537   | 8597 | 8657 | 8718 | 8778 | 8838  | 8898  | 8958  | 9018  | 9078  |
| 723 | 9138   | 9198 | 9258 | 9318 | 9379 | 9439  | 9499  | 9559  | 9619  | 9679  |
| 724 | 9739   | 9799 | 9859 | 9918 | 9978 | ..38  | ..98  | ..158 | ..218 | ..278 |
| 725 | 860338 | 0398 | 0458 | 0518 | 0578 | 0637  | 0697  | 0757  | 0817  | 0877  |
| 726 | 0937   | 0996 | 1056 | 1116 | 1176 | 1236  | 1295  | 1355  | 1415  | 1475  |
| 727 | 1534   | 1594 | 1654 | 1714 | 1773 | 1833  | 1893  | 1952  | 2012  | 2072  |
| 728 | 2131   | 2191 | 2251 | 2310 | 2370 | 2430  | 2489  | 2549  | 2608  | 2668  |
| 729 | 2728   | 2787 | 2847 | 2906 | 2966 | 3025  | 3085  | 3144  | 3204  | 3263  |
| 730 | 3323   | 3382 | 3442 | 3501 | 3561 | 3620  | 3680  | 3739  | 3799  | 3858  |
| 731 | 3917   | 3977 | 4036 | 4096 | 4155 | 4214  | 4274  | 4333  | 4392  | 4452  |
| 732 | 4511   | 4570 | 4630 | 4689 | 4148 | 4808  | 4867  | 4926  | 4985  | 5045  |
| 733 | 5104   | 5163 | 5222 | 5282 | 5341 | 5400  | 5459  | 5519  | 5578  | 5637  |
| 734 | 5696   | 5755 | 5814 | 5874 | 5933 | 5992  | 6051  | 6110  | 6169  | 6228  |
| 735 | 6287   | 6346 | 6405 | 6465 | 6524 | 6583  | 6642  | 6701  | 6760  | 6819  |
| 736 | 6878   | 6937 | 6996 | 7055 | 7114 | 7173  | 7232  | 7291  | 7350  | 7409  |
| 737 | 7467   | 7526 | 7585 | 7644 | 7703 | 7762  | 7821  | 7880  | 7939  | 7998  |
| 738 | 8056   | 8115 | 8174 | 8233 | 8292 | 8350  | 8409  | 8468  | 8527  | 8586  |
| 739 | 8644   | 8703 | 8762 | 8821 | 8879 | 8938  | 8997  | 9056  | 9114  | 9173  |
| 740 | 9232   | 9290 | 9349 | 9408 | 9466 | 9525  | 9584  | 9642  | 9701  | 9760  |
| 741 | 9818   | 9877 | 9935 | 9994 | ..53 | ..111 | ..170 | ..228 | ..287 | ..345 |
| 742 | 870404 | 0462 | 0521 | 0579 | 0638 | 0696  | 0755  | 0813  | 0872  | 0930  |
| 743 | 0989   | 1047 | 1106 | 1164 | 1223 | 1281  | 1339  | 1398  | 1456  | 1515  |
| 744 | 1573   | 1631 | 1690 | 1748 | 1806 | 1865  | 1923  | 1981  | 2040  | 2098  |
| 745 | 2156   | 2215 | 2273 | 2331 | 2389 | 2448  | 2506  | 2564  | 2622  | 2681  |
| 746 | 2739   | 2797 | 2855 | 2913 | 2972 | 3030  | 3088  | 3146  | 3204  | 3262  |
| 747 | 3321   | 3379 | 3437 | 3495 | 3553 | 3611  | 3669  | 3727  | 3785  | 3844  |
| 748 | 3902   | 3960 | 4018 | 4076 | 4134 | 4192  | 4250  | 4308  | 4366  | 4424  |
| 749 | 4482   | 4540 | 4598 | 4656 | 4714 | 4772  | 4830  | 4888  | 4946  | 5003  |

| N.  | 0      | 1    | 2    | 3     | 4     | 5     | 6     | 7     | 8    | 9    |
|-----|--------|------|------|-------|-------|-------|-------|-------|------|------|
| 750 | 875061 | 5119 | 5177 | 5235  | 5293  | 5351  | 5409  | 5466  | 5524 | 5582 |
| 751 | 5640   | 5698 | 5756 | 5813  | 5871  | 5929  | 5987  | 6045  | 6102 | 6160 |
| 752 | 6218   | 6276 | 6333 | 6391  | 6449  | 6507  | 6564  | 6622  | 6680 | 6737 |
| 753 | 6795   | 6853 | 6910 | 6968  | 7026  | 7083  | 7141  | 7199  | 7256 | 7314 |
| 754 | 7371   | 7429 | 7487 | 7544  | 7602  | 7659  | 7717  | 7774  | 7832 | 7889 |
| 755 | 7947   | 8004 | 8062 | 8119  | 8177  | 8234  | 8292  | 8349  | 8407 | 8464 |
| 756 | 8522   | 8579 | 8637 | 8694  | 8752  | 8809  | 8866  | 8924  | 8981 | 9039 |
| 757 | 9096   | 9153 | 9211 | 9268  | 9325  | 9383  | 9440  | 9497  | 9555 | 9612 |
| 758 | 9669   | 9726 | 9784 | 9841  | 9898  | 9956  | . .13 | . .70 | .127 | .185 |
| 759 | 880242 | 0299 | 0356 | 0413  | 0471  | 0528  | 0580  | 0642  | 0699 | 0756 |
| 760 | 0814   | 0871 | 0928 | 0985  | 1042  | 1099  | 1156  | 1213  | 1271 | 1328 |
| 761 | 1385   | 1442 | 1499 | 1556  | 1613  | 1670  | 1727  | 1784  | 1841 | 1898 |
| 762 | 1955   | 2012 | 2069 | 2126  | 2183  | 2240  | 2297  | 2354  | 2411 | 2468 |
| 763 | 2525   | 2581 | 2638 | 2695  | 2752  | 2809  | 2866  | 2923  | 2980 | 3037 |
| 764 | 3093   | 3150 | 3207 | 3264  | 3321  | 3377  | 3434  | 3491  | 3548 | 3605 |
| 765 | 3661   | 3718 | 3775 | 3832  | 3888  | 3945  | 4002  | 4059  | 4115 | 4172 |
| 766 | 4229   | 4285 | 4342 | 4399  | 4455  | 4512  | 4569  | 4625  | 4682 | 4739 |
| 767 | 4795   | 4852 | 4909 | 4965  | 5022  | 5078  | 5135  | 5192  | 5248 | 5305 |
| 768 | 5361   | 5418 | 5474 | 5531  | 5587  | 5644  | 5700  | 5757  | 5813 | 5870 |
| 769 | 5926   | 5983 | 6039 | 6096  | 6152  | 6209  | 6265  | 6321  | 6378 | 6434 |
| 770 | 6491   | 6547 | 6604 | 6660  | 6716  | 6773  | 6829  | 6885  | 6942 | 6998 |
| 771 | 7054   | 7111 | 7167 | 7223  | 7280  | 7336  | 7392  | 7449  | 7505 | 7561 |
| 772 | 7617   | 7674 | 7730 | 7786  | 7842  | 7898  | 7955  | 8011  | 8067 | 8123 |
| 773 | 8179   | 8236 | 8292 | 8348  | 8404  | 8460  | 8516  | 8573  | 8629 | 8685 |
| 774 | 8741   | 8797 | 8853 | 8909  | 8965  | 9021  | 9077  | 9134  | 9190 | 9246 |
| 775 | 9302   | 9358 | 9414 | 9470  | 9526  | 9582  | 9638  | 9694  | 9750 | 9806 |
| 776 | 9862   | 9918 | 0974 | . .30 | . .86 | .141  | .197  | .253  | .309 | .365 |
| 777 | 890421 | 0477 | 0533 | 0589  | 0645  | 0700  | 0756  | 0812  | 0868 | 0924 |
| 778 | 0980   | 1035 | 1091 | 1147  | 1203  | 1259  | 1314  | 1370  | 1426 | 1482 |
| 779 | 1537   | 1593 | 1649 | 1705  | 1760  | 1816  | 1872  | 1928  | 1983 | 2039 |
| 780 | 2095   | 2150 | 2206 | 2262  | 2317  | 2373  | 2429  | 2484  | 2540 | 2595 |
| 781 | 2651   | 2707 | 2762 | 2818  | 2873  | 2929  | 2985  | 3040  | 3096 | 3151 |
| 782 | 3207   | 3262 | 3318 | 3373  | 3429  | 3484  | 3540  | 3595  | 3651 | 3706 |
| 783 | 3762   | 3817 | 3873 | 3928  | 3984  | 4039  | 4094  | 4150  | 4205 | 4261 |
| 784 | 4316   | 4371 | 4427 | 4482  | 4538  | 4593  | 4648  | 4704  | 4759 | 4814 |
| 785 | 4870   | 4925 | 4980 | 5036  | 5091  | 5146  | 5201  | 5257  | 5312 | 5367 |
| 786 | 5423   | 5478 | 5533 | 5588  | 5644  | 5699  | 5754  | 5809  | 5864 | 5920 |
| 787 | 5975   | 6030 | 6085 | 6140  | 6195  | 6251  | 6306  | 6361  | 6416 | 6471 |
| 788 | 6526   | 6581 | 6636 | 6692  | 6747  | 6802  | 6857  | 6912  | 6967 | 7022 |
| 789 | 7077   | 7132 | 7187 | 7242  | 7297  | 7352  | 7407  | 7462  | 7517 | 7572 |
| 790 | 7627   | 7683 | 7737 | 7792  | 7847  | 7902  | 7957  | 8012  | 8067 | 8122 |
| 791 | 8176   | 8231 | 8286 | 8341  | 8396  | 8451  | 8506  | 8561  | 8615 | 8670 |
| 792 | 8725   | 8780 | 8835 | 8890  | 8944  | 8999  | 9054  | 9109  | 9164 | 9218 |
| 793 | 9273   | 9328 | 9383 | 9437  | 9492  | 9547  | 9602  | 9656  | 9711 | 9766 |
| 794 | 9821   | 9875 | 9930 | 9985  | . .39 | . .94 | .149  | .203  | .258 | .312 |
| 795 | 900367 | 0422 | 0476 | 0531  | 0586  | 0640  | 0695  | 0749  | 0804 | 0859 |
| 796 | 0913   | 0968 | 1022 | 1077  | 1131  | 1186  | 1240  | 1295  | 1349 | 1404 |
| 797 | 1458   | 1513 | 1567 | 1622  | 1676  | 1736  | 1785  | 1840  | 1894 | 1948 |
| 798 | 2003   | 2057 | 2112 | 2166  | 2221  | 2275  | 2329  | 2384  | 2438 | 2492 |
| 799 | 2547   | 2601 | 2655 | 2710  | 2764  | 2818  | 2873  | 2927  | 2981 | 3036 |

## OF NUMBERS.

17

| N.  | 0       | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|---------|------|------|------|------|------|------|------|------|------|
| 800 | 993090  | 3144 | 3199 | 3253 | 3307 | 3361 | 3416 | 3470 | 3524 | 3578 |
| 801 | 3633    | 3687 | 3741 | 3795 | 3849 | 3904 | 3958 | 4012 | 4066 | 4120 |
| 802 | 4174    | 4229 | 4283 | 4337 | 4391 | 4445 | 4499 | 4553 | 4607 | 4661 |
| 803 | 4716    | 4770 | 4824 | 4878 | 4932 | 4986 | 5040 | 5094 | 5148 | 5202 |
| 804 | 5256    | 5310 | 5364 | 5418 | 5472 | 5526 | 5580 | 5634 | 5688 | 5742 |
| 805 | 5796    | 5850 | 5904 | 5958 | 6012 | 6066 | 6119 | 6173 | 6227 | 6281 |
| 806 | 6335    | 6389 | 6443 | 6497 | 6551 | 6604 | 6658 | 6712 | 6766 | 6820 |
| 807 | 6874    | 6927 | 6981 | 7035 | 7089 | 7143 | 7196 | 7250 | 7304 | 7358 |
| 808 | 7411    | 7465 | 7519 | 7573 | 7626 | 7680 | 7734 | 7787 | 7841 | 7895 |
| 809 | 7949    | 8002 | 8056 | 8110 | 8163 | 8217 | 8270 | 8324 | 8378 | 8431 |
| 810 | 8485    | 8539 | 8592 | 8646 | 8699 | 8753 | 8807 | 8860 | 8914 | 8967 |
| 811 | 9021    | 9074 | 9128 | 9181 | 9235 | 9289 | 9342 | 9396 | 9449 | 9503 |
| 812 | 9556    | 9610 | 9663 | 9716 | 9770 | 9823 | 9877 | 9930 | 9984 | . 37 |
| 813 | 910091  | 0144 | 0197 | 0251 | 0304 | 0358 | 0411 | 0464 | 0518 | 0571 |
| 814 | 0624    | 0678 | 0731 | 0784 | 0838 | 0891 | 0944 | 0998 | 1051 | 1104 |
| 815 | 1158    | 1211 | 1264 | 1317 | 1371 | 1424 | 1477 | 1530 | 1584 | 1637 |
| 816 | 1690    | 1743 | 1797 | 1850 | 1903 | 1956 | 2009 | 2063 | 2115 | 2169 |
| 817 | 2222    | 2275 | 2328 | 2381 | 2435 | 2488 | 2541 | 2594 | 2648 | 2700 |
| 818 | 2753    | 2806 | 2859 | 2913 | 2966 | 3019 | 3072 | 3125 | 3178 | 3231 |
| 819 | 3284    | 3337 | 3390 | 3443 | 3496 | 3549 | 3602 | 3655 | 3708 | 3761 |
| 820 | 3814    | 3867 | 3920 | 3973 | 4026 | 4079 | 4132 | 4184 | 4237 | 4290 |
| 821 | 4343    | 4396 | 4449 | 4502 | 4555 | 4608 | 4660 | 4713 | 4766 | 4819 |
| 822 | 4872    | 4925 | 4977 | 5030 | 5083 | 5136 | 5189 | 5241 | 5294 | 5347 |
| 823 | 5400    | 5453 | 5505 | 5558 | 5611 | 5664 | 5716 | 5769 | 5822 | 5875 |
| 824 | 5927    | 5980 | 6033 | 6085 | 6138 | 6191 | 6243 | 6296 | 6349 | 6401 |
| 825 | 6454    | 6507 | 6559 | 6612 | 6664 | 6717 | 6770 | 6822 | 6875 | 6927 |
| 826 | 6980    | 7033 | 7085 | 7138 | 7190 | 7243 | 7295 | 7348 | 7400 | 7453 |
| 827 | 7506    | 7558 | 7611 | 7663 | 7716 | 7768 | 7820 | 7873 | 7925 | 7978 |
| 828 | 8030    | 8083 | 8135 | 8188 | 8240 | 8293 | 8345 | 8397 | 8450 | 8502 |
| 829 | 8555    | 8607 | 8659 | 8712 | 8764 | 8816 | 8869 | 8921 | 8973 | 9026 |
| 830 | 9078    | 9130 | 9183 | 9235 | 9287 | 9340 | 9392 | 9444 | 9496 | 9549 |
| 831 | 9601    | 9653 | 9706 | 9758 | 9810 | 9862 | 9914 | 9967 | . 19 | . 71 |
| 832 | 9920123 | 0176 | 0228 | 0280 | 0332 | 0384 | 0436 | 0489 | 0541 | 0593 |
| 833 | 0645    | 0697 | 0749 | 0801 | 0853 | 0906 | 0958 | 1010 | 1062 | 1114 |
| 834 | 1166    | 1218 | 1270 | 1322 | 1374 | 1426 | 1478 | 1530 | 1582 | 1634 |
| 835 | 1686    | 1738 | 1790 | 1842 | 1894 | 1946 | 1998 | 2050 | 2102 | 2154 |
| 836 | 2206    | 2258 | 2310 | 2362 | 2414 | 2466 | 2518 | 2570 | 2622 | 2674 |
| 837 | 2725    | 2777 | 2829 | 2881 | 2933 | 2985 | 3037 | 3089 | 3140 | 3192 |
| 838 | 3244    | 3296 | 3348 | 3399 | 3451 | 3503 | 3555 | 3607 | 3658 | 3710 |
| 839 | 3762    | 3814 | 3865 | 3917 | 3969 | 4021 | 4072 | 4124 | 4174 | 4228 |
| 840 | 4279    | 4331 | 4383 | 4434 | 4486 | 4538 | 4589 | 4641 | 4693 | 4744 |
| 841 | 4796    | 4848 | 4899 | 4951 | 5003 | 5054 | 5106 | 5157 | 5209 | 5261 |
| 842 | 5312    | 5364 | 5415 | 5467 | 5518 | 5570 | 5621 | 5673 | 5725 | 5776 |
| 843 | 5828    | 5874 | 5931 | 5982 | 6034 | 6085 | 6137 | 6188 | 6240 | 6291 |
| 844 | 6342    | 6394 | 6445 | 6497 | 6548 | 6600 | 6651 | 6702 | 6754 | 6805 |
| 845 | 6857    | 6908 | 6959 | 7011 | 7062 | 7114 | 7165 | 7216 | 7268 | 7319 |
| 846 | 7370    | 7422 | 7473 | 7524 | 7576 | 7627 | 7678 | 7730 | 7783 | 7832 |
| 847 | 7883    | 7935 | 7986 | 8037 | 8088 | 8140 | 8191 | 8242 | 8293 | 8345 |
| 848 | 8396    | 8447 | 8498 | 8549 | 8601 | 8652 | 8703 | 8754 | 8805 | 8857 |
| 849 | 8908    | 8959 | 9010 | 9061 | 9112 | 9163 | 9216 | 9266 | 9317 | 9368 |

| N.  | 0      | 1    | 2      | 3      | 4      | 5     | 6     | 7     | 8     | 9     |
|-----|--------|------|--------|--------|--------|-------|-------|-------|-------|-------|
| 850 | 929419 | 9473 | 9521   | 9572   | 9623   | 9674  | 9725  | 9776  | 9827  | 9879  |
| 851 | 9930   | 9981 | . . 32 | . . 83 | . 134  | . 185 | . 236 | . 287 | . 338 | . 389 |
| 852 | 930440 | 0491 | 0542   | 0592   | 0643   | 0694  | 0745  | 0796  | 0847  | 0898  |
| 853 | 0949   | 1000 | 1051   | 1102   | 1153   | 1204  | 1254  | 1305  | 1356  | 1407  |
| 854 | 1458   | 1509 | 1560   | 1610   | 1661   | 1712  | 1763  | 1814  | 1865  | 1915  |
| 855 | 1966   | 2017 | 2068   | 2118   | 2169   | 2220  | 2271  | 2322  | 2372  | 2423  |
| 856 | 2474   | 2524 | 2575   | 2626   | 2677   | 2727  | 2778  | 2829  | 2879  | 2930  |
| 857 | 2981   | 3031 | 3082   | 3133   | 3183   | 3234  | 3285  | 3335  | 3386  | 3437  |
| 858 | 3487   | 3538 | 3589   | 3639   | 3690   | 3740  | 3791  | 3841  | 3892  | 3943  |
| 859 | 3993   | 4044 | 4094   | 4145   | 4195   | 4246  | 4296  | 4347  | 4397  | 4448  |
| 860 | 4498   | 4549 | 4599   | 4650   | 4700   | 4751  | 4801  | 4852  | 4902  | 4953  |
| 861 | 5003   | 5054 | 5104   | 5154   | 5205   | 5255  | 5306  | 5356  | 5406  | 5457  |
| 862 | 5507   | 5558 | 5608   | 5658   | 5709   | 5759  | 5809  | 5860  | 5910  | 5960  |
| 863 | 6011   | 6061 | 6111   | 6162   | 6212   | 6262  | 6313  | 6363  | 6413  | 6463  |
| 864 | 6514   | 6564 | 6614   | 6665   | 6715   | 6765  | 6815  | 6865  | 6916  | 6966  |
| 865 | 7016   | 7066 | 7117   | 7167   | 7217   | 7267  | 7317  | 7367  | 7418  | 7468  |
| 866 | 7518   | 7568 | 7618   | 7668   | 7718   | 7769  | 7819  | 7869  | 7919  | 7969  |
| 867 | 8019   | 8069 | 8119   | 8169   | 8219   | 8269  | 8320  | 8370  | 8420  | 8470  |
| 868 | 8520   | 8570 | 8620   | 8670   | 8720   | 8770  | 8820  | 8870  | 8919  | 8970  |
| 869 | 9020   | 9070 | 9120   | 9170   | 9220   | 9270  | 9320  | 9369  | 9419  | 9469  |
| 870 | 9519   | 9569 | 9616   | 9669   | 9719   | 9769  | 9819  | 9869  | 9918  | 9968  |
| 871 | 940018 | 0068 | 0118   | 0168   | 0218   | 0267  | 0317  | 0367  | 0417  | 0467  |
| 872 | 0516   | 0566 | 0616   | 0666   | 0716   | 0765  | 0815  | 0865  | 0915  | 0964  |
| 873 | 1014   | 1064 | 1114   | 1163   | 1213   | 1263  | 1313  | 1362  | 1412  | 1462  |
| 874 | 1511   | 1561 | 1611   | 1660   | 1710   | 1760  | 1809  | 1859  | 1909  | 1958  |
| 875 | 2008   | 2058 | 2107   | 2157   | 2207   | 2256  | 2306  | 2355  | 2405  | 2455  |
| 876 | 2504   | 2554 | 2603   | 2653   | 2702   | 2752  | 2801  | 2851  | 2901  | 2950  |
| 877 | 3000   | 3049 | 3099   | 3148   | 3198   | 3247  | 3297  | 3346  | 3396  | 3445  |
| 878 | 3495   | 3544 | 3593   | 3643   | 3692   | 3742  | 3791  | 3841  | 3890  | 3939  |
| 879 | 3989   | 4038 | 4088   | 4137   | 4186   | 4236  | 4285  | 4335  | 4384  | 4433  |
| 880 | 4483   | 4532 | 4581   | 4631   | 4680   | 4729  | 4779  | 4828  | 4877  | 4927  |
| 881 | 4976   | 5025 | 5074   | 5124   | 5173   | 5222  | 5272  | 5321  | 5370  | 5419  |
| 882 | 5469   | 5518 | 5567   | 5616   | 5665   | 5715  | 5764  | 5813  | 5862  | 5912  |
| 883 | 5961   | 6010 | 6059   | 6108   | 6157   | 6207  | 6256  | 6305  | 6354  | 6403  |
| 884 | 6452   | 6501 | 6551   | 6600   | 6649   | 6698  | 6747  | 6796  | 6845  | 6894  |
| 885 | 6943   | 6992 | 7041   | 7090   | 7140   | 7189  | 7238  | 7287  | 7336  | 7385  |
| 886 | 7434   | 7483 | 7532   | 7581   | 7630   | 7679  | 7728  | 7777  | 7826  | 7875  |
| 887 | 7924   | 7973 | 8022   | 8070   | 8119   | 8168  | 8217  | 8266  | 8315  | 8365  |
| 888 | 8413   | 8462 | 8511   | 8560   | 8609   | 8657  | 8706  | 8755  | 8804  | 8853  |
| 889 | 8902   | 8951 | 8999   | 9048   | 9097   | 9146  | 9195  | 9244  | 9292  | 9341  |
| 890 | 9390   | 9439 | 9488   | 9536   | 9585   | 9634  | 9683  | 9731  | 9780  | 9829  |
| 891 | 9878   | 9926 | 9975   | . . 24 | . . 73 | . 121 | . 170 | . 219 | . 267 | . 316 |
| 892 | 950365 | 0414 | 0462   | 0511   | 0560   | 0608  | 0657  | 0706  | 0754  | 0803  |
| 893 | 0851   | 0900 | 0949   | 0997   | 1046   | 1095  | 1143  | 1192  | 1240  | 1289  |
| 894 | 1338   | 1386 | 1435   | 1483   | 1532   | 1580  | 1629  | 1677  | 1726  | 1775  |
| 895 | 1823   | 1872 | 1920   | 1969   | 2017   | 2066  | 2114  | 2163  | 2211  | 2260  |
| 896 | 2308   | 2356 | 2405   | 2453   | 2502   | 2550  | 2599  | 2647  | 2696  | 2744  |
| 897 | 2792   | 2841 | 2889   | 2938   | 2986   | 3034  | 3083  | 3131  | 3180  | 3228  |
| 898 | 3276   | 3325 | 3373   | 3421   | 3470   | 3518  | 3566  | 3615  | 3663  | 3711  |
| 899 | 3760   | 3808 | 3856   | 3905   | 3953   | 4001  | 4049  | 4098  | 4146  | 4194  |

## OF NUMBERS.

19

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 900 | 954243 | 4291 | 4339 | 4387 | 4435 | 4484 | 4532 | 4580 | 4628 | 4677 |
| 901 | 4725   | 4773 | 4821 | 4869 | 4918 | 4966 | 5014 | 5062 | 5110 | 5158 |
| 902 | 5207   | 5255 | 5303 | 5351 | 5399 | 5447 | 5495 | 5543 | 5592 | 5640 |
| 903 | 5688   | 5736 | 5784 | 5832 | 5880 | 5928 | 5976 | 6024 | 6072 | 6120 |
| 904 | 6168   | 6216 | 6265 | 6313 | 6361 | 6409 | 6457 | 6505 | 6553 | 6601 |
| 905 | 6649   | 6697 | 6745 | 6793 | 6840 | 6888 | 6936 | 6984 | 7032 | 7080 |
| 906 | 7128   | 7176 | 7224 | 7272 | 7320 | 7368 | 7416 | 7464 | 7512 | 7559 |
| 907 | 7607   | 7655 | 7703 | 7751 | 7799 | 7847 | 7894 | 7942 | 7990 | 8038 |
| 908 | 8086   | 8134 | 8181 | 8229 | 8277 | 8325 | 8373 | 8421 | 8468 | 8516 |
| 909 | 8564   | 8612 | 8659 | 8707 | 8755 | 8803 | 8850 | 8898 | 8946 | 8994 |
| 910 | 9041   | 9089 | 9137 | 9185 | 9232 | 9280 | 9328 | 9375 | 9423 | 9471 |
| 911 | 9518   | 9566 | 9614 | 9661 | 9709 | 9757 | 9804 | 9852 | 9900 | 9947 |
| 912 | 9995   | .42  | .90  | .138 | .185 | .233 | .280 | .328 | .376 | .423 |
| 913 | 960471 | 0518 | 0566 | 0613 | 0661 | 0709 | 0756 | 0804 | 0851 | 0899 |
| 914 | 0946   | 0994 | 1041 | 1089 | 1136 | 1184 | 1231 | 1279 | 1326 | 1374 |
| 915 | 1421   | 1469 | 1516 | 1563 | 1611 | 1658 | 1706 | 1753 | 1801 | 1848 |
| 916 | 1895   | 1943 | 1990 | 2038 | 2085 | 2132 | 2180 | 2227 | 2275 | 2322 |
| 917 | 2369   | 2417 | 2464 | 2511 | 2559 | 2606 | 2653 | 2701 | 2748 | 2795 |
| 918 | 2843   | 2890 | 2937 | 2985 | 3032 | 3079 | 3126 | 3174 | 3221 | 3268 |
| 919 | 3316   | 3363 | 3410 | 3457 | 3504 | 3552 | 3599 | 3646 | 3693 | 3741 |
| 920 | 3788   | 3835 | 3882 | 3929 | 3977 | 4024 | 4071 | 4118 | 4165 | 4212 |
| 921 | 4260   | 4307 | 4354 | 4401 | 4448 | 4495 | 4542 | 4590 | 4637 | 4684 |
| 922 | 4731   | 4778 | 4825 | 4872 | 4919 | 4966 | 5013 | 5061 | 5108 | 5155 |
| 923 | 5202   | 5249 | 5296 | 5343 | 5390 | 5437 | 5484 | 5531 | 5578 | 5625 |
| 924 | 5672   | 5719 | 5766 | 5813 | 5860 | 5907 | 5954 | 6001 | 6048 | 6095 |
| 925 | 6142   | 6189 | 6236 | 6283 | 6329 | 6376 | 6423 | 6470 | 6517 | 6564 |
| 926 | 6611   | 6658 | 6705 | 6752 | 6799 | 6845 | 6892 | 6939 | 6986 | 7033 |
| 927 | 7080   | 7127 | 7173 | 7220 | 7267 | 7314 | 7361 | 7408 | 7454 | 7501 |
| 928 | 7548   | 7595 | 7642 | 7688 | 7735 | 7782 | 7829 | 7875 | 7922 | 7969 |
| 929 | 8016   | 8062 | 8109 | 8156 | 8203 | 8249 | 8296 | 8343 | 8390 | 8436 |
| 930 | 8483   | 8530 | 8576 | 8623 | 8670 | 8716 | 8763 | 8810 | 8856 | 8903 |
| 931 | 8950   | 8996 | 9043 | 9090 | 9136 | 9183 | 9229 | 9276 | 9323 | 9369 |
| 932 | 9416   | 9463 | 9509 | 9556 | 9602 | 9649 | 9695 | 9742 | 9789 | 9835 |
| 933 | 9882   | 9928 | 9975 | .21  | .68  | .114 | .161 | .207 | .254 | .300 |
| 934 | 970347 | 0393 | 0440 | 0486 | 0533 | 0579 | 0626 | 0672 | 0719 | 0765 |
| 935 | 0812   | 0858 | 0904 | 0951 | 0997 | 1044 | 1090 | 1137 | 1183 | 1229 |
| 936 | 1276   | 1322 | 1369 | 1415 | 1461 | 1508 | 1554 | 1601 | 1647 | 1693 |
| 937 | 1740   | 1786 | 1832 | 1879 | 1925 | 1971 | 2018 | 2064 | 2110 | 2157 |
| 938 | 2203   | 2249 | 2295 | 2342 | 2388 | 2434 | 2481 | 2527 | 2573 | 2619 |
| 939 | 2666   | 2712 | 2758 | 2804 | 2851 | 2897 | 2943 | 2989 | 3035 | 3082 |
| 940 | 3128   | 3174 | 3220 | 3266 | 3313 | 3359 | 3405 | 3451 | 3497 | 3543 |
| 941 | 3590   | 3636 | 3682 | 3728 | 3774 | 3820 | 3866 | 3913 | 3959 | 4005 |
| 942 | 4051   | 4097 | 4143 | 4189 | 4235 | 4281 | 4327 | 4374 | 4420 | 4466 |
| 943 | 4512   | 4558 | 4604 | 4650 | 4696 | 4742 | 4788 | 4834 | 4880 | 4926 |
| 944 | 4972   | 5018 | 5064 | 5110 | 5156 | 5202 | 5248 | 5294 | 5340 | 5386 |
| 945 | 5432   | 5478 | 5524 | 5570 | 5616 | 5662 | 5707 | 5753 | 5799 | 5845 |
| 946 | 5891   | 5937 | 5983 | 6029 | 6075 | 6121 | 6167 | 6212 | 6258 | 6304 |
| 947 | 6350   | 6396 | 6442 | 6488 | 6533 | 6579 | 6625 | 6671 | 6717 | 6763 |
| 948 | 6808   | 6854 | 6900 | 6946 | 6992 | 7037 | 7083 | 7129 | 7175 | 7220 |
| 949 | 7266   | 7312 | 7358 | 7403 | 7449 | 7495 | 7541 | 7586 | 7632 | 7678 |

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|-----|--------|------|------|------|------|------|------|------|------|------|
| 950 | 977724 | 7769 | 7815 | 7861 | 7906 | 7952 | 7998 | 8043 | 8089 | 8135 |
| 951 | 8181   | 8226 | 8272 | 8317 | 8363 | 8409 | 8454 | 8500 | 8546 | 8591 |
| 952 | 8637   | 8683 | 8728 | 8774 | 8819 | 8865 | 8911 | 8956 | 9002 | 9047 |
| 953 | 9093   | 9138 | 9184 | 9230 | 9275 | 9321 | 9366 | 9412 | 9457 | 9503 |
| 954 | 9548   | 9594 | 9639 | 9685 | 9730 | 9776 | 9821 | 9867 | 9912 | 9958 |
| 955 | 980003 | 0049 | 0094 | 0140 | 0185 | 0231 | 0276 | 0322 | 0367 | 0412 |
| 956 | 0458   | 0503 | 0549 | 0594 | 0640 | 0685 | 0730 | 0776 | 0821 | 0867 |
| 957 | 0912   | 0957 | 1003 | 1048 | 1093 | 1139 | 1184 | 1229 | 1275 | 1320 |
| 958 | 1366   | 1411 | 1456 | 1501 | 1547 | 1592 | 1637 | 1683 | 1728 | 1773 |
| 959 | 1819   | 1864 | 1909 | 1954 | 2000 | 2045 | 2090 | 2135 | 2181 | 2226 |
| 960 | 2271   | 2316 | 2362 | 2407 | 2452 | 2497 | 2543 | 2588 | 2633 | 2678 |
| 961 | 2723   | 2769 | 2814 | 2859 | 2904 | 2949 | 2994 | 3040 | 3085 | 3130 |
| 962 | 3175   | 3220 | 3265 | 3310 | 3356 | 3401 | 3446 | 3491 | 3536 | 3581 |
| 963 | 3626   | 3671 | 3716 | 3762 | 3807 | 3852 | 3897 | 3942 | 3987 | 4032 |
| 964 | 4077   | 4122 | 4167 | 4212 | 4257 | 4302 | 4347 | 4392 | 4437 | 4482 |
| 965 | 4527   | 4572 | 4617 | 4662 | 4707 | 4752 | 4797 | 4842 | 4887 | 4932 |
| 966 | 4977   | 5022 | 5067 | 5112 | 5157 | 5202 | 5247 | 5292 | 5337 | 5382 |
| 967 | 5426   | 5471 | 5516 | 5561 | 5606 | 5651 | 5696 | 5741 | 5786 | 5831 |
| 968 | 5875   | 5920 | 5965 | 6010 | 6055 | 6100 | 6144 | 6189 | 6234 | 6279 |
| 969 | 6324   | 6369 | 6413 | 6458 | 6503 | 6548 | 6593 | 6637 | 6682 | 6727 |
| 970 | 6772   | 6817 | 6861 | 6906 | 6951 | 6996 | 7040 | 7085 | 7130 | 7175 |
| 971 | 7219   | 7264 | 7309 | 7353 | 7398 | 7443 | 7488 | 7532 | 7577 | 7622 |
| 972 | 7666   | 7711 | 7756 | 7800 | 7845 | 7890 | 7934 | 7979 | 8024 | 8068 |
| 973 | 8113   | 8157 | 8202 | 8247 | 8291 | 8336 | 8381 | 8425 | 8470 | 8514 |
| 974 | 8559   | 8604 | 8648 | 8693 | 8737 | 8782 | 8826 | 8871 | 8916 | 8960 |
| 975 | 9005   | 9049 | 9093 | 9138 | 9183 | 9227 | 9272 | 9316 | 9361 | 9405 |
| 976 | 9450   | 9494 | 9539 | 9583 | 9628 | 9672 | 9717 | 9761 | 9806 | 9850 |
| 977 | 9895   | 9939 | 9983 | .28  | .72  | .117 | .161 | .206 | .250 | .294 |
| 978 | 990339 | 0383 | 0428 | 0472 | 0516 | 0561 | 0605 | 0650 | 0694 | 0738 |
| 979 | 0783   | 0827 | 0871 | 0916 | 0960 | 1004 | 1049 | 1093 | 1137 | 1182 |
| 980 | 1226   | 1270 | 1315 | 1359 | 1403 | 1448 | 1492 | 1536 | 1580 | 1625 |
| 981 | 1669   | 1713 | 1758 | 1802 | 1846 | 1890 | 1935 | 1979 | 2023 | 2067 |
| 982 | 2111   | 2156 | 2200 | 2244 | 2288 | 2333 | 2377 | 2421 | 2465 | 2509 |
| 983 | 2554   | 2598 | 2642 | 2686 | 2730 | 2774 | 2819 | 2863 | 2907 | 2951 |
| 984 | 2995   | 3039 | 3083 | 3127 | 3172 | 3216 | 3260 | 3304 | 3348 | 3392 |
| 985 | 3436   | 3480 | 3524 | 3568 | 3613 | 3657 | 3701 | 3745 | 3789 | 3833 |
| 986 | 3877   | 3921 | 3965 | 4009 | 4053 | 4097 | 4141 | 4185 | 4229 | 4273 |
| 987 | 4317   | 4361 | 4405 | 4449 | 4493 | 4537 | 4581 | 4625 | 4669 | 4713 |
| 988 | 4757   | 4801 | 4845 | 4889 | 4933 | 4977 | 5021 | 5065 | 5109 | 5152 |
| 989 | 5196   | 5240 | 5284 | 5328 | 5372 | 5416 | 5460 | 5504 | 5547 | 5591 |
| 990 | 5635   | 5679 | 5723 | 5767 | 5811 | 5854 | 5898 | 5942 | 5986 | 6030 |
| 991 | 6074   | 6117 | 6161 | 6205 | 6249 | 6293 | 6337 | 6380 | 6424 | 6468 |
| 992 | 6512   | 6555 | 6599 | 6643 | 6687 | 6731 | 6774 | 6818 | 6862 | 6906 |
| 993 | 6949   | 6993 | 7037 | 7080 | 7124 | 7168 | 7212 | 7255 | 7299 | 7343 |
| 994 | 7386   | 7430 | 7474 | 7517 | 7561 | 7605 | 7648 | 7692 | 7736 | 7779 |
| 995 | 7823   | 7867 | 7910 | 7954 | 7998 | 8041 | 8085 | 8129 | 8172 | 8216 |
| 996 | 8259   | 8303 | 8347 | 8390 | 8434 | 8477 | 8521 | 8564 | 8608 | 8652 |
| 997 | 8695   | 8739 | 8792 | 8836 | 8880 | 8923 | 8966 | 9009 | 9053 | 9097 |
| 998 | 9131   | 9174 | 9218 | 9261 | 9305 | 9348 | 9392 | 9435 | 9479 | 9522 |
| 999 | 9565   | 9609 | 9652 | 9696 | 9739 | 9783 | 9826 | 9870 | 9913 | 9957 |

TABLE II. Log. Sines and Tangents. (0°) Natural Sines.

21

|    | Sine.    | D.10''   | Cosine.   | D.10'' | Tang.    | D.10'' | Cotang.   | N.sine. | N. cos. |    |
|----|----------|----------|-----------|--------|----------|--------|-----------|---------|---------|----|
| 0  | 0.000000 |          | 10.000000 |        | 0.000000 |        | Infinite. | 00000   | 100000  | 60 |
| 1  | 6.463726 |          | 000000    |        | 6.463726 |        | 13.536274 | 00029   | 100000  | 59 |
| 2  | 764756   |          | 000000    |        | 764756   |        | 235244    | 00058   | 100000  | 58 |
| 3  | 940847   |          | 000000    |        | 940847   |        | 059153    | 00087   | 100000  | 57 |
| 4  | 7.065786 |          | 000000    |        | 7.065786 |        | 12.934214 | 00116   | 100000  | 56 |
| 5  | 162696   |          | 000000    |        | 162696   |        | 837304    | 00145   | 100000  | 55 |
| 6  | 241877   | 9.999999 |           |        | 241878   |        | 758122    | 00175   | 100000  | 54 |
| 7  | 308824   | 999999   |           |        | 308825   |        | 691175    | 00204   | 100000  | 53 |
| 8  | 366816   | 999999   |           |        | 366817   |        | 633183    | 00233   | 100000  | 52 |
| 9  | 417968   | 999999   |           |        | 417970   |        | 582030    | 00262   | 100000  | 51 |
| 10 | 463725   | 999998   |           |        | 463727   |        | 536273    | 00291   | 100000  | 50 |
| 11 | 7.505118 | 9.999998 |           |        | 7.505120 |        | 12.494880 | 00320   | 99999   | 49 |
| 12 | 542906   | 999997   |           |        | 542909   |        | 457091    | 00349   | 99999   | 48 |
| 13 | 577668   | 999997   |           |        | 577672   |        | 422328    | 00378   | 99999   | 47 |
| 14 | 609853   | 999996   |           |        | 609857   |        | 390143    | 00407   | 99999   | 46 |
| 15 | 639816   | 999996   |           |        | 639820   |        | 360180    | 00436   | 99999   | 45 |
| 16 | 667845   | 999995   |           |        | 667849   |        | 332151    | 00465   | 99999   | 44 |
| 17 | 694173   | 999995   |           |        | 694179   |        | 305821    | 00495   | 99999   | 43 |
| 18 | 718997   | 999994   |           |        | 719003   |        | 280997    | 00524   | 99999   | 42 |
| 19 | 742477   | 999993   |           |        | 742484   |        | 257516    | 00553   | 99998   | 41 |
| 20 | 764754   | 999993   |           |        | 764761   |        | 235239    | 00582   | 99998   | 40 |
| 21 | 7.785943 | 9.999992 |           |        | 7.785951 |        | 12.214049 | 00611   | 99998   | 39 |
| 22 | 806146   | 999991   |           |        | 806155   |        | 193845    | 00640   | 99998   | 38 |
| 23 | 825451   | 999990   |           |        | 825460   |        | 174540    | 00669   | 99998   | 37 |
| 24 | 843934   | 999989   |           |        | 843944   |        | 156056    | 00698   | 99998   | 36 |
| 25 | 861663   | 999988   |           |        | 861674   |        | 138326    | 00727   | 99997   | 35 |
| 26 | 878695   | 999988   |           |        | 878708   |        | 121292    | 00756   | 99997   | 34 |
| 27 | 895085   | 999987   |           |        | 895099   |        | 104901    | 00785   | 99997   | 33 |
| 28 | 910879   | 999986   |           |        | 910894   |        | 089106    | 00814   | 99997   | 32 |
| 29 | 926119   | 999985   |           |        | 926134   |        | 073866    | 00844   | 99996   | 31 |
| 30 | 940842   | 999983   |           |        | 940858   |        | 059142    | 00873   | 99996   | 30 |
| 31 | 7.955082 | 9.999982 |           |        | 7.955100 |        | 12.044900 | 00902   | 99996   | 29 |
| 32 | 968870   | 999981   | 0.2       |        | 968889   | 2298   | 031111    | 00931   | 99996   | 28 |
| 33 | 982233   | 999980   | 0.2       |        | 982253   | 2227   | 017747    | 00960   | 99995   | 27 |
| 34 | 995198   | 999979   | 0.2       |        | 995219   | 2161   | 004781    | 00989   | 99995   | 26 |
| 35 | 8.007787 | 999977   | 0.2       |        | 8.007809 | 2098   | 11.992191 | 01018   | 99995   | 25 |
| 36 | 020021   | 999976   | 0.2       |        | 020045   | 2039   | 979955    | 01047   | 99995   | 24 |
| 37 | 031919   | 999975   | 0.2       |        | 031945   | 1983   | 968055    | 01076   | 99994   | 23 |
| 38 | 043501   | 999973   | 0.2       |        | 043527   | 1930   | 956473    | 01105   | 99994   | 22 |
| 39 | 054781   | 999972   | 0.2       |        | 054809   | 1880   | 945191    | 01134   | 99994   | 21 |
| 40 | 065776   | 999971   | 0.2       |        | 065806   | 1833   | 934194    | 01164   | 99993   | 20 |
| 41 | 8.076500 | 9.999969 | 0.2       |        | 8.076531 | 1787   | 11.923469 | 01193   | 99993   | 19 |
| 42 | 086965   | 999968   | 0.2       |        | 086997   | 1744   | 913003    | 01222   | 99993   | 18 |
| 43 | 097183   | 999966   | 0.2       |        | 097217   | 1703   | 902783    | 01251   | 99992   | 17 |
| 44 | 107167   | 999964   | 0.2       |        | 107202   | 1664   | 892797    | 01280   | 99992   | 16 |
| 45 | 116926   | 999963   | 0.3       |        | 116963   | 1627   | 883037    | 01309   | 99991   | 15 |
| 46 | 126471   | 999961   | 0.3       |        | 126510   | 1591   | 873490    | 01338   | 99991   | 14 |
| 47 | 135810   | 999959   | 0.3       |        | 135851   | 1557   | 864149    | 01367   | 99991   | 13 |
| 48 | 144953   | 999958   | 0.3       |        | 144996   | 1524   | 855004    | 01396   | 99990   | 12 |
| 49 | 153907   | 999956   | 0.3       |        | 153952   | 1493   | 846048    | 01425   | 99990   | 11 |
| 50 | 162681   | 999954   | 0.3       |        | 162727   | 1463   | 837273    | 01454   | 99989   | 10 |
| 51 | 8.171280 | 9.999952 | 0.3       |        | 8.171328 | 1434   | 11.828672 | 01483   | 99989   | 9  |
| 52 | 179713   | 999950   | 0.3       |        | 179763   | 1406   | 820237    | 01513   | 99989   | 8  |
| 53 | 187985   | 999948   | 0.3       |        | 188036   | 1379   | 811964    | 01542   | 99988   | 7  |
| 54 | 196102   | 999946   | 0.3       |        | 196156   | 1353   | 803544    | 01571   | 99988   | 6  |
| 55 | 204070   | 999944   | 0.3       |        | 204126   | 1328   | 795874    | 01600   | 99987   | 5  |
| 56 | 211895   | 999942   | 0.3       |        | 211953   | 1304   | 788047    | 01629   | 99987   | 4  |
| 57 | 219581   | 999940   | 0.4       |        | 219641   | 1281   | 780359    | 01658   | 99986   | 3  |
| 58 | 227134   | 999938   | 0.4       |        | 227195   | 1259   | 772505    | 01687   | 99986   | 2  |
| 59 | 234557   | 999936   | 0.4       |        | 234621   | 1238   | 765379    | 01716   | 99985   | 1  |
| 60 | 241855   | 999934   | 0.4       |        | 241921   | 1217   | 758079    | 01745   | 99985   | 0  |
|    | Cosine.  |          | Sine.     |        | Cotang.  |        | Tang.     | N. cos. | N. sine |    |

89 Degrees.



|    | Sine.    | D. 10'' | Cosine.  | D. 10'' | Tang.    | D. 10'' | Cotang.   | N. sine. | N. cos.  |
|----|----------|---------|----------|---------|----------|---------|-----------|----------|----------|
| 0  | 8.241855 |         | 9.999934 |         | 8.241921 |         | 11.758079 | 01742    | 99985    |
| 1  | 249033   | 1196    | 999932   | 0.4     | 249102   | 1197    | 758089.   | 01774    | 99984    |
| 2  | 256094   | 1177    | 999929   | 0.4     | 256165   | 1177    | 743835    | 01803    | 99984    |
| 3  | 263042   | 1158    | 999927   | 0.4     | 263115   | 1158    | 736885    | 01832    | 99983    |
| 4  | 269881   | 1140    | 999925   | 0.4     | 269956   | 1140    | 730044    | 01862    | 99983    |
| 5  | 276614   | 1122    | 999922   | 0.4     | 276691   | 1122    | 723309    | 01891    | 99982    |
| 6  | 283243   | 1105    | 999920   | 0.4     | 283323   | 1105    | 716677    | 01920    | 99982    |
| 7  | 289773   | 1088    | 999918   | 0.4     | 289856   | 1089    | 710144    | 01949    | 99981    |
| 8  | 296207   | 1072    | 999915   | 0.4     | 296292   | 1073    | 703708    | 01978    | 99980    |
| 9  | 302546   | 1056    | 999913   | 0.4     | 302634   | 1057    | 697366    | 02007    | 99980    |
| 10 | 308794   | 1041    | 999910   | 0.4     | 308884   | 1042    | 691116    | 02036    | 99979    |
| 11 | 8.314954 | 1027    | 9.999907 | 0.4     | 8.315046 | 1013    | 11.684954 | 02065    | 99979    |
| 12 | 321027   | 998     | 999905   | 0.4     | 321122   | 999     | 678878    | 02094    | 99978    |
| 13 | 327016   | 985     | 999902   | 0.4     | 327114   | 985     | 672886    | 02123    | 99977    |
| 14 | 332924   | 971     | 999899   | 0.4     | 333025   | 972     | 666975    | 02152    | 99977    |
| 15 | 338753   | 959     | 999897   | 0.5     | 333856   | 959     | 661144    | 02181    | 99976    |
| 16 | 344504   | 946     | 999894   | 0.5     | 344610   | 946     | 655390    | 02211    | 99976    |
| 17 | 350181   | 934     | 999891   | 0.5     | 350289   | 934     | 649711    | 02240    | 99975    |
| 18 | 355783   | 922     | 999888   | 0.5     | 355895   | 922     | 644105    | 02269    | 99974    |
| 19 | 361315   | 910     | 999885   | 0.5     | 361430   | 911     | 638570    | 02298    | 99974    |
| 20 | 366777   | 899     | 999882   | 0.5     | 366895   | 899     | 633105    | 02327    | 99973    |
| 21 | 8.372171 | 888     | 9.999879 | 0.5     | 8.372292 | 888     | 11.627708 | 02356    | 99972    |
| 22 | 377499   | 877     | 999876   | 0.5     | 377622   | 879     | 622378    | 02385    | 99972    |
| 23 | 382762   | 867     | 999873   | 0.5     | 382889   | 867     | 617111    | 02414    | 99971    |
| 24 | 387962   | 856     | 999870   | 0.5     | 388092   | 867     | 611908    | 02443    | 99970    |
| 25 | 393101   | 846     | 999867   | 0.5     | 393234   | 847     | 606766    | 02472    | 99969    |
| 26 | 398179   | 837     | 999864   | 0.5     | 398315   | 837     | 601685    | 02501    | 99969    |
| 27 | 403199   | 827     | 999861   | 0.5     | 403338   | 828     | 596662    | 02530    | 99968    |
| 28 | 408161   | 818     | 999858   | 0.5     | 408304   | 818     | 591696    | 02560    | 99967    |
| 29 | 413068   | 809     | 999854   | 0.5     | 413213   | 809     | 586787    | 02589    | 99966    |
| 30 | 417919   | 800     | 999851   | 0.6     | 418068   | 800     | 581932    | 02618    | 99966    |
| 31 | 8.422717 | 791     | 9.999848 | 0.6     | 8.422869 | 791     | 11.577131 | 02647    | 99965    |
| 32 | 427462   | 782     | 999844   | 0.6     | 427618   | 783     | 572382    | 02676    | 99964    |
| 33 | 432156   | 774     | 999841   | 0.6     | 432315   | 774     | 567685    | 02705    | 99963    |
| 34 | 436800   | 766     | 999838   | 0.6     | 436962   | 766     | 563038    | 02734    | 99963    |
| 35 | 441394   | 758     | 999834   | 0.6     | 441560   | 758     | 558440    | 02763    | 99962    |
| 36 | 445941   | 750     | 999831   | 0.6     | 446110   | 750     | 553890    | 02792    | 99961    |
| 37 | 450440   | 742     | 999827   | 0.6     | 450613   | 743     | 549387    | 02821    | 99960    |
| 38 | 454893   | 735     | 999823   | 0.6     | 455070   | 735     | 544930    | 02850    | 99959    |
| 39 | 459301   | 727     | 999820   | 0.6     | 459481   | 728     | 540519    | 02879    | 99959    |
| 40 | 463665   | 720     | 999816   | 0.6     | 463849   | 720     | 536151    | 02908    | 99958    |
| 41 | 8.467985 | 712     | 9.999812 | 0.6     | 8.468172 | 713     | 11.531828 | 02938    | 99957    |
| 42 | 472263   | 706     | 999809   | 0.6     | 472454   | 707     | 527546    | 02967    | 99956    |
| 43 | 476498   | 699     | 999805   | 0.6     | 476693   | 700     | 523307    | 02996    | 99955    |
| 44 | 480693   | 692     | 999801   | 0.6     | 480892   | 693     | 519108    | 03025    | 99954    |
| 45 | 484848   | 686     | 999797   | 0.7     | 485050   | 686     | 514950    | 03054    | 99953    |
| 46 | 488963   | 679     | 999793   | 0.7     | 489170   | 680     | 510830    | 03083    | 99952    |
| 47 | 493040   | 673     | 999790   | 0.7     | 493250   | 674     | 506750    | 03112    | 99952    |
| 48 | 497078   | 667     | 999786   | 0.7     | 497293   | 668     | 502707    | 03141    | 99951    |
| 49 | 501080   | 661     | 999782   | 0.7     | 501298   | 661     | 498702    | 03170    | 99950    |
| 50 | 505045   | 655     | 999778   | 0.7     | 505267   | 655     | 494733    | 03199    | 99949    |
| 51 | 8.508974 | 649     | 9.999774 | 0.7     | 8.509200 | 650     | 11.490800 | 03228    | 99948    |
| 52 | 512867   | 643     | 999769   | 0.7     | 513098   | 644     | 486902    | 03257    | 99947    |
| 53 | 516726   | 637     | 999765   | 0.7     | 516961   | 638     | 483039    | 03286    | 99946    |
| 54 | 520551   | 632     | 999761   | 0.7     | 520790   | 633     | 479210    | 03316    | 99945    |
| 55 | 524343   | 626     | 999757   | 0.7     | 524586   | 627     | 475414    | 03345    | 99944    |
| 56 | 528102   | 621     | 999753   | 0.7     | 528349   | 622     | 471651    | 03374    | 99943    |
| 57 | 531828   | 616     | 999748   | 0.7     | 532080   | 616     | 467920    | 03403    | 99942    |
| 58 | 535523   | 611     | 999744   | 0.7     | 535779   | 611     | 464221    | 03432    | 99941    |
| 59 | 539186   | 605     | 999740   | 0.7     | 539447   | 606     | 460553    | 03461    | 99940    |
| 60 | 542819   | 605     | 999735   | 0.7     | 543084   | 606     | 456916    | 03490    | 99939    |
|    | Cosine.  |         | Sine.    |         | Cotang.  |         | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (2°) Natural Sines.

23

|    | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.    | D. 10' | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 8.542819 | 600    | 9.999735 | 0.7    | 8.543084 | 602    | 11.456916 | 03490    | 99939    |
| 1  | 546422   | 595    | 999731   | 0.7    | 546691   | 596    | 453309    | 03519    | 99938    |
| 2  | 549995   | 591    | 999726   | 0.7    | 550268   | 591    | 449732    | 03548    | 99937    |
| 3  | 553539   | 586    | 999722   | 0.8    | 553817   | 587    | 446183    | 03577    | 99936    |
| 4  | 557054   | 581    | 999717   | 0.8    | 557335   | 582    | 442664    | 03606    | 99935    |
| 5  | 560540   | 576    | 999713   | 0.8    | 560828   | 577    | 439172    | 03635    | 99934    |
| 6  | 563999   | 572    | 999708   | 0.8    | 564291   | 573    | 435709    | 03664    | 99933    |
| 7  | 567431   | 567    | 999704   | 0.8    | 567727   | 568    | 432273    | 03693    | 99932    |
| 8  | 570836   | 563    | 999699   | 0.8    | 571137   | 564    | 428863    | 03723    | 99931    |
| 9  | 574214   | 559    | 999694   | 0.8    | 574520   | 559    | 425480    | 03752    | 99930    |
| 10 | 577566   | 554    | 999689   | 0.8    | 577877   | 555    | 422123    | 03781    | 99929    |
| 11 | 8.580892 | 550    | 9.999685 | 0.8    | 8.581208 | 551    | 11.418792 | 03810    | 99927    |
| 12 | 584193   | 546    | 999680   | 0.8    | 584514   | 547    | 415486    | 03839    | 99926    |
| 13 | 587469   | 542    | 999675   | 0.8    | 587795   | 543    | 412205    | 03868    | 99925    |
| 14 | 590721   | 538    | 999670   | 0.8    | 591051   | 539    | 408949    | 03897    | 99924    |
| 15 | 593948   | 534    | 999665   | 0.8    | 594283   | 535    | 405717    | 03926    | 99923    |
| 16 | 597152   | 530    | 999660   | 0.8    | 597492   | 531    | 402508    | 03955    | 99922    |
| 17 | 600332   | 526    | 999655   | 0.8    | 600677   | 527    | 399323    | 03984    | 99921    |
| 18 | 603489   | 522    | 999650   | 0.8    | 603839   | 523    | 396161    | 04013    | 99919    |
| 19 | 606623   | 519    | 999645   | 0.8    | 606978   | 519    | 393022    | 04042    | 99918    |
| 20 | 609734   | 515    | 999640   | 0.8    | 610094   | 516    | 389906    | 04071    | 99917    |
| 21 | 8.612823 | 511    | 9.999635 | 0.9    | 8.613189 | 512    | 11.386811 | 04100    | 99916    |
| 22 | 615891   | 508    | 999629   | 0.9    | 616262   | 508    | 383738    | 03129    | 99915    |
| 23 | 618937   | 504    | 999624   | 0.9    | 619813   | 505    | 380687    | 04159    | 99913    |
| 24 | 621962   | 501    | 999619   | 0.9    | 622343   | 501    | 377657    | 04188    | 99912    |
| 25 | 624965   | 497    | 999614   | 0.9    | 625352   | 498    | 374648    | 04217    | 99911    |
| 26 | 627948   | 494    | 999608   | 0.9    | 628340   | 495    | 371660    | 04246    | 99910    |
| 27 | 630911   | 490    | 999603   | 0.9    | 631308   | 491    | 368692    | 04275    | 99909    |
| 28 | 633854   | 487    | 999597   | 0.9    | 634256   | 488    | 365744    | 04304    | 99907    |
| 29 | 636776   | 484    | 999592   | 0.9    | 637184   | 485    | 362816    | 04333    | 99906    |
| 30 | 639680   | 481    | 999586   | 0.9    | 640093   | 482    | 359907    | 04362    | 99905    |
| 31 | 8.642563 | 477    | 9.999581 | 0.9    | 8.642982 | 478    | 11.357018 | 04391    | 99904    |
| 32 | 645428   | 474    | 999575   | 0.9    | 645853   | 475    | 354147    | 04420    | 99902    |
| 33 | 648274   | 471    | 999570   | 0.9    | 648704   | 472    | 351296    | 04449    | 99901    |
| 34 | 651102   | 468    | 999564   | 0.9    | 651537   | 469    | 348463    | 04478    | 99900    |
| 35 | 653911   | 465    | 999558   | 1.0    | 654352   | 466    | 345648    | 04507    | 99898    |
| 36 | 656702   | 462    | 999553   | 1.0    | 657149   | 463    | 342851    | 04536    | 99897    |
| 37 | 659475   | 459    | 999547   | 1.0    | 659928   | 460    | 340072    | 04565    | 99896    |
| 38 | 662230   | 456    | 999541   | 1.0    | 662689   | 457    | 337311    | 04594    | 99894    |
| 39 | 664968   | 453    | 999535   | 1.0    | 665433   | 454    | 334567    | 04623    | 99893    |
| 40 | 667689   | 451    | 999529   | 1.0    | 668160   | 453    | 331840    | 04652    | 99892    |
| 41 | 8.670393 | 448    | 9.999524 | 1.0    | 8.670870 | 453    | 11.329130 | 04682    | 99890    |
| 42 | 673080   | 445    | 999518   | 1.0    | 673563   | 449    | 326437    | 04711    | 99889    |
| 43 | 675751   | 442    | 999512   | 1.0    | 676239   | 446    | 323761    | 04740    | 99888    |
| 44 | 678405   | 440    | 999506   | 1.0    | 678900   | 442    | 321100    | 04769    | 99886    |
| 45 | 681043   | 437    | 999500   | 1.0    | 681544   | 438    | 318456    | 04798    | 99885    |
| 46 | 683665   | 434    | 999493   | 1.0    | 684172   | 435    | 315828    | 04827    | 99883    |
| 47 | 686272   | 432    | 999487   | 1.0    | 686784   | 433    | 313216    | 04856    | 99882    |
| 48 | 688863   | 429    | 999481   | 1.0    | 689381   | 430    | 310619    | 04885    | 99881    |
| 49 | 691438   | 427    | 999475   | 1.0    | 691963   | 428    | 308037    | 04914    | 99879    |
| 50 | 693998   | 424    | 999469   | 1.0    | 694529   | 425    | 305471    | 04943    | 99878    |
| 51 | 8.696543 | 422    | 9.999463 | 1.1    | 8.697081 | 423    | 11.302919 | 04972    | 99876    |
| 52 | 699073   | 419    | 999456   | 1.1    | 699617   | 420    | 300383    | 05001    | 99875    |
| 53 | 701589   | 417    | 999450   | 1.1    | 702139   | 418    | 297861    | 05030    | 99873    |
| 54 | 704090   | 414    | 999443   | 1.1    | 704246   | 415    | 295354    | 05059    | 99872    |
| 55 | 706577   | 412    | 999437   | 1.1    | 707140   | 413    | 292850    | 05088    | 99870    |
| 56 | 709049   | 410    | 999431   | 1.1    | 709618   | 411    | 290382    | 05117    | 99869    |
| 57 | 711507   | 407    | 999424   | 1.1    | 710208   | 408    | 287917    | 05146    | 99867    |
| 58 | 713952   | 405    | 999418   | 1.1    | 714534   | 406    | 285465    | 05175    | 99866    |
| 59 | 716383   | 403    | 999411   | 1.1    | 716972   | 404    | 283028    | 05205    | 99864    |
| 60 | 718800   |        | 999404   |        | 719396   |        | 280604    | 05234    | 99863    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

87 Degrees.

|    | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.    | D. 10' | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 8.718800 | 401    | 9.999404 | 1.1    | 8.719396 | 402    | 11.280604 | 05234    | 99863    |
| 1  | 721204   | 398    | 999398   | 1.1    | 721806   | 399    | 278194    | 05263    | 99861    |
| 2  | 723595   | 396    | 999391   | 1.1    | 724204   | 397    | 275796    | 05292    | 99860    |
| 3  | 725972   | 394    | 999384   | 1.1    | 726588   | 395    | 273412    | 05321    | 99858    |
| 4  | 728337   | 392    | 999378   | 1.1    | 728959   | 393    | 271041    | 05350    | 99857    |
| 5  | 730688   | 390    | 999371   | 1.1    | 731317   | 391    | 268683    | 05379    | 99855    |
| 6  | 733027   | 388    | 999364   | 1.1    | 733663   | 389    | 266337    | 05408    | 99854    |
| 7  | 735354   | 386    | 999357   | 1.2    | 735996   | 387    | 264004    | 05437    | 99852    |
| 8  | 737667   | 384    | 999350   | 1.2    | 738317   | 385    | 261683    | 05466    | 99851    |
| 9  | 739969   | 382    | 999343   | 1.2    | 740626   | 383    | 259374    | 05495    | 99849    |
| 10 | 742259   | 380    | 999336   | 1.2    | 742922   | 381    | 257078    | 05524    | 99847    |
| 11 | 8.744536 | 378    | 9.999329 | 1.2    | 8.745207 | 379    | 11.254798 | 05553    | 99846    |
| 12 | 746802   | 376    | 999322   | 1.2    | 747479   | 377    | 252521    | 05582    | 99844    |
| 13 | 749055   | 374    | 999315   | 1.2    | 749740   | 375    | 250260    | 05611    | 99842    |
| 14 | 751297   | 372    | 999308   | 1.2    | 751989   | 373    | 248011    | 05640    | 99841    |
| 15 | 753528   | 370    | 999301   | 1.2    | 754227   | 371    | 245773    | 05669    | 99839    |
| 16 | 755747   | 368    | 999294   | 1.2    | 756453   | 369    | 243547    | 05698    | 99838    |
| 17 | 757955   | 366    | 999286   | 1.2    | 758668   | 367    | 241382    | 05727    | 99836    |
| 18 | 760151   | 364    | 999279   | 1.2    | 760872   | 365    | 239128    | 05756    | 99834    |
| 19 | 762337   | 362    | 999272   | 1.2    | 763065   | 364    | 236935    | 05785    | 99833    |
| 20 | 764511   | 360    | 999265   | 1.2    | 765246   | 362    | 234764    | 05814    | 99831    |
| 21 | 8.766675 | 359    | 9.999257 | 1.2    | 8.767417 | 360    | 11.232583 | 05844    | 99829    |
| 22 | 768828   | 357    | 999250   | 1.2    | 769578   | 358    | 230422    | 05873    | 99827    |
| 23 | 770970   | 355    | 999242   | 1.3    | 771727   | 356    | 228273    | 05902    | 99826    |
| 24 | 773101   | 353    | 999235   | 1.3    | 773866   | 355    | 226134    | 05931    | 99824    |
| 25 | 775223   | 352    | 999227   | 1.3    | 775995   | 353    | 224005    | 05960    | 99822    |
| 26 | 777333   | 350    | 999220   | 1.3    | 778114   | 351    | 221886    | 05989    | 99821    |
| 27 | 779434   | 348    | 999212   | 1.3    | 780222   | 350    | 219778    | 06018    | 99819    |
| 28 | 781524   | 347    | 999205   | 1.3    | 782320   | 348    | 217680    | 06047    | 99817    |
| 29 | 783605   | 345    | 999197   | 1.3    | 784408   | 346    | 215592    | 06076    | 99815    |
| 30 | 785675   | 344    | 999189   | 1.3    | 786486   | 345    | 213514    | 06105    | 99813    |
| 31 | 8.787796 | 343    | 9.999181 | 1.3    | 8.788554 | 343    | 11.211446 | 06134    | 99812    |
| 32 | 789787   | 342    | 999174   | 1.3    | 790613   | 341    | 209887    | 06163    | 99810    |
| 33 | 791828   | 340    | 999166   | 1.3    | 792662   | 340    | 207388    | 06192    | 99808    |
| 34 | 793859   | 339    | 999158   | 1.3    | 794701   | 338    | 205299    | 06221    | 99806    |
| 35 | 795881   | 338    | 999150   | 1.3    | 796731   | 337    | 203269    | 06250    | 99804    |
| 36 | 797894   | 336    | 999142   | 1.3    | 798752   | 335    | 201248    | 06279    | 99803    |
| 37 | 799897   | 334    | 999134   | 1.3    | 800763   | 334    | 199237    | 06308    | 99801    |
| 38 | 801892   | 332    | 999126   | 1.3    | 802765   | 332    | 197235    | 06337    | 99799    |
| 39 | 803876   | 331    | 999118   | 1.3    | 804858   | 331    | 195242    | 06366    | 99797    |
| 40 | 805852   | 329    | 999110   | 1.3    | 806742   | 328    | 193258    | 06395    | 99795    |
| 41 | 8.807819 | 328    | 9.999102 | 1.3    | 8.808717 | 329    | 11.191283 | 06424    | 99793    |
| 42 | 809777   | 326    | 999094   | 1.4    | 810683   | 326    | 189317    | 06453    | 99792    |
| 43 | 811726   | 323    | 999086   | 1.4    | 812641   | 325    | 187359    | 06482    | 99790    |
| 44 | 813667   | 322    | 999077   | 1.4    | 814589   | 323    | 185411    | 06511    | 99788    |
| 45 | 815599   | 320    | 999069   | 1.4    | 816529   | 322    | 183471    | 06540    | 99786    |
| 46 | 817522   | 319    | 999061   | 1.4    | 818461   | 320    | 181539    | 06569    | 99784    |
| 47 | 819436   | 318    | 999053   | 1.4    | 820384   | 319    | 179616    | 06598    | 99782    |
| 48 | 821343   | 316    | 999044   | 1.4    | 822298   | 318    | 177702    | 06627    | 99780    |
| 49 | 823240   | 315    | 999036   | 1.4    | 824205   | 316    | 175795    | 06656    | 99778    |
| 50 | 825130   | 313    | 999027   | 1.4    | 826103   | 315    | 173897    | 06685    | 99776    |
| 51 | 8.827011 | 312    | 9.999019 | 1.4    | 8.827992 | 314    | 11.172008 | 06714    | 99774    |
| 52 | 828884   | 311    | 999010   | 1.4    | 829874   | 312    | 170126    | 06743    | 99772    |
| 53 | 830749   | 309    | 999002   | 1.4    | 831748   | 311    | 168252    | 06773    | 99770    |
| 54 | 832607   | 308    | 998993   | 1.4    | 833613   | 310    | 166387    | 06802    | 99768    |
| 55 | 834456   | 307    | 998984   | 1.4    | 835471   | 308    | 164529    | 06831    | 99766    |
| 56 | 836297   | 306    | 998976   | 1.4    | 837321   | 307    | 162679    | 06860    | 99764    |
| 57 | 838130   | 304    | 998967   | 1.5    | 839163   | 306    | 160837    | 06889    | 99762    |
| 58 | 839956   | 303    | 998958   | 1.5    | 840998   | 304    | 159002    | 06918    | 99760    |
| 59 | 841774   | 302    | 998950   | 1.5    | 842825   | 303    | 157175    | 06947    | 99758    |
| 60 | 843585   |        | 998941   | 1.5    | 844644   |        | 155356    | 06976    | 99756    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. ( $4^{\circ}$ ) Natural Sines.

25

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 8.843585 |        | 9.998941 |        | 8.844644 |        | 11.155356 | 06976    | 99756 60 |
| 1  | 845397   | 300    | 998932   | 1.5    | 846455   | 301    | 153545    | 07005    | 99754 59 |
| 2  | 847183   | 299    | 998923   | 1.5    | 848260   | 302    | 151740    | 07034    | 99752 58 |
| 3  | 848971   | 298    | 998914   | 1.5    | 850057   | 299    | 149943    | 07063    | 99750 57 |
| 4  | 850751   | 295    | 998905   | 1.5    | 851846   | 297    | 148154    | 07092    | 99748 56 |
| 5  | 852525   | 294    | 998896   | 1.5    | 853628   | 296    | 146372    | 07121    | 99746 55 |
| 6  | 854291   | 293    | 998887   | 1.5    | 855408   | 295    | 144597    | 07150    | 99744 54 |
| 7  | 856049   | 292    | 998878   | 1.5    | 857171   | 293    | 142829    | 07179    | 99742 53 |
| 8  | 857801   | 291    | 998869   | 1.5    | 858932   | 292    | 141068    | 07208    | 99740 52 |
| 9  | 859546   | 290    | 998860   | 1.5    | 860686   | 291    | 139314    | 07237    | 99738 51 |
| 10 | 861283   | 288    | 998851   | 1.5    | 862433   | 290    | 137567    | 07266    | 99736 50 |
| 11 | 8.863074 | 287    | 9.998841 | 1.5    | 8.864173 | 289    | 11.135827 | 07295    | 99734 49 |
| 12 | 864738   | 286    | 998832   | 1.5    | 865906   | 288    | 134094    | 07324    | 99731 48 |
| 13 | 866455   | 285    | 998823   | 1.5    | 867632   | 287    | 132368    | 07353    | 99729 47 |
| 14 | 868165   | 284    | 998813   | 1.6    | 869351   | 286    | 130649    | 07382    | 99727 46 |
| 15 | 869868   | 283    | 998804   | 1.6    | 871064   | 285    | 128936    | 07411    | 99725 45 |
| 16 | 871565   | 282    | 998795   | 1.6    | 872770   | 284    | 127230    | 07440    | 99723 44 |
| 17 | 873255   | 281    | 998785   | 1.6    | 874469   | 283    | 125531    | 07469    | 99721 43 |
| 18 | 874938   | 279    | 998776   | 1.6    | 876162   | 282    | 123838    | 07498    | 99719 42 |
| 19 | 876615   | 277    | 998766   | 1.6    | 877849   | 281    | 122151    | 07527    | 99716 41 |
| 20 | 878285   | 279    | 998757   | 1.6    | 879529   | 280    | 120471    | 07556    | 99714 40 |
| 21 | 8.879949 | 276    | 9.998747 | 1.6    | 8.881202 | 279    | 11.18798  | 07585    | 99712 39 |
| 22 | 881607   | 275    | 998738   | 1.6    | 882869   | 278    | 117131    | 07614    | 99710 38 |
| 23 | 883258   | 274    | 998728   | 1.6    | 884530   | 277    | 115470    | 07643    | 99708 37 |
| 24 | 884903   | 273    | 998718   | 1.6    | 886185   | 276    | 113815    | 07672    | 99705 36 |
| 25 | 886542   | 272    | 998708   | 1.6    | 887833   | 275    | 112167    | 07701    | 99703 35 |
| 26 | 888174   | 271    | 998699   | 1.6    | 889476   | 274    | 110524    | 07730    | 99701 34 |
| 27 | 889801   | 270    | 998689   | 1.6    | 891112   | 273    | 108888    | 07759    | 99699 33 |
| 28 | 891421   | 269    | 998679   | 1.6    | 892742   | 272    | 107258    | 07788    | 99696 32 |
| 29 | 893035   | 268    | 998669   | 1.6    | 894366   | 271    | 105634    | 07817    | 99694 31 |
| 30 | 894643   | 267    | 998659   | 1.7    | 895984   | 270    | 104010    | 07846    | 99692 30 |
| 31 | 8.896246 | 266    | 9.998649 | 1.7    | 8.897596 | 269    | 11.102404 | 07875    | 99689 29 |
| 32 | 897842   | 265    | 998639   | 1.7    | 899203   | 268    | 100797    | 07904    | 99687 28 |
| 33 | 899432   | 264    | 998629   | 1.7    | 900808   | 267    | 999197    | 07933    | 99685 27 |
| 34 | 901017   | 263    | 998619   | 1.7    | 902398   | 266    | 997602    | 07962    | 99683 26 |
| 35 | 902596   | 262    | 998609   | 1.7    | 903987   | 265    | 996013    | 07991    | 99680 25 |
| 36 | 904169   | 261    | 998599   | 1.7    | 905570   | 264    | 994430    | 08020    | 99678 24 |
| 37 | 905736   | 260    | 998589   | 1.7    | 907147   | 263    | 992853    | 08049    | 99676 23 |
| 38 | 907297   | 259    | 998578   | 1.7    | 908719   | 262    | 991281    | 08078    | 99673 22 |
| 39 | 908853   | 258    | 998568   | 1.7    | 910285   | 261    | 989715    | 08107    | 99671 21 |
| 40 | 910404   | 257    | 998558   | 1.7    | 911846   | 260    | 988154    | 08136    | 99668 20 |
| 41 | 8.911949 | 256    | 9.998548 | 1.7    | 8.913401 | 259    | 11.086599 | 08165    | 99666 19 |
| 42 | 913488   | 255    | 998537   | 1.7    | 914951   | 258    | 985049    | 08194    | 99664 18 |
| 43 | 915022   | 255    | 998527   | 1.7    | 916495   | 257    | 983505    | 08223    | 99661 17 |
| 44 | 916559   | 254    | 998516   | 1.8    | 918034   | 256    | 981966    | 08252    | 99659 16 |
| 45 | 918073   | 253    | 998506   | 1.8    | 919568   | 255    | 980432    | 08281    | 99657 15 |
| 46 | 919591   | 252    | 998495   | 1.8    | 921096   | 254    | 978904    | 08310    | 99654 14 |
| 47 | 921103   | 251    | 998485   | 1.8    | 922619   | 253    | 977381    | 08339    | 99652 13 |
| 48 | 922610   | 250    | 998474   | 1.8    | 924136   | 252    | 975864    | 08368    | 99649 12 |
| 49 | 924112   | 249    | 998464   | 1.8    | 925649   | 251    | 974351    | 08397    | 99647 11 |
| 50 | 925609   | 249    | 998453   | 1.8    | 927156   | 250    | 972844    | 08426    | 99644 10 |
| 51 | 8.927100 | 248    | 9.998442 | 1.8    | 8.928658 | 249    | 11.071342 | 08455    | 99642 9  |
| 52 | 928587   | 247    | 998431   | 1.8    | 930155   | 248    | 969845    | 08484    | 99639 8  |
| 53 | 930068   | 246    | 998421   | 1.8    | 931647   | 247    | 968353    | 08513    | 99637 7  |
| 54 | 931544   | 245    | 998410   | 1.8    | 933134   | 246    | 966866    | 08542    | 99635 6  |
| 55 | 933015   | 244    | 998399   | 1.8    | 934616   | 245    | 965384    | 08571    | 99632 5  |
| 56 | 934481   | 243    | 998388   | 1.8    | 936093   | 244    | 963907    | 08600    | 99630 4  |
| 57 | 935942   | 243    | 998377   | 1.8    | 937565   | 243    | 962435    | 08629    | 99627 3  |
| 58 | 937398   | 242    | 998366   | 1.8    | 939032   | 242    | 960968    | 08658    | 99625 2  |
| 59 | 938850   | 241    | 998355   | 1.8    | 940494   | 241    | 959506    | 08687    | 99622 1  |
| 60 | 940296   |        | 998344   |        | 941952   |        | 958048    | 08716    | 99619 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

85 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 8.940296 | 240    | 9.998344 | 1.9    | 8.941952 | 242    | 11.058048 | 08716    | 99619    | 60 |
| 1  | 941738   | 239    | 998333   | 1.9    | 943404   | 241    | 056596    | 08745    | 99617    | 59 |
| 2  | 943174   | 239    | 998322   | 1.9    | 944852   | 241    | 055148    | 08774    | 99614    | 58 |
| 3  | 944606   | 238    | 998311   | 1.9    | 946295   | 240    | 053705    | 08803    | 99612    | 57 |
| 4  | 946034   | 237    | 998300   | 1.9    | 947734   | 239    | 052266    | 08831    | 99609    | 56 |
| 5  | 947456   | 236    | 998289   | 1.9    | 949168   | 238    | 050832    | 08860    | 99607    | 55 |
| 6  | 948874   | 235    | 998277   | 1.9    | 950597   | 237    | 049403    | 08889    | 99604    | 54 |
| 7  | 950287   | 235    | 998266   | 1.9    | 952021   | 237    | 047979    | 08918    | 99602    | 53 |
| 8  | 951696   | 235    | 998255   | 1.9    | 953441   | 237    | 046559    | 08947    | 99599    | 52 |
| 9  | 953100   | 234    | 998243   | 1.9    | 954856   | 236    | 045144    | 08976    | 99596    | 51 |
| 10 | 954499   | 233    | 998232   | 1.9    | 956267   | 234    | 043733    | 09005    | 99594    | 50 |
| 11 | 8.955894 | 232    | 9.998220 | 1.9    | 8.957674 | 234    | 11.042326 | 09034    | 99591    | 49 |
| 12 | 957284   | 231    | 998209   | 1.9    | 959075   | 233    | 040925    | 09063    | 99588    | 48 |
| 13 | 958670   | 231    | 998197   | 1.9    | 960473   | 232    | 039527    | 09092    | 99586    | 47 |
| 14 | 960052   | 229    | 998186   | 1.9    | 961866   | 231    | 038134    | 09121    | 99583    | 46 |
| 15 | 961429   | 229    | 998174   | 1.9    | 963255   | 231    | 036745    | 09150    | 99580    | 45 |
| 16 | 962801   | 228    | 998163   | 1.9    | 964639   | 230    | 035361    | 09179    | 99578    | 44 |
| 17 | 964170   | 227    | 998151   | 1.9    | 966019   | 229    | 033981    | 09208    | 99575    | 43 |
| 18 | 965534   | 227    | 998139   | 2.0    | 967394   | 229    | 032606    | 09237    | 99572    | 42 |
| 19 | 966893   | 226    | 998128   | 2.0    | 968766   | 228    | 031234    | 09266    | 99570    | 41 |
| 20 | 968249   | 225    | 998116   | 2.0    | 970133   | 227    | 029867    | 09295    | 99567    | 40 |
| 21 | 8.969600 | 224    | 9.998104 | 2.0    | 8.971496 | 226    | 11.028504 | 09324    | 99564    | 39 |
| 22 | 970947   | 224    | 998092   | 2.0    | 972855   | 226    | 027145    | 09353    | 99562    | 38 |
| 23 | 972289   | 223    | 998080   | 2.0    | 974209   | 225    | 025791    | 09382    | 99559    | 37 |
| 24 | 973628   | 222    | 998068   | 2.0    | 975560   | 224    | 024440    | 09411    | 99556    | 36 |
| 25 | 974962   | 222    | 998056   | 2.0    | 976906   | 224    | 023094    | 09440    | 99553    | 35 |
| 26 | 976293   | 221    | 998044   | 2.0    | 978248   | 223    | 021752    | 09469    | 99551    | 34 |
| 27 | 977619   | 220    | 998032   | 2.0    | 979586   | 222    | 020414    | 09498    | 99548    | 33 |
| 28 | 978941   | 220    | 998020   | 2.0    | 980921   | 222    | 019079    | 09527    | 99545    | 32 |
| 29 | 980259   | 219    | 998008   | 2.0    | 982251   | 221    | 017749    | 09556    | 99542    | 31 |
| 30 | 981573   | 218    | 997996   | 2.0    | 983577   | 220    | 016423    | 09585    | 99540    | 30 |
| 31 | 8.982883 | 218    | 9.997984 | 2.0    | 8.984899 | 219    | 11.015101 | 09614    | 99537    | 29 |
| 32 | 984189   | 217    | 997972   | 2.0    | 986217   | 219    | 013783    | 09642    | 99534    | 28 |
| 33 | 985491   | 216    | 997959   | 2.0    | 987532   | 219    | 012468    | 09671    | 99531    | 27 |
| 34 | 986789   | 216    | 997947   | 2.0    | 988842   | 218    | 011158    | 09700    | 99528    | 26 |
| 35 | 988083   | 215    | 997935   | 2.1    | 990149   | 217    | 009851    | 09729    | 99526    | 25 |
| 36 | 989374   | 214    | 997922   | 2.1    | 991451   | 216    | 008549    | 09758    | 99523    | 24 |
| 37 | 990660   | 214    | 997910   | 2.1    | 992750   | 216    | 007250    | 09787    | 99520    | 23 |
| 38 | 991943   | 213    | 997897   | 2.1    | 994045   | 215    | 005955    | 09816    | 99517    | 22 |
| 39 | 993222   | 213    | 997885   | 2.1    | 995337   | 215    | 004663    | 09845    | 99514    | 21 |
| 40 | 994497   | 212    | 997872   | 2.1    | 996624   | 215    | 003376    | 09874    | 99511    | 20 |
| 41 | 8.995768 | 211    | 9.997860 | 2.1    | 8.997908 | 214    | 11.002092 | 09903    | 99508    | 19 |
| 42 | 997036   | 211    | 997847   | 2.1    | 999188   | 213    | 000812    | 09932    | 99506    | 18 |
| 43 | 998299   | 210    | 997835   | 2.1    | 9.000465 | 212    | 10.999535 | 09961    | 99503    | 17 |
| 44 | 999560   | 209    | 997822   | 2.1    | 001738   | 211    | 998262    | 09990    | 99500    | 16 |
| 45 | 9.000816 | 209    | 997809   | 2.1    | 003007   | 211    | 996993    | 10019    | 99497    | 15 |
| 46 | 002069   | 208    | 997797   | 2.1    | 004272   | 210    | 995728    | 10048    | 99494    | 14 |
| 47 | 003318   | 208    | 997784   | 2.1    | 005534   | 210    | 994466    | 10077    | 99491    | 13 |
| 48 | 004563   | 207    | 997771   | 2.1    | 006792   | 209    | 993208    | 10106    | 99488    | 12 |
| 49 | 005805   | 206    | 997758   | 2.1    | 008047   | 208    | 991953    | 10135    | 99485    | 11 |
| 50 | 007044   | 206    | 997745   | 2.1    | 009298   | 208    | 990702    | 10164    | 99482    | 10 |
| 51 | 9.008278 | 205    | 9.997732 | 2.1    | 9.010546 | 207    | 10.989454 | 10192    | 99479    | 9  |
| 52 | 009510   | 205    | 997719   | 2.1    | 011790   | 207    | 988210    | 10221    | 99476    | 8  |
| 53 | 010737   | 204    | 997706   | 2.1    | 013031   | 207    | 986969    | 10250    | 99473    | 7  |
| 54 | 011962   | 203    | 997693   | 2.2    | 014268   | 206    | 985732    | 10279    | 99470    | 6  |
| 55 | 013182   | 203    | 997680   | 2.2    | 015502   | 205    | 984498    | 10308    | 99467    | 5  |
| 56 | 014400   | 202    | 997667   | 2.2    | 016732   | 204    | 983268    | 10337    | 99464    | 4  |
| 57 | 015613   | 202    | 997654   | 2.2    | 017959   | 204    | 982041    | 10366    | 99461    | 3  |
| 58 | 016824   | 201    | 997641   | 2.2    | 019183   | 203    | 980817    | 10395    | 99458    | 2  |
| 59 | 018031   | 201    | 997628   | 2.2    | 020403   | 203    | 979597    | 10424    | 99455    | 1  |
| 60 | 019235   | 201    | 997614   | 2.2    | 021620   | 203    | 978380    | 10453    | 99452    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (6°) Natural Sines.

27

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.019235 | 200    | 9.997614 | 2.2    | 9.021620 | 202    | 10.978380 | 10453    | 99452    |
| 1  | 020435   | 199    | 997601   | 2.2    | 022834   | 203    | 977166    | 10482    | 99449    |
| 2  | 021632   | 199    | 997588   | 2.2    | 024044   | 201    | 975956    | 10511    | 99446    |
| 3  | 022825   | 198    | 997574   | 2.2    | 025251   | 201    | 974749    | 10540    | 99443    |
| 4  | 024016   | 198    | 997561   | 2.2    | 026455   | 200    | 973545    | 10569    | 99440    |
| 5  | 025203   | 198    | 997547   | 2.2    | 027655   | 199    | 972345    | 10597    | 99437    |
| 6  | 026386   | 197    | 997534   | 2.2    | 028852   | 199    | 971148    | 10626    | 99434    |
| 7  | 027567   | 196    | 997520   | 2.3    | 030046   | 198    | 969954    | 10655    | 99431    |
| 8  | 028744   | 196    | 997507   | 2.3    | 031237   | 198    | 968763    | 10684    | 99428    |
| 9  | 029918   | 195    | 997493   | 2.3    | 032425   | 197    | 967575    | 10713    | 99424    |
| 10 | 031089   | 195    | 997480   | 2.3    | 033609   | 197    | 966391    | 10742    | 99421    |
| 11 | 9.032257 | 194    | 9.997466 | 2.3    | 9.034791 | 197    | 10.965209 | 10771    | 99418    |
| 12 | 033421   | 194    | 997452   | 2.3    | 035969   | 196    | 964031    | 10800    | 99415    |
| 13 | 034582   | 193    | 997439   | 2.3    | 037144   | 195    | 962856    | 10829    | 99412    |
| 14 | 035741   | 192    | 997425   | 2.3    | 038316   | 195    | 961684    | 10858    | 99409    |
| 15 | 036896   | 192    | 997411   | 2.3    | 039485   | 194    | 960515    | 10887    | 99406    |
| 16 | 038048   | 191    | 997397   | 2.3    | 040651   | 194    | 959349    | 10916    | 99402    |
| 17 | 039197   | 191    | 997383   | 2.3    | 041813   | 193    | 958187    | 10945    | 99399    |
| 18 | 040342   | 190    | 997369   | 2.3    | 042973   | 193    | 957027    | 10973    | 99396    |
| 19 | 041485   | 190    | 997355   | 2.3    | 044130   | 192    | 955870    | 11002    | 99393    |
| 20 | 042625   | 189    | 997341   | 2.3    | 045284   | 192    | 954716    | 11031    | 99390    |
| 21 | 9.043762 | 189    | 9.997327 | 2.4    | 9.046434 | 191    | 10.953566 | 11060    | 99386    |
| 22 | 044895   | 188    | 997313   | 2.4    | 047582   | 191    | 952418    | 11089    | 99383    |
| 23 | 046026   | 188    | 997299   | 2.4    | 048727   | 190    | 951273    | 11118    | 99380    |
| 24 | 047154   | 187    | 997285   | 2.4    | 049869   | 190    | 950131    | 11147    | 99377    |
| 25 | 048279   | 187    | 997271   | 2.4    | 051008   | 189    | 948992    | 11176    | 99374    |
| 26 | 049400   | 186    | 997257   | 2.4    | 052144   | 189    | 947856    | 11205    | 99370    |
| 27 | 050519   | 186    | 997242   | 2.4    | 053277   | 188    | 946723    | 11234    | 99367    |
| 28 | 051635   | 185    | 997228   | 2.4    | 054407   | 188    | 945593    | 11263    | 99364    |
| 29 | 052749   | 185    | 997214   | 2.4    | 055536   | 187    | 944465    | 11291    | 99360    |
| 30 | 053859   | 184    | 997199   | 2.4    | 056659   | 187    | 943341    | 11320    | 99357    |
| 31 | 9.054966 | 184    | 9.997185 | 2.4    | 9.057781 | 186    | 10.942219 | 11349    | 99354    |
| 32 | 056071   | 184    | 997170   | 2.4    | 058900   | 186    | 941100    | 11378    | 99351    |
| 33 | 057172   | 183    | 997156   | 2.4    | 060016   | 186    | 939984    | 11407    | 99347    |
| 34 | 058271   | 183    | 997141   | 2.4    | 061130   | 185    | 938870    | 11436    | 99344    |
| 35 | 059367   | 182    | 997127   | 2.4    | 062240   | 185    | 937760    | 11465    | 99341    |
| 36 | 060460   | 182    | 997112   | 2.4    | 063348   | 184    | 936652    | 11494    | 99337    |
| 37 | 061551   | 181    | 997098   | 2.4    | 064453   | 184    | 935547    | 11523    | 99334    |
| 38 | 062639   | 180    | 997083   | 2.5    | 065556   | 183    | 934444    | 11552    | 99331    |
| 39 | 063724   | 180    | 997068   | 2.5    | 066655   | 183    | 933345    | 11580    | 99327    |
| 40 | 064806   | 180    | 997053   | 2.5    | 067752   | 182    | 932248    | 11609    | 99324    |
| 41 | 9.065885 | 179    | 9.997039 | 2.5    | 9.068846 | 182    | 10.931154 | 11638    | 99320    |
| 42 | 066962   | 179    | 997024   | 2.5    | 069038   | 181    | 930062    | 11667    | 99317    |
| 43 | 068036   | 179    | 997009   | 2.5    | 071027   | 181    | 928973    | 11696    | 99314    |
| 44 | 069107   | 178    | 996994   | 2.5    | 072113   | 181    | 927887    | 11725    | 99310    |
| 45 | 070176   | 178    | 996979   | 2.5    | 073197   | 180    | 926803    | 11754    | 99307    |
| 46 | 071242   | 178    | 996964   | 2.5    | 074278   | 180    | 925732    | 11783    | 99303    |
| 47 | 072306   | 177    | 996949   | 2.5    | 075356   | 179    | 924644    | 11812    | 99300    |
| 48 | 073366   | 176    | 996934   | 2.5    | 076432   | 179    | 923568    | 11840    | 99297    |
| 49 | 074424   | 176    | 996919   | 2.5    | 077505   | 178    | 922495    | 11869    | 99293    |
| 50 | 075480   | 175    | 996904   | 2.5    | 078576   | 178    | 921424    | 11898    | 99290    |
| 51 | 9.076533 | 175    | 9.996889 | 2.5    | 9.079644 | 178    | 10.920356 | 11927    | 99286    |
| 52 | 077583   | 175    | 996874   | 2.5    | 080710   | 177    | 919290    | 11956    | 99283    |
| 53 | 078631   | 174    | 996858   | 2.5    | 081773   | 177    | 918227    | 11985    | 99279    |
| 54 | 079676   | 174    | 996843   | 2.5    | 082833   | 177    | 917167    | 12014    | 99276    |
| 55 | 080719   | 173    | 996828   | 2.5    | 083891   | 176    | 916109    | 12043    | 99272    |
| 56 | 081759   | 173    | 996812   | 2.6    | 084947   | 175    | 915053    | 12071    | 99269    |
| 57 | 082797   | 172    | 996797   | 2.6    | 086000   | 175    | 914000    | 12100    | 99265    |
| 58 | 083832   | 172    | 996782   | 2.6    | 087050   | 175    | 912950    | 12129    | 99262    |
| 59 | 084864   | 172    | 996766   | 2.6    | 088098   | 174    | 911902    | 12158    | 99258    |
| 60 | 085894   |        | 996751   | 2.6    | 089144   |        | 910856    | 12187    | 99255    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

83 Degrees.

| $\angle$ | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.    | D. 10' | Cotang.   | N. sine. | N. cos.  |
|----------|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0        | 9.085894 | 171    | 9.996751 | 2.6    | 9.089144 | 174    | 10.910856 | 12187    | 99255    |
| 1        | 086922   | 171    | 996735   | 2.6    | 090187   | 173    | 909813    | 12216    | 99251    |
| 2        | 087947   | 170    | 996720   | 2.6    | 091228   | 173    | 908772    | 12245    | 99248    |
| 3        | 088970   | 170    | 996704   | 2.6    | 092266   | 173    | 907734    | 12274    | 99244    |
| 4        | 089990   | 170    | 996688   | 2.6    | 093302   | 173    | 906698    | 12302    | 99240    |
| 5        | 091008   | 170    | 996673   | 2.6    | 094336   | 172    | 905664    | 12331    | 99237    |
| 6        | 092024   | 169    | 996657   | 2.6    | 095367   | 172    | 904633    | 12360    | 99233    |
| 7        | 093037   | 169    | 996641   | 2.6    | 096395   | 171    | 903605    | 12389    | 99230    |
| 8        | 094047   | 168    | 996625   | 2.6    | 097422   | 171    | 902578    | 12418    | 99226    |
| 9        | 095056   | 168    | 996610   | 2.6    | 098446   | 170    | 901554    | 12447    | 99222    |
| 10       | 096062   | 167    | 996594   | 2.6    | 099468   | 170    | 900532    | 12476    | 99219    |
| 11       | 9.097065 | 167    | 9.996578 | 2.6    | 9.100487 | 170    | 10.899513 | 12504    | 99215    |
| 12       | 098066   | 167    | 996562   | 2.7    | 101504   | 169    | 898496    | 12533    | 99211    |
| 13       | 099065   | 166    | 996546   | 2.7    | 102519   | 169    | 897481    | 12562    | 99208    |
| 14       | 100062   | 166    | 996530   | 2.7    | 103532   | 169    | 896468    | 12591    | 99204    |
| 15       | 101056   | 165    | 996514   | 2.7    | 104542   | 168    | 895458    | 12620    | 99200    |
| 16       | 102048   | 165    | 996498   | 2.7    | 105550   | 168    | 894450    | 12649    | 99197    |
| 17       | 103037   | 164    | 996482   | 2.7    | 106556   | 167    | 893444    | 12678    | 99193    |
| 18       | 104025   | 164    | 996465   | 2.7    | 107559   | 167    | 892441    | 12706    | 99189    |
| 19       | 105010   | 164    | 996449   | 2.7    | 108560   | 167    | 891440    | 12735    | 99186    |
| 20       | 105992   | 163    | 996433   | 2.7    | 109559   | 166    | 890441    | 12764    | 99182    |
| 21       | 9.106973 | 163    | 9.996417 | 2.7    | 9.110556 | 166    | 10.889444 | 12793    | 99178    |
| 22       | 107951   | 163    | 996400   | 2.7    | 111561   | 166    | 888449    | 12822    | 99175    |
| 23       | 108927   | 162    | 996384   | 2.7    | 112543   | 165    | 887457    | 12851    | 99171    |
| 24       | 109901   | 162    | 996368   | 2.7    | 113533   | 165    | 886467    | 12880    | 99167    |
| 25       | 110873   | 162    | 996351   | 2.7    | 114521   | 165    | 885479    | 12908    | 99163    |
| 26       | 111842   | 162    | 996335   | 2.7    | 115507   | 164    | 884493    | 12937    | 99160    |
| 27       | 112809   | 161    | 996318   | 2.7    | 116491   | 164    | 883509    | 12966    | 99156    |
| 28       | 113774   | 161    | 996302   | 2.7    | 117472   | 163    | 882528    | 12995    | 99152    |
| 29       | 114737   | 160    | 996285   | 2.8    | 118452   | 163    | 881548    | 13024    | 99148    |
| 30       | 115698   | 160    | 996269   | 2.8    | 119429   | 162    | 880571    | 13053    | 99144    |
| 31       | 9.116656 | 159    | 9.996252 | 2.8    | 9.120404 | 162    | 10.879596 | 13081    | 99141    |
| 32       | 117613   | 159    | 996235   | 2.8    | 121377   | 162    | 878623    | 13110    | 99137    |
| 33       | 118567   | 159    | 996219   | 2.8    | 122348   | 161    | 877652    | 13139    | 99133    |
| 34       | 119519   | 158    | 996202   | 2.8    | 123317   | 161    | 876683    | 13168    | 99129    |
| 35       | 120469   | 158    | 996185   | 2.8    | 124284   | 161    | 875716    | 13197    | 99125    |
| 36       | 121417   | 158    | 996168   | 2.8    | 125249   | 160    | 874751    | 13226    | 99122    |
| 37       | 122362   | 157    | 996151   | 2.8    | 126211   | 160    | 873789    | 13254    | 99118    |
| 38       | 123306   | 157    | 996134   | 2.8    | 127172   | 160    | 872828    | 13283    | 99114    |
| 39       | 124248   | 157    | 996117   | 2.8    | 128130   | 160    | 871870    | 13312    | 99110    |
| 40       | 125187   | 157    | 996100   | 2.8    | 129087   | 159    | 870913    | 13341    | 99106    |
| 41       | 9.126125 | 156    | 9.996083 | 2.9    | 9.130041 | 159    | 10.869959 | 13370    | 99102    |
| 42       | 127060   | 156    | 996066   | 2.9    | 130994   | 158    | 869006    | 13399    | 99098    |
| 43       | 127993   | 155    | 996049   | 2.9    | 131944   | 158    | 868056    | 13427    | 99094    |
| 44       | 128925   | 155    | 996032   | 2.9    | 132893   | 158    | 867107    | 13456    | 99091    |
| 45       | 129854   | 155    | 996015   | 2.9    | 133839   | 157    | 866161    | 13485    | 99087    |
| 46       | 130781   | 154    | 995998   | 2.9    | 134784   | 157    | 865216    | 13514    | 99083    |
| 47       | 131706   | 154    | 995980   | 2.9    | 135726   | 157    | 864274    | 13543    | 99079    |
| 48       | 132630   | 153    | 995963   | 2.9    | 136667   | 156    | 863333    | 13572    | 99075    |
| 49       | 133551   | 153    | 995946   | 2.9    | 137605   | 156    | 862395    | 13600    | 99071    |
| 50       | 134470   | 153    | 995928   | 2.9    | 138542   | 156    | 861458    | 13629    | 99067    |
| 51       | 9.135387 | 152    | 9.995911 | 2.9    | 9.139476 | 155    | 10.860524 | 13658    | 99063    |
| 52       | 136303   | 152    | 995894   | 2.9    | 140409   | 155    | 859591    | 13687    | 99059    |
| 53       | 137216   | 152    | 995876   | 2.9    | 141340   | 155    | 858660    | 13716    | 99055    |
| 54       | 138128   | 152    | 995859   | 2.9    | 142269   | 154    | 857731    | 13744    | 99051    |
| 55       | 139037   | 151    | 995841   | 2.9    | 143196   | 154    | 856804    | 13773    | 99047    |
| 56       | 139944   | 151    | 995823   | 2.9    | 144121   | 154    | 855879    | 13802    | 99043    |
| 57       | 140850   | 151    | 995806   | 2.9    | 145044   | 153    | 854956    | 13831    | 99039    |
| 58       | 141754   | 150    | 995788   | 2.9    | 145966   | 153    | 854034    | 13860    | 99035    |
| 59       | 142655   | 150    | 995771   | 2.9    | 146885   | 153    | 853115    | 13889    | 99031    |
| 60       | 143555   | 150    | 995753   | 2.9    | 147803   | 153    | 852197    | 13917    | 99027    |
|          | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (8°) Natural Sines.

29

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos. |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|---------|----|
| 0  | 9.143555 | 150    | 9.995753 | 3.0    | 9.147803 | 153    | 10.852197 | 13917    | 99027   | 60 |
| 1  | 144453   | 149    | 995735   | 3.0    | 148718   | 152    | 851282    | 13946    | 99023   | 59 |
| 2  | 145349   | 149    | 995717   | 3.0    | 149632   | 152    | 850368    | 13975    | 99019   | 58 |
| 3  | 146243   | 149    | 995699   | 3.0    | 150544   | 152    | 849456    | 14004    | 99015   | 57 |
| 4  | 147136   | 149    | 995681   | 3.0    | 151454   | 152    | 848546    | 14033    | 99011   | 56 |
| 5  | 148026   | 148    | 995664   | 3.0    | 152363   | 151    | 847637    | 14061    | 99006   | 55 |
| 6  | 148915   | 148    | 995646   | 3.0    | 153269   | 151    | 846731    | 14090    | 99002   | 54 |
| 7  | 149802   | 148    | 995628   | 3.0    | 154174   | 151    | 845826    | 14119    | 98998   | 53 |
| 8  | 150686   | 147    | 995610   | 3.0    | 155077   | 150    | 844923    | 14148    | 98994   | 52 |
| 9  | 151569   | 147    | 995591   | 3.0    | 155978   | 150    | 844022    | 14177    | 98990   | 51 |
| 10 | 152451   | 147    | 995573   | 3.0    | 156877   | 150    | 843123    | 14205    | 98986   | 50 |
| 11 | 9.153330 | 147    | 9.995555 | 3.0    | 9.157775 | 150    | 10.842225 | 14234    | 98982   | 49 |
| 12 | 154208   | 146    | 995537   | 3.0    | 158671   | 149    | 841329    | 14263    | 98978   | 48 |
| 13 | 155083   | 146    | 995519   | 3.0    | 159565   | 149    | 840435    | 14292    | 98973   | 47 |
| 14 | 155957   | 146    | 995501   | 3.1    | 160467   | 149    | 839543    | 14320    | 98969   | 46 |
| 15 | 156830   | 145    | 995482   | 3.1    | 161347   | 148    | 838653    | 14349    | 98965   | 45 |
| 16 | 157700   | 145    | 995464   | 3.1    | 162236   | 148    | 837764    | 14378    | 98961   | 44 |
| 17 | 158569   | 145    | 995446   | 3.1    | 163123   | 148    | 836877    | 14407    | 98957   | 43 |
| 18 | 159435   | 144    | 995427   | 3.1    | 164008   | 148    | 835992    | 14436    | 98953   | 42 |
| 19 | 160301   | 144    | 995409   | 3.1    | 164892   | 147    | 835108    | 14464    | 98948   | 41 |
| 20 | 161164   | 144    | 995390   | 3.1    | 165774   | 147    | 834226    | 14493    | 98944   | 40 |
| 21 | 9.162025 | 143    | 9.995372 | 3.1    | 9.166654 | 146    | 10.833346 | 14522    | 98940   | 39 |
| 22 | 162885   | 143    | 995353   | 3.1    | 167532   | 146    | 832468    | 14551    | 98936   | 38 |
| 23 | 163743   | 143    | 995334   | 3.1    | 168409   | 146    | 831591    | 14580    | 98931   | 37 |
| 24 | 164600   | 143    | 995316   | 3.1    | 169284   | 146    | 830716    | 14608    | 98927   | 36 |
| 25 | 165454   | 142    | 995297   | 3.1    | 170157   | 145    | 829843    | 14637    | 98923   | 35 |
| 26 | 166307   | 142    | 995278   | 3.1    | 171029   | 145    | 828971    | 14666    | 98919   | 34 |
| 27 | 167159   | 142    | 995260   | 3.1    | 171899   | 145    | 828101    | 14695    | 98914   | 33 |
| 28 | 168008   | 141    | 995241   | 3.2    | 172767   | 144    | 827233    | 14723    | 98910   | 32 |
| 29 | 168856   | 141    | 995222   | 3.2    | 173634   | 144    | 826366    | 14752    | 98906   | 31 |
| 30 | 169702   | 141    | 995203   | 3.2    | 174499   | 144    | 825501    | 14781    | 98902   | 30 |
| 31 | 9.170547 | 140    | 9.995184 | 3.2    | 9.175362 | 144    | 10.824638 | 14810    | 98897   | 29 |
| 32 | 171389   | 140    | 995165   | 3.2    | 176224   | 143    | 823776    | 14838    | 98893   | 28 |
| 33 | 172230   | 140    | 995146   | 3.2    | 177084   | 143    | 822916    | 14867    | 98889   | 27 |
| 34 | 173070   | 140    | 995127   | 3.2    | 177942   | 143    | 822058    | 14896    | 98884   | 26 |
| 35 | 173908   | 139    | 995108   | 3.2    | 178799   | 142    | 821201    | 14925    | 98880   | 25 |
| 36 | 174744   | 139    | 995089   | 3.2    | 179655   | 142    | 820345    | 14954    | 98876   | 24 |
| 37 | 175578   | 139    | 995070   | 3.2    | 180508   | 142    | 819492    | 14982    | 98871   | 23 |
| 38 | 176411   | 139    | 995051   | 3.2    | 181360   | 142    | 818640    | 15011    | 98867   | 22 |
| 39 | 177242   | 139    | 995032   | 3.2    | 182211   | 141    | 817789    | 15040    | 98863   | 21 |
| 40 | 178072   | 138    | 995013   | 3.2    | 183059   | 141    | 816941    | 15069    | 98858   | 20 |
| 41 | 9.178900 | 138    | 9.994993 | 3.2    | 9.183907 | 141    | 10.816093 | 15097    | 98854   | 19 |
| 42 | 179726   | 137    | 994974   | 3.2    | 184752   | 141    | 815248    | 15126    | 98849   | 18 |
| 43 | 180551   | 137    | 994955   | 3.2    | 185597   | 140    | 814403    | 15155    | 98845   | 17 |
| 44 | 181374   | 137    | 994935   | 3.2    | 186439   | 140    | 813561    | 15184    | 98841   | 16 |
| 45 | 182196   | 137    | 994916   | 3.2    | 187280   | 140    | 812720    | 15212    | 98836   | 15 |
| 46 | 183016   | 137    | 994896   | 3.3    | 188120   | 140    | 811880    | 15241    | 98832   | 14 |
| 47 | 183834   | 136    | 994877   | 3.3    | 188958   | 139    | 811042    | 15270    | 98827   | 13 |
| 48 | 184651   | 136    | 994857   | 3.3    | 189794   | 139    | 810206    | 15299    | 98823   | 12 |
| 49 | 185466   | 136    | 994838   | 3.3    | 190629   | 139    | 809371    | 15327    | 98818   | 11 |
| 50 | 186280   | 135    | 994818   | 3.3    | 191462   | 139    | 808538    | 15356    | 98814   | 10 |
| 51 | 9.187092 | 135    | 9.994798 | 3.3    | 9.192294 | 138    | 10.807706 | 15385    | 98809   | 9  |
| 52 | 187903   | 135    | 994779   | 3.3    | 193124   | 138    | 806876    | 15414    | 98805   | 8  |
| 53 | 188712   | 135    | 994759   | 3.3    | 193953   | 138    | 806047    | 15442    | 98800   | 7  |
| 54 | 189519   | 134    | 994739   | 3.3    | 194780   | 138    | 805220    | 15471    | 98796   | 6  |
| 55 | 190325   | 134    | 994719   | 3.3    | 195606   | 137    | 804394    | 15500    | 98791   | 5  |
| 56 | 191130   | 134    | 994700   | 3.3    | 196430   | 137    | 803570    | 15529    | 98787   | 4  |
| 57 | 191933   | 134    | 994680   | 3.3    | 197253   | 137    | 802747    | 15557    | 98782   | 3  |
| 58 | 192734   | 133    | 994660   | 3.3    | 198074   | 137    | 801926    | 15586    | 98778   | 2  |
| 59 | 193534   | 133    | 994640   | 3.3    | 198894   | 136    | 801106    | 15615    | 98773   | 1  |
| 60 | 194332   | 133    | 994620   | 3.3    | 199713   | 136    | 800287    | 15643    | 98769   | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine |    |

81 Degrees.



|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.194332 | 133    | 9.994620 | 3.3    | 9.199713 | 136    | 10.800287 | 15643    | 98769    | 60 |
| 1  | 195129   | 133    | 994600   | 3.3    | 200529   | 136    | 799471    | 15672    | 98764    | 59 |
| 2  | 195925   | 132    | 994580   | 3.3    | 201345   | 136    | 798655    | 15701    | 98760    | 58 |
| 3  | 196719   | 132    | 994560   | 3.3    | 202159   | 135    | 797841    | 15730    | 98755    | 57 |
| 4  | 197511   | 132    | 994540   | 3.4    | 202971   | 135    | 797029    | 15758    | 98751    | 56 |
| 5  | 198302   | 132    | 994519   | 3.4    | 203782   | 135    | 796218    | 15787    | 98746    | 55 |
| 6  | 199091   | 131    | 994499   | 3.4    | 204592   | 135    | 795408    | 15816    | 98741    | 54 |
| 7  | 199879   | 131    | 994479   | 3.4    | 205400   | 134    | 794600    | 15845    | 98737    | 53 |
| 8  | 200666   | 131    | 994459   | 3.4    | 206207   | 134    | 793793    | 15873    | 98732    | 52 |
| 9  | 201451   | 131    | 994438   | 3.4    | 207013   | 134    | 792987    | 15902    | 98728    | 51 |
| 10 | 202234   | 131    | 994418   | 3.4    | 207817   | 134    | 792183    | 15931    | 98723    | 50 |
| 11 | 9.203017 | 130    | 9.994397 | 3.4    | 9.208619 | 133    | 10.791381 | 15959    | 98718    | 49 |
| 12 | 203797   | 130    | 994377   | 3.4    | 209420   | 133    | 790580    | 15988    | 98714    | 48 |
| 13 | 204577   | 130    | 994357   | 3.4    | 210220   | 133    | 789780    | 16017    | 98709    | 47 |
| 14 | 205354   | 129    | 994336   | 3.4    | 211018   | 133    | 788982    | 16046    | 98704    | 46 |
| 15 | 206131   | 129    | 994316   | 3.4    | 211815   | 133    | 788185    | 16074    | 98700    | 45 |
| 16 | 206906   | 129    | 994295   | 3.4    | 212611   | 132    | 787389    | 16103    | 98695    | 44 |
| 17 | 207679   | 129    | 994274   | 3.5    | 213405   | 132    | 786595    | 16132    | 98690    | 43 |
| 18 | 208452   | 128    | 994254   | 3.5    | 214198   | 132    | 785802    | 16160    | 98686    | 42 |
| 19 | 209222   | 128    | 994233   | 3.5    | 214989   | 132    | 785011    | 16189    | 98681    | 41 |
| 20 | 209992   | 128    | 994212   | 3.5    | 215780   | 131    | 784220    | 16218    | 98676    | 40 |
| 21 | 9.210760 | 128    | 9.994191 | 3.5    | 9.216568 | 131    | 10.783432 | 16246    | 98671    | 39 |
| 22 | 211526   | 127    | 994171   | 3.5    | 217356   | 131    | 782644    | 16275    | 98667    | 38 |
| 23 | 212291   | 127    | 994150   | 3.5    | 218142   | 131    | 781858    | 16304    | 98662    | 37 |
| 24 | 213055   | 127    | 994129   | 3.5    | 218926   | 130    | 781074    | 16333    | 98657    | 36 |
| 25 | 213818   | 127    | 994108   | 3.5    | 219710   | 130    | 780290    | 16361    | 98652    | 35 |
| 26 | 214579   | 127    | 994087   | 3.5    | 220492   | 130    | 779508    | 16390    | 98648    | 34 |
| 27 | 215338   | 126    | 994066   | 3.5    | 221272   | 130    | 778728    | 16419    | 98643    | 33 |
| 28 | 216097   | 126    | 994045   | 3.5    | 222052   | 130    | 777948    | 16447    | 98638    | 32 |
| 29 | 216854   | 126    | 994024   | 3.5    | 222830   | 129    | 777170    | 16476    | 98633    | 31 |
| 30 | 217609   | 126    | 994003   | 3.5    | 223606   | 129    | 776394    | 16505    | 98629    | 30 |
| 31 | 9.218363 | 125    | 9.993981 | 3.5    | 9.224382 | 129    | 10.775618 | 16533    | 98624    | 29 |
| 32 | 219116   | 125    | 993960   | 3.5    | 225156   | 129    | 774844    | 16562    | 98619    | 28 |
| 33 | 219868   | 125    | 993939   | 3.5    | 225929   | 129    | 774071    | 16591    | 98614    | 27 |
| 34 | 220618   | 125    | 993918   | 3.5    | 226700   | 128    | 773300    | 16620    | 98609    | 26 |
| 35 | 221367   | 125    | 993896   | 3.6    | 227471   | 128    | 772529    | 16648    | 98604    | 25 |
| 36 | 222115   | 124    | 993875   | 3.6    | 228239   | 128    | 771761    | 16677    | 98600    | 24 |
| 37 | 222861   | 124    | 993854   | 3.6    | 229007   | 128    | 770993    | 16706    | 98595    | 23 |
| 38 | 223606   | 124    | 993832   | 3.6    | 229773   | 127    | 770227    | 16734    | 98590    | 22 |
| 39 | 224349   | 124    | 993811   | 3.6    | 230539   | 127    | 769461    | 16763    | 98585    | 21 |
| 40 | 225092   | 123    | 993789   | 3.6    | 231302   | 127    | 768698    | 16792    | 98580    | 20 |
| 41 | 9.225833 | 123    | 9.993768 | 3.6    | 9.232065 | 127    | 10.767935 | 16820    | 98575    | 19 |
| 42 | 226573   | 123    | 993746   | 3.6    | 232826   | 127    | 767174    | 16849    | 98570    | 18 |
| 43 | 227311   | 123    | 993725   | 3.6    | 233586   | 126    | 766414    | 16878    | 98565    | 17 |
| 44 | 228048   | 123    | 993703   | 3.6    | 234345   | 126    | 765655    | 16906    | 98561    | 16 |
| 45 | 228784   | 122    | 993681   | 3.6    | 235103   | 126    | 764897    | 16935    | 98556    | 15 |
| 46 | 229518   | 122    | 993660   | 3.6    | 235859   | 126    | 764141    | 16964    | 98551    | 14 |
| 47 | 230252   | 122    | 993638   | 3.6    | 236614   | 126    | 763386    | 16992    | 98546    | 13 |
| 48 | 230984   | 122    | 993616   | 3.6    | 237368   | 125    | 762632    | 17021    | 98541    | 12 |
| 49 | 231714   | 122    | 993594   | 3.7    | 238120   | 125    | 761880    | 17050    | 98536    | 11 |
| 50 | 232444   | 121    | 993572   | 3.7    | 238872   | 125    | 761128    | 17078    | 98531    | 10 |
| 51 | 9.233172 | 121    | 9.993550 | 3.7    | 9.239622 | 125    | 10.760378 | 17107    | 98526    | 9  |
| 52 | 233899   | 121    | 993528   | 3.7    | 240371   | 125    | 759629    | 17136    | 98521    | 8  |
| 53 | 234625   | 121    | 993506   | 3.7    | 241118   | 124    | 758882    | 17164    | 98516    | 7  |
| 54 | 235349   | 120    | 993484   | 3.7    | 241865   | 124    | 758135    | 17193    | 98511    | 6  |
| 55 | 236073   | 120    | 993462   | 3.7    | 242610   | 124    | 757389    | 17222    | 98506    | 5  |
| 56 | 236795   | 120    | 993440   | 3.7    | 243354   | 124    | 756646    | 17250    | 98501    | 4  |
| 57 | 237515   | 120    | 993418   | 3.7    | 244097   | 124    | 755903    | 17279    | 98496    | 3  |
| 58 | 238235   | 120    | 993396   | 3.7    | 244839   | 123    | 755161    | 17308    | 98491    | 2  |
| 59 | 238953   | 120    | 993374   | 3.7    | 245579   | 123    | 754421    | 17336    | 98486    | 1  |
| 60 | 239670   | 119    | 993351   | 3.7    | 246319   | 123    | 753681    | 17365    | 98481    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (10°) Natural Sines.

31

| <i>i</i> | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |          |
|----------|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----------|
| 0        | 9.239670 |        | 9.993351 |        | 9.246319 |        | 10.753681 | 17365    | 98481    | 60       |
| 1        | 240386   | 119    | 993329   | 3.7    | 247057   | 123    | 752943    | 17393    | 98476    | 59       |
| 2        | 241101   | 119    | 993307   | 3.7    | 247794   | 123    | 752206    | 17422    | 98471    | 58       |
| 3        | 241814   | 119    | 993285   | 3.7    | 248530   | 122    | 751470    | 17451    | 98466    | 57       |
| 4        | 242526   | 118    | 993262   | 3.7    | 249264   | 122    | 750736    | 17479    | 98461    | 56       |
| 5        | 243237   | 118    | 993240   | 3.7    | 249998   | 122    | 750002    | 17508    | 98455    | 55       |
| 6        | 243947   | 118    | 993217   | 3.7    | 250730   | 122    | 749270    | 17537    | 98450    | 54       |
| 7        | 244656   | 118    | 993195   | 3.8    | 251461   | 122    | 748539    | 17565    | 98445    | 53       |
| 8        | 245363   | 118    | 993172   | 3.8    | 252191   | 122    | 747809    | 17594    | 98440    | 52       |
| 9        | 246069   | 118    | 993149   | 3.8    | 252920   | 121    | 747080    | 17623    | 98435    | 51       |
| 10       | 246775   | 117    | 993127   | 3.8    | 253648   | 121    | 746352    | 17651    | 98430    | 50       |
| 11       | 9.247478 | 117    | 9.993104 | 3.8    | 9.254374 | 121    | 10.745626 | 17680    | 98425    | 49       |
| 12       | 248181   | 117    | 993081   | 3.8    | 255100   | 121    | 744900    | 17708    | 98420    | 48       |
| 13       | 248883   | 117    | 993059   | 3.8    | 255824   | 120    | 744176    | 17737    | 98414    | 47       |
| 14       | 249583   | 117    | 993036   | 3.8    | 256547   | 120    | 743453    | 17766    | 98409    | 46       |
| 15       | 250282   | 116    | 993013   | 3.8    | 257269   | 120    | 742731    | 17794    | 98404    | 45       |
| 16       | 250980   | 116    | 992990   | 3.8    | 257990   | 120    | 742010    | 17823    | 98399    | 44       |
| 17       | 251677   | 116    | 992967   | 3.8    | 258710   | 120    | 741290    | 17852    | 98394    | 43       |
| 18       | 252373   | 116    | 992944   | 3.8    | 259429   | 120    | 740571    | 17880    | 98389    | 42       |
| 19       | 253067   | 116    | 992921   | 3.8    | 260146   | 119    | 739854    | 17909    | 98383    | 41       |
| 20       | 253761   | 115    | 992898   | 3.8    | 260863   | 119    | 739137    | 17937    | 98378    | 40       |
| 21       | 9.254453 | 115    | 9.992875 | 3.8    | 9.261578 | 119    | 10.738422 | 17966    | 98373    | 39       |
| 22       | 255144   | 115    | 992852   | 3.8    | 262292   | 119    | 737708    | 17995    | 98368    | 38       |
| 23       | 255834   | 115    | 992829   | 3.8    | 263005   | 119    | 736995    | 18023    | 98362    | 37       |
| 24       | 256523   | 115    | 992806   | 3.9    | 263717   | 119    | 736283    | 18052    | 98357    | 36       |
| 25       | 257211   | 115    | 992783   | 3.9    | 264428   | 118    | 735572    | 18081    | 98352    | 35       |
| 26       | 257898   | 114    | 992759   | 3.9    | 265138   | 118    | 734862    | 18109    | 98347    | 34       |
| 27       | 258583   | 114    | 992736   | 3.9    | 265847   | 118    | 734153    | 18138    | 98341    | 33       |
| 28       | 259268   | 114    | 992713   | 3.9    | 266555   | 118    | 733445    | 18166    | 98336    | 32       |
| 29       | 259951   | 114    | 992690   | 3.9    | 267261   | 118    | 732739    | 18195    | 98331    | 31       |
| 30       | 260633   | 114    | 992666   | 3.9    | 267967   | 118    | 732033    | 18224    | 98325    | 30       |
| 31       | 9.261314 | 113    | 9.992643 | 3.9    | 9.268671 | 117    | 10.731329 | 18252    | 98320    | 29       |
| 32       | 261994   | 113    | 992619   | 3.9    | 269375   | 117    | 730625    | 18281    | 98315    | 28       |
| 33       | 262673   | 113    | 992596   | 3.9    | 270077   | 117    | 729923    | 18309    | 98310    | 27       |
| 34       | 263351   | 113    | 992572   | 3.9    | 270779   | 117    | 729221    | 18338    | 98304    | 26       |
| 35       | 264027   | 113    | 992549   | 3.9    | 271479   | 117    | 728521    | 18367    | 98299    | 25       |
| 36       | 264703   | 112    | 992525   | 3.9    | 272178   | 116    | 727822    | 18395    | 98294    | 24       |
| 37       | 265377   | 112    | 992501   | 3.9    | 272876   | 116    | 727124    | 18424    | 98288    | 23       |
| 38       | 266051   | 112    | 992478   | 3.9    | 273573   | 116    | 726427    | 18452    | 98283    | 22       |
| 39       | 266723   | 112    | 992454   | 4.0    | 274269   | 116    | 725731    | 18481    | 98277    | 21       |
| 40       | 267395   | 112    | 992430   | 4.0    | 274964   | 116    | 725036    | 18509    | 98272    | 20       |
| 41       | 9.268065 | 111    | 9.992406 | 4.0    | 9.275658 | 115    | 10.724342 | 18538    | 98267    | 19       |
| 42       | 268734   | 111    | 992382   | 4.0    | 276351   | 115    | 723649    | 18567    | 98261    | 18       |
| 43       | 269402   | 111    | 992359   | 4.0    | 277043   | 115    | 722957    | 18595    | 98256    | 17       |
| 44       | 270069   | 111    | 992335   | 4.0    | 277734   | 115    | 722266    | 18624    | 98250    | 16       |
| 45       | 270735   | 111    | 992311   | 4.0    | 278424   | 115    | 721576    | 18652    | 98245    | 15       |
| 46       | 271400   | 111    | 992287   | 4.0    | 279113   | 115    | 720887    | 18681    | 98240    | 14       |
| 47       | 272064   | 110    | 992263   | 4.0    | 279801   | 114    | 720199    | 18710    | 98234    | 13       |
| 48       | 272726   | 110    | 992239   | 4.0    | 280488   | 114    | 719512    | 18738    | 98229    | 12       |
| 49       | 273388   | 110    | 992214   | 4.0    | 281174   | 114    | 718826    | 18767    | 98223    | 11       |
| 50       | 274049   | 110    | 992190   | 4.0    | 281858   | 114    | 718142    | 18795    | 98218    | 10       |
| 51       | 9.274708 | 110    | 9.992166 | 4.0    | 9.282542 | 114    | 10.717458 | 18824    | 98212    | 9        |
| 52       | 275367   | 110    | 992142   | 4.0    | 283225   | 114    | 716775    | 18852    | 98207    | 8        |
| 53       | 276024   | 109    | 992117   | 4.1    | 283907   | 113    | 716093    | 18881    | 98201    | 7        |
| 54       | 276681   | 109    | 992093   | 4.1    | 284588   | 113    | 715412    | 18910    | 98196    | 6        |
| 55       | 277337   | 109    | 992069   | 4.1    | 285268   | 113    | 714732    | 18938    | 98190    | 5        |
| 56       | 277991   | 109    | 992044   | 4.1    | 285947   | 113    | 714053    | 18967    | 98185    | 4        |
| 57       | 278644   | 109    | 992020   | 4.1    | 286624   | 113    | 713376    | 18995    | 98179    | 3        |
| 58       | 279297   | 109    | 991996   | 4.1    | 287301   | 113    | 712699    | 19024    | 98174    | 2        |
| 59       | 279948   | 109    | 991971   | 4.1    | 287977   | 113    | 712023    | 19052    | 98168    | 1        |
| 60       | 280599   | 108    | 991947   | 4.1    | 288652   | 112    | 711348    | 19081    | 98163    | 0        |
|          | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. | <i>i</i> |

79 Degrees.

|    | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.    | D. 10' | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.280599 | 108    | 9.991947 | 4.1    | 9.288652 | 112    | 10.711348 | 19061    | 98163    | 60 |
| 1  | 281248   | 108    | 991922   | 4.1    | 289326   | 112    | 710674    | 19109    | 98157    | 59 |
| 2  | 281897   | 108    | 991897   | 4.1    | 289999   | 112    | 710001    | 19138    | 98152    | 58 |
| 3  | 282544   | 108    | 991873   | 4.1    | 290671   | 112    | 709329    | 19167    | 98146    | 57 |
| 4  | 283190   | 108    | 991848   | 4.1    | 291342   | 112    | 708658    | 19195    | 98140    | 56 |
| 5  | 283836   | 107    | 991823   | 4.1    | 292013   | 111    | 707987    | 19224    | 98135    | 55 |
| 6  | 284480   | 107    | 991799   | 4.1    | 292682   | 111    | 707318    | 19252    | 98129    | 54 |
| 7  | 285124   | 107    | 991774   | 4.1    | 293350   | 111    | 706650    | 19281    | 98124    | 53 |
| 8  | 285766   | 107    | 991749   | 4.2    | 294017   | 111    | 705983    | 19309    | 98118    | 52 |
| 9  | 286408   | 107    | 991724   | 4.2    | 294684   | 111    | 705316    | 19338    | 98112    | 51 |
| 10 | 287048   | 107    | 991699   | 4.2    | 295349   | 111    | 704651    | 19366    | 98107    | 50 |
| 11 | 9.287687 | 106    | 9.991674 | 4.2    | 9.296013 | 111    | 10.703987 | 19395    | 98101    | 49 |
| 12 | 288326   | 106    | 991649   | 4.2    | 296677   | 111    | 703323    | 19423    | 98096    | 48 |
| 13 | 288964   | 106    | 991624   | 4.2    | 297339   | 110    | 702661    | 19452    | 98090    | 47 |
| 14 | 289600   | 106    | 991599   | 4.2    | 298001   | 110    | 701999    | 19481    | 98084    | 46 |
| 15 | 290236   | 106    | 991574   | 4.2    | 298662   | 110    | 701338    | 19509    | 98079    | 45 |
| 16 | 290870   | 106    | 991549   | 4.2    | 299322   | 110    | 700678    | 19538    | 98073    | 44 |
| 17 | 291504   | 105    | 991524   | 4.2    | 299980   | 110    | 700020    | 19566    | 98067    | 43 |
| 18 | 292137   | 105    | 991498   | 4.2    | 300638   | 109    | 699362    | 19595    | 98061    | 42 |
| 19 | 292768   | 105    | 991473   | 4.2    | 301295   | 109    | 698705    | 19623    | 98056    | 41 |
| 20 | 293399   | 105    | 991448   | 4.2    | 301951   | 109    | 698049    | 19652    | 98050    | 40 |
| 21 | 9.294029 | 105    | 9.991422 | 4.2    | 9.302607 | 109    | 10.697393 | 19680    | 98044    | 39 |
| 22 | 294658   | 105    | 991397   | 4.2    | 303261   | 109    | 696739    | 19709    | 98039    | 38 |
| 23 | 295286   | 105    | 991372   | 4.3    | 303914   | 109    | 696086    | 19737    | 98033    | 37 |
| 24 | 295913   | 104    | 991346   | 4.3    | 304567   | 109    | 695433    | 19766    | 98027    | 36 |
| 25 | 296539   | 104    | 991321   | 4.3    | 305218   | 108    | 694782    | 19794    | 98021    | 35 |
| 26 | 297164   | 104    | 991295   | 4.3    | 305869   | 108    | 694131    | 19823    | 98016    | 34 |
| 27 | 297788   | 104    | 991270   | 4.3    | 306519   | 108    | 693481    | 19851    | 98010    | 33 |
| 28 | 298412   | 104    | 991244   | 4.3    | 307168   | 108    | 692832    | 19880    | 98004    | 32 |
| 29 | 299034   | 104    | 991218   | 4.3    | 307815   | 108    | 692185    | 19908    | 97998    | 31 |
| 30 | 299655   | 103    | 991193   | 4.3    | 308463   | 108    | 691537    | 19937    | 97992    | 30 |
| 31 | 9.300276 | 103    | 9.991167 | 4.3    | 9.309109 | 107    | 10.690891 | 19965    | 97987    | 29 |
| 32 | 300895   | 103    | 991141   | 4.3    | 309754   | 107    | 690246    | 19994    | 97981    | 28 |
| 33 | 301514   | 103    | 991115   | 4.3    | 310398   | 107    | 689602    | 20022    | 97975    | 27 |
| 34 | 302132   | 103    | 991090   | 4.3    | 311042   | 107    | 688958    | 20051    | 97969    | 26 |
| 35 | 302748   | 103    | 991064   | 4.3    | 311685   | 107    | 688315    | 20079    | 97963    | 25 |
| 36 | 303364   | 102    | 991038   | 4.3    | 312327   | 107    | 687673    | 20108    | 97958    | 24 |
| 37 | 303979   | 102    | 991012   | 4.3    | 312967   | 107    | 687033    | 20136    | 97952    | 23 |
| 38 | 304593   | 102    | 990986   | 4.3    | 313608   | 106    | 686392    | 20165    | 97946    | 22 |
| 39 | 305207   | 102    | 990960   | 4.3    | 314247   | 106    | 685753    | 20193    | 97940    | 21 |
| 40 | 305819   | 102    | 990934   | 4.4    | 314885   | 106    | 685115    | 20222    | 97934    | 20 |
| 41 | 9.306430 | 102    | 9.990908 | 4.4    | 9.315523 | 106    | 10.684477 | 20250    | 97928    | 19 |
| 42 | 307041   | 102    | 990882   | 4.4    | 316159   | 106    | 683841    | 20279    | 97922    | 18 |
| 43 | 307650   | 102    | 990855   | 4.4    | 316795   | 106    | 683205    | 20307    | 97916    | 17 |
| 44 | 308259   | 101    | 990829   | 4.4    | 317430   | 106    | 682570    | 20336    | 97910    | 16 |
| 45 | 308867   | 101    | 990803   | 4.4    | 318064   | 105    | 681936    | 20364    | 97905    | 15 |
| 46 | 309474   | 101    | 990777   | 4.4    | 318697   | 105    | 681303    | 20393    | 97899    | 14 |
| 47 | 310080   | 101    | 990750   | 4.4    | 319329   | 105    | 680671    | 20421    | 97893    | 13 |
| 48 | 310685   | 101    | 990724   | 4.4    | 319961   | 105    | 680039    | 20450    | 97887    | 12 |
| 49 | 311289   | 100    | 990697   | 4.4    | 320592   | 105    | 679408    | 20478    | 97881    | 11 |
| 50 | 311893   | 100    | 990671   | 4.4    | 321222   | 105    | 678778    | 20507    | 97875    | 10 |
| 51 | 9.312495 | 100    | 9.990644 | 4.4    | 9.321851 | 105    | 10.678149 | 20535    | 97869    | 9  |
| 52 | 313097   | 100    | 990618   | 4.4    | 322479   | 104    | 677521    | 20563    | 97863    | 8  |
| 53 | 313698   | 100    | 990591   | 4.4    | 323106   | 104    | 676894    | 20592    | 97857    | 7  |
| 54 | 314297   | 100    | 990565   | 4.4    | 323733   | 104    | 676267    | 20620    | 97851    | 6  |
| 55 | 314897   | 100    | 990538   | 4.4    | 324358   | 104    | 675642    | 20649    | 97845    | 5  |
| 56 | 315495   | 100    | 990511   | 4.5    | 324983   | 104    | 675017    | 20677    | 97839    | 4  |
| 57 | 316092   | 99     | 990485   | 4.5    | 325607   | 104    | 674393    | 20706    | 97833    | 3  |
| 58 | 316689   | 99     | 990458   | 4.5    | 326231   | 104    | 673769    | 20734    | 97827    | 2  |
| 59 | 317284   | 99     | 990431   | 4.5    | 326853   | 104    | 673147    | 20763    | 97821    | 1  |
| 60 | 317879   |        | 990404   | 4.5    | 327475   |        | 672525    | 20791    | 97815    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (12°) Natural Sines.

33

| <i>i</i> | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine.    | N. cos.  |
|----------|----------|--------|----------|--------|----------|--------|-----------|-------------|----------|
| 0        | 9.317879 |        | 9.990404 |        | 9.327474 |        | 10.672526 | 20791 97815 | 60       |
| 1        | 318473   | 99.0   | 990378   | 4.5    | 328095   | 103    | 671905    | 20820 97809 | 59       |
| 2        | 319066   | 98.8   | 990351   | 4.5    | 328715   | 103    | 671285    | 20848 97803 | 58       |
| 3        | 319658   | 98.7   | 990324   | 4.5    | 329334   | 103    | 670666    | 20877 97797 | 57       |
| 4        | 320249   | 98.6   | 990297   | 4.5    | 329953   | 103    | 670047    | 20906 97791 | 56       |
| 5        | 320840   | 98.4   | 990270   | 4.5    | 330570   | 103    | 669430    | 20933 97784 | 55       |
| 6        | 321430   | 98.3   | 990243   | 4.5    | 331187   | 103    | 668813    | 20962 97778 | 54       |
| 7        | 322019   | 98.2   | 990215   | 4.5    | 331803   | 103    | 668197    | 20990 97772 | 53       |
| 8        | 322607   | 98.0   | 990188   | 4.5    | 332418   | 102    | 667582    | 21019 97766 | 52       |
| 9        | 323194   | 97.7   | 990161   | 4.5    | 333033   | 102    | 666967    | 21047 97760 | 51       |
| 10       | 323780   | 97.6   | 990134   | 4.5    | 333646   | 102    | 666354    | 21076 97754 | 50       |
| 11       | 9.324366 | 97.5   | 9.990107 | 4.5    | 9.334259 | 102    | 10.665741 | 21104 97748 | 49       |
| 12       | 324950   | 97.3   | 990079   | 4.6    | 334871   | 102    | 665129    | 21132 97742 | 48       |
| 13       | 325534   | 97.2   | 990052   | 4.6    | 335482   | 102    | 664518    | 21161 97735 | 47       |
| 14       | 326117   | 97.0   | 990025   | 4.6    | 336093   | 102    | 663907    | 21189 97729 | 46       |
| 15       | 326700   | 96.9   | 989997   | 4.6    | 336702   | 101    | 663298    | 21218 97723 | 45       |
| 16       | 327281   | 96.8   | 989970   | 4.6    | 337311   | 101    | 662689    | 21246 97717 | 44       |
| 17       | 327862   | 96.6   | 989942   | 4.6    | 337919   | 101    | 662081    | 21275 97711 | 43       |
| 18       | 328442   | 96.5   | 989915   | 4.6    | 338527   | 101    | 661473    | 21303 97705 | 42       |
| 19       | 329021   | 96.5   | 989887   | 4.6    | 339133   | 101    | 660867    | 21331 97698 | 41       |
| 20       | 329599   | 96.4   | 989860   | 4.6    | 339739   | 101    | 660261    | 21360 97692 | 40       |
| 21       | 9.330176 | 96.2   | 9.989832 | 4.6    | 9.340344 | 101    | 10.659656 | 21388 97686 | 39       |
| 22       | 330753   | 96.1   | 989804   | 4.6    | 340948   | 101    | 659052    | 21417 97680 | 38       |
| 23       | 331329   | 96.0   | 989777   | 4.6    | 341552   | 100    | 658448    | 21445 97673 | 37       |
| 24       | 331903   | 95.8   | 989749   | 4.6    | 342155   | 100    | 657845    | 21474 97667 | 36       |
| 25       | 332478   | 95.7   | 989721   | 4.7    | 342757   | 100    | 657243    | 21502 97661 | 35       |
| 26       | 333051   | 95.6   | 989693   | 4.7    | 343358   | 100    | 656642    | 21530 97655 | 34       |
| 27       | 333624   | 95.4   | 989665   | 4.7    | 343958   | 100    | 656042    | 21559 97648 | 33       |
| 28       | 334195   | 95.3   | 989637   | 4.7    | 344558   | 100    | 655442    | 21587 97642 | 32       |
| 29       | 334766   | 95.2   | 989609   | 4.7    | 345157   | 100    | 654843    | 21616 97636 | 31       |
| 30       | 335337   | 95.0   | 989582   | 4.7    | 345755   | 100    | 654245    | 21644 97630 | 30       |
| 31       | 9.335906 | 94.9   | 9.989553 | 4.7    | 9.346353 | 99.4   | 10.653647 | 21672 97623 | 29       |
| 32       | 336475   | 94.8   | 989525   | 4.7    | 346949   | 99.3   | 653051    | 21701 97617 | 28       |
| 33       | 337043   | 94.6   | 989497   | 4.7    | 347545   | 99.2   | 652455    | 21729 97611 | 27       |
| 34       | 337610   | 94.4   | 989469   | 4.7    | 348141   | 99.1   | 651859    | 21758 97604 | 26       |
| 35       | 338176   | 94.3   | 989441   | 4.7    | 348735   | 99.0   | 651265    | 21786 97598 | 25       |
| 36       | 338742   | 94.1   | 989413   | 4.7    | 349329   | 98.8   | 650671    | 21814 97592 | 24       |
| 37       | 339306   | 94.0   | 989384   | 4.7    | 349922   | 98.7   | 650078    | 21843 97585 | 23       |
| 38       | 339871   | 93.9   | 989356   | 4.7    | 350514   | 98.6   | 649486    | 21871 97579 | 22       |
| 39       | 340434   | 93.7   | 989328   | 4.7    | 351106   | 98.5   | 648894    | 21899 97573 | 21       |
| 40       | 340996   | 93.6   | 989300   | 4.7    | 351697   | 98.3   | 648303    | 21928 97566 | 20       |
| 41       | 9.341558 | 93.5   | 9.989271 | 4.7    | 9.352287 | 98.2   | 10.647713 | 21956 97560 | 19       |
| 42       | 342119   | 93.4   | 989243   | 4.7    | 352876   | 98.1   | 647124    | 21985 97553 | 18       |
| 43       | 342679   | 93.2   | 989214   | 4.7    | 353465   | 98.0   | 646535    | 22013 97547 | 17       |
| 44       | 343239   | 93.1   | 989186   | 4.7    | 354053   | 97.9   | 645947    | 22041 97541 | 16       |
| 45       | 343797   | 93.0   | 989157   | 4.7    | 354640   | 97.7   | 645360    | 22070 97534 | 15       |
| 46       | 344355   | 92.9   | 989128   | 4.8    | 355227   | 97.6   | 644773    | 22098 97528 | 14       |
| 47       | 344912   | 92.7   | 989100   | 4.8    | 355813   | 97.5   | 644187    | 22126 97521 | 13       |
| 48       | 345469   | 92.6   | 989071   | 4.8    | 356398   | 97.4   | 643602    | 22155 97515 | 12       |
| 49       | 346024   | 92.5   | 989042   | 4.8    | 356982   | 97.3   | 643018    | 22183 97508 | 11       |
| 50       | 346579   | 92.4   | 989014   | 4.8    | 357566   | 97.1   | 642434    | 22212 97502 | 10       |
| 51       | 9.347134 | 92.2   | 9.988985 | 4.8    | 9.358149 | 97.0   | 10.641851 | 22240 97496 | 9        |
| 52       | 347687   | 92.1   | 988956   | 4.8    | 358731   | 96.9   | 641269    | 22268 97489 | 8        |
| 53       | 348240   | 92.0   | 988927   | 4.8    | 359313   | 96.8   | 640687    | 22297 97483 | 7        |
| 54       | 348792   | 91.9   | 988898   | 4.8    | 359893   | 96.7   | 640107    | 22325 97476 | 6        |
| 55       | 349343   | 91.7   | 988869   | 4.8    | 360474   | 96.6   | 639526    | 22353 97470 | 5        |
| 56       | 349893   | 91.6   | 988840   | 4.8    | 361053   | 96.5   | 638947    | 22382 97463 | 4        |
| 57       | 350443   | 91.5   | 988811   | 4.9    | 361632   | 96.4   | 638368    | 22410 97457 | 3        |
| 58       | 350992   | 91.4   | 988782   | 4.9    | 362210   | 96.3   | 637790    | 22438 97450 | 2        |
| 59       | 351540   | 91.3   | 988753   | 4.9    | 362787   | 96.2   | 637213    | 22467 97444 | 1        |
| 60       | 352088   |        | 988724   |        | 363364   | 96.1   | 636636    | 22495 97437 | 0        |
|          | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.     | N. sine. |

77 Degrees.

| <i>i</i> | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N.sine. | N. cos.  |          |
|----------|----------|--------|----------|--------|----------|--------|-----------|---------|----------|----------|
| 0        | 9.352088 |        | 9.988724 |        | 9.363364 |        | 10.636636 | 22495   | 97437    | 60       |
| 1        | 352635   | 91.1   | 988695   | 4.9    | 363940   | 96.0   | 636060    | 22523   | 97430    | 59       |
| 2        | 353181   | 90.9   | 988666   | 4.9    | 364515   | 95.9   | 635485    | 22552   | 97424    | 58       |
| 3        | 353726   | 90.8   | 988636   | 4.9    | 365090   | 95.8   | 634910    | 22580   | 97417    | 57       |
| 4        | 354271   | 90.7   | 988607   | 4.9    | 365664   | 95.7   | 634336    | 22608   | 97411    | 56       |
| 5        | 354815   | 90.5   | 988578   | 4.9    | 366237   | 95.5   | 633763    | 22637   | 97404    | 55       |
| 6        | 355358   | 90.4   | 988548   | 4.9    | 366810   | 95.4   | 633190    | 22665   | 97398    | 54       |
| 7        | 355901   | 90.3   | 988519   | 4.9    | 367382   | 95.3   | 632618    | 22693   | 97391    | 53       |
| 8        | 356443   | 90.2   | 988489   | 4.9    | 367953   | 95.2   | 632047    | 22722   | 97384    | 52       |
| 9        | 356984   | 90.1   | 988460   | 4.9    | 368524   | 95.1   | 631476    | 22750   | 97378    | 51       |
| 10       | 357524   | 89.9   | 988430   | 4.9    | 369094   | 95.0   | 630906    | 22778   | 97371    | 50       |
| 11       | 9.358064 |        | 9.988401 |        | 9.369663 |        | 10.630337 | 22807   | 97365    | 49       |
| 12       | 358603   | 89.8   | 988371   | 4.9    | 370232   | 94.8   | 629768    | 22835   | 97358    | 48       |
| 13       | 359141   | 89.7   | 988342   | 4.9    | 370799   | 94.6   | 629201    | 22863   | 97351    | 47       |
| 14       | 359678   | 89.6   | 988312   | 4.9    | 371367   | 94.5   | 628633    | 22892   | 97345    | 46       |
| 15       | 360215   | 89.5   | 988282   | 5.0    | 371933   | 94.4   | 628067    | 22920   | 97338    | 45       |
| 16       | 360752   | 89.3   | 988252   | 5.0    | 372499   | 94.3   | 627501    | 22948   | 97331    | 44       |
| 17       | 361287   | 89.2   | 988223   | 5.0    | 373064   | 94.2   | 626936    | 22977   | 97325    | 43       |
| 18       | 361822   | 89.1   | 988193   | 5.0    | 373629   | 94.1   | 626371    | 23005   | 97318    | 42       |
| 19       | 362356   | 89.0   | 988163   | 5.0    | 374193   | 94.0   | 625807    | 23033   | 97311    | 41       |
| 20       | 362889   | 88.9   | 988133   | 5.0    | 374756   | 93.9   | 625244    | 23062   | 97304    | 40       |
| 21       | 9.363422 |        | 9.988103 |        | 9.375319 |        | 10.624681 | 23090   | 97298    | 39       |
| 22       | 363954   | 88.7   | 988073   | 5.0    | 375881   | 93.8   | 624681    | 23118   | 97291    | 38       |
| 23       | 364485   | 88.5   | 988043   | 5.0    | 376442   | 93.7   | 624119    | 23146   | 97284    | 37       |
| 24       | 365016   | 88.3   | 988013   | 5.0    | 377003   | 93.6   | 623558    | 23175   | 97278    | 36       |
| 25       | 365546   | 88.2   | 987983   | 5.0    | 377563   | 93.5   | 622997    | 23203   | 97271    | 35       |
| 26       | 366075   | 88.1   | 987953   | 5.0    | 378122   | 93.4   | 622437    | 23231   | 97264    | 34       |
| 27       | 366604   | 88.0   | 987922   | 5.0    | 378681   | 93.3   | 621878    | 23259   | 97257    | 33       |
| 28       | 367131   | 87.9   | 987892   | 5.0    | 379239   | 93.2   | 621319    | 23288   | 97251    | 32       |
| 29       | 367659   | 87.7   | 987862   | 5.0    | 379797   | 93.1   | 620761    | 23316   | 97244    | 31       |
| 30       | 368185   | 87.6   | 987832   | 5.1    | 380354   | 93.0   | 620203    | 23345   | 97237    | 30       |
| 31       | 9.368711 |        | 9.987801 |        | 9.380910 |        | 10.619090 | 23373   | 97230    | 29       |
| 32       | 369236   | 87.5   | 987771   | 5.1    | 381466   | 92.9   | 619546    | 23401   | 97223    | 28       |
| 33       | 369761   | 87.4   | 987740   | 5.1    | 382020   | 92.8   | 618990    | 23429   | 97217    | 27       |
| 34       | 370285   | 87.3   | 987710   | 5.1    | 382575   | 92.7   | 618434    | 23458   | 97210    | 26       |
| 35       | 370808   | 87.2   | 987679   | 5.1    | 383129   | 92.6   | 617880    | 23487   | 97203    | 25       |
| 36       | 371330   | 87.1   | 987649   | 5.1    | 383682   | 92.5   | 617326    | 23516   | 97196    | 24       |
| 37       | 371852   | 87.0   | 987618   | 5.1    | 384234   | 92.4   | 616776    | 23545   | 97189    | 23       |
| 38       | 372373   | 86.9   | 987588   | 5.1    | 384786   | 92.3   | 616224    | 23574   | 97182    | 22       |
| 39       | 372894   | 86.8   | 987557   | 5.1    | 385337   | 92.2   | 615676    | 23603   | 97175    | 21       |
| 40       | 373414   | 86.6   | 987526   | 5.1    | 385888   | 92.1   | 615124    | 23632   | 97169    | 20       |
| 41       | 9.373933 |        | 9.987496 |        | 9.386438 |        | 10.613562 | 23661   | 97162    | 19       |
| 42       | 374444   | 86.4   | 987465   | 5.1    | 386438   | 92.0   | 614576    | 23690   | 97155    | 18       |
| 43       | 374970   | 86.3   | 987434   | 5.1    | 386987   | 91.9   | 614024    | 23719   | 97148    | 17       |
| 44       | 375487   | 86.2   | 987403   | 5.1    | 387536   | 91.8   | 613476    | 23748   | 97141    | 16       |
| 45       | 376003   | 86.1   | 987372   | 5.2    | 388084   | 91.7   | 612924    | 23777   | 97134    | 15       |
| 46       | 376519   | 86.0   | 987341   | 5.2    | 388631   | 91.6   | 612376    | 23806   | 97127    | 14       |
| 47       | 377035   | 85.9   | 987310   | 5.2    | 389178   | 91.5   | 611822    | 23835   | 97120    | 13       |
| 48       | 377549   | 85.8   | 987279   | 5.2    | 389724   | 91.4   | 611276    | 23864   | 97113    | 12       |
| 49       | 378063   | 85.7   | 987248   | 5.2    | 390270   | 91.3   | 610730    | 23893   | 97106    | 11       |
| 50       | 378577   | 85.6   | 987217   | 5.2    | 390815   | 91.2   | 610182    | 23922   | 97100    | 10       |
| 51       | 9.379089 |        | 9.987186 |        | 9.391903 |        | 10.608097 | 23951   | 97093    | 9        |
| 52       | 379601   | 85.5   | 987186   | 5.2    | 391360   | 91.1   | 609640    | 23980   | 97086    | 8        |
| 53       | 380113   | 85.4   | 987155   | 5.2    | 391903   | 91.0   | 609185    | 24009   | 97079    | 7        |
| 54       | 380624   | 85.1   | 987124   | 5.2    | 392447   | 90.9   | 608730    | 24038   | 97072    | 6        |
| 55       | 381134   | 85.0   | 987092   | 5.2    | 392989   | 90.8   | 608276    | 24067   | 97065    | 5        |
| 56       | 381643   | 84.9   | 987061   | 5.2    | 393531   | 90.7   | 607824    | 24096   | 97058    | 4        |
| 57       | 382152   | 84.8   | 987030   | 5.2    | 394073   | 90.6   | 607376    | 24125   | 97051    | 3        |
| 58       | 382661   | 84.7   | 986998   | 5.2    | 394614   | 90.5   | 606924    | 24154   | 97044    | 2        |
| 59       | 383168   | 84.6   | 986967   | 5.2    | 395154   | 90.4   | 606476    | 24183   | 97037    | 1        |
| 60       | 383675   | 84.5   | 986936   | 5.2    | 395694   | 90.3   | 606024    | 24212   | 97030    | 0        |
|          |          |        | 986904   | 5.2    | 396233   | 90.2   | 605576    | 24241   |          |          |
|          |          |        |          |        | 396771   | 90.1   | 605124    | 24270   |          |          |
|          |          |        |          |        |          | 90.0   | 604676    | 24299   |          |          |
|          |          |        |          |        |          | 89.9   | 604224    | 24328   |          |          |
|          |          |        |          |        |          | 89.8   | 603776    | 24357   |          |          |
|          |          |        |          |        |          | 89.7   | 603324    | 24386   |          |          |
|          |          |        |          |        |          |        |           |         |          |          |
|          | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos. | N. sine. | <i>i</i> |

TABLE II.

Log. Sines and Tangents. (14°) Natural Sines.

35

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.333675 |        | 9.986904 |        | 9.396771 |        | 10.603229 | 24192    | 97030    |
| 1  | 384182   | 84.4   | 986873   | 5.2    | 397309   | 89.6   | 602691    | 24220    | 97023    |
| 2  | 384687   | 84.3   | 986841   | 5.3    | 397846   | 89.6   | 602154    | 24249    | 97015    |
| 3  | 385192   | 84.2   | 986809   | 5.3    | 398383   | 89.5   | 601617    | 24277    | 97008    |
| 4  | 385697   | 84.1   | 986778   | 5.3    | 398919   | 89.4   | 601081    | 24305    | 97001    |
| 5  | 386201   | 84.0   | 986746   | 5.3    | 399455   | 89.3   | 600545    | 24333    | 96994    |
| 6  | 386704   | 83.9   | 986714   | 5.3    | 399990   | 89.2   | 600010    | 24362    | 96987    |
| 7  | 387207   | 83.8   | 986683   | 5.3    | 400524   | 89.1   | 599476    | 24390    | 96980    |
| 8  | 387709   | 83.7   | 986651   | 5.3    | 401058   | 89.0   | 598942    | 24418    | 96973    |
| 9  | 388210   | 83.6   | 986619   | 5.3    | 401591   | 88.9   | 598409    | 24446    | 96966    |
| 10 | 388711   | 83.5   | 986587   | 5.3    | 402124   | 88.8   | 597876    | 24474    | 96959    |
| 11 | 9.389211 | 83.4   | 9.986555 | 5.3    | 9.402656 | 88.7   | 10.597344 | 24503    | 96952    |
| 12 | 389711   | 83.3   | 986523   | 5.3    | 403187   | 88.6   | 596813    | 24531    | 96945    |
| 13 | 390210   | 83.2   | 986491   | 5.3    | 403718   | 88.5   | 596282    | 24559    | 96937    |
| 14 | 390708   | 83.1   | 986459   | 5.3    | 404249   | 88.4   | 595751    | 24587    | 96930    |
| 15 | 391206   | 83.0   | 986427   | 5.3    | 404778   | 88.3   | 595222    | 24615    | 96923    |
| 16 | 391703   | 82.8   | 986395   | 5.3    | 405308   | 88.2   | 594692    | 24644    | 96916    |
| 17 | 392199   | 82.7   | 986363   | 5.3    | 405836   | 88.1   | 594164    | 24672    | 96909    |
| 18 | 392695   | 82.6   | 986331   | 5.4    | 406364   | 88.0   | 593636    | 24700    | 96902    |
| 19 | 393191   | 82.5   | 986299   | 5.4    | 406892   | 87.9   | 593108    | 24728    | 96894    |
| 20 | 393685   | 82.4   | 986266   | 5.4    | 407419   | 87.8   | 592581    | 24756    | 96887    |
| 21 | 9.394179 | 82.3   | 9.986234 | 5.4    | 9.407945 | 87.7   | 10.592055 | 24784    | 96880    |
| 22 | 394673   | 82.2   | 986202   | 5.4    | 408471   | 87.6   | 591529    | 24813    | 96873    |
| 23 | 395166   | 82.0   | 986169   | 5.4    | 408997   | 87.5   | 591003    | 24841    | 96866    |
| 24 | 395658   | 81.9   | 986137   | 5.4    | 409521   | 87.4   | 590479    | 24869    | 96858    |
| 25 | 396150   | 81.8   | 986104   | 5.4    | 410045   | 87.3   | 589955    | 24897    | 96851    |
| 26 | 396641   | 81.7   | 986072   | 5.4    | 410569   | 87.2   | 589431    | 24925    | 96844    |
| 27 | 397132   | 81.6   | 986039   | 5.4    | 411092   | 87.1   | 588908    | 24954    | 96837    |
| 28 | 397621   | 81.5   | 986007   | 5.4    | 411615   | 87.0   | 588385    | 24982    | 96830    |
| 29 | 398111   | 81.4   | 985974   | 5.4    | 412137   | 86.9   | 587863    | 25010    | 96822    |
| 30 | 398600   | 81.3   | 985942   | 5.4    | 412658   | 86.8   | 587342    | 25038    | 96815    |
| 31 | 9.399088 | 81.2   | 9.985909 | 5.5    | 9.413179 | 86.7   | 10.586821 | 25066    | 96808    |
| 32 | 399575   | 81.1   | 985876   | 5.5    | 413699   | 86.6   | 586801    | 25094    | 96801    |
| 33 | 400062   | 81.0   | 985843   | 5.5    | 414219   | 86.5   | 586278    | 25122    | 96794    |
| 34 | 400549   | 80.9   | 985811   | 5.5    | 414738   | 86.4   | 585756    | 25150    | 96787    |
| 35 | 401035   | 80.8   | 985778   | 5.5    | 415257   | 86.3   | 585232    | 25178    | 96780    |
| 36 | 401520   | 80.7   | 985745   | 5.5    | 415775   | 86.2   | 584709    | 25206    | 96773    |
| 37 | 402005   | 80.6   | 985712   | 5.5    | 416293   | 86.1   | 584185    | 25234    | 96766    |
| 38 | 402489   | 80.5   | 985679   | 5.5    | 416810   | 86.0   | 583661    | 25262    | 96759    |
| 39 | 402972   | 80.4   | 985646   | 5.5    | 417326   | 85.9   | 583136    | 25290    | 96752    |
| 40 | 403455   | 80.3   | 985613   | 5.5    | 417842   | 85.8   | 582612    | 25318    | 96745    |
| 41 | 9.403938 | 80.2   | 9.985580 | 5.5    | 9.418358 | 85.7   | 10.581642 | 25346    | 96738    |
| 42 | 404420   | 80.1   | 985547   | 5.5    | 418873   | 85.6   | 581127    | 25374    | 96731    |
| 43 | 404901   | 80.0   | 985514   | 5.5    | 419387   | 85.5   | 580603    | 25402    | 96724    |
| 44 | 405382   | 79.9   | 985480   | 5.5    | 419901   | 85.4   | 580079    | 25430    | 96717    |
| 45 | 405862   | 79.8   | 985447   | 5.5    | 420415   | 85.3   | 579555    | 25458    | 96710    |
| 46 | 406341   | 79.7   | 985414   | 5.5    | 420927   | 85.2   | 579031    | 25486    | 96703    |
| 47 | 406820   | 79.6   | 985380   | 5.5    | 421440   | 85.1   | 578506    | 25514    | 96696    |
| 48 | 407299   | 79.5   | 985347   | 5.5    | 421952   | 85.0   | 577981    | 25542    | 96689    |
| 49 | 407777   | 79.4   | 985314   | 5.5    | 422463   | 84.9   | 577456    | 25570    | 96682    |
| 50 | 408254   | 79.3   | 985280   | 5.5    | 422974   | 84.8   | 576931    | 25598    | 96675    |
| 51 | 9.408731 | 79.2   | 9.985247 | 5.5    | 9.423484 | 84.7   | 10.576516 | 25626    | 96668    |
| 52 | 409207   | 79.1   | 985213   | 5.5    | 423993   | 84.6   | 576407    | 25654    | 96661    |
| 53 | 409682   | 79.0   | 985180   | 5.5    | 424503   | 84.5   | 575881    | 25682    | 96654    |
| 54 | 410157   | 78.9   | 985146   | 5.5    | 425011   | 84.4   | 575356    | 25710    | 96647    |
| 55 | 410632   | 78.8   | 985113   | 5.5    | 425519   | 84.3   | 574831    | 25738    | 96640    |
| 56 | 411106   | 78.7   | 985079   | 5.5    | 426027   | 84.2   | 574305    | 25766    | 96633    |
| 57 | 411579   | 78.6   | 985045   | 5.5    | 426534   | 84.1   | 573780    | 25794    | 96626    |
| 58 | 412052   |        | 985011   | 5.5    | 427041   |        | 573254    | 25822    | 96619    |
| 59 | 412524   |        | 984978   | 5.5    | 427547   |        | 572728    | 25850    | 96612    |
| 60 | 412996   |        | 984944   | 5.5    | 428052   |        | 572202    | 25878    | 96605    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

75 Degrees.

|    | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.    | D. 10' | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.412996 | 78.5   | 9.984944 | 5.7    | 9.428062 | 84.2   | 10.571948 | 25882    | 96593    |
| 1  | 413467   | 78.4   | 984910   | 5.7    | 428557   | 84.1   | 571443    | 25910    | 96585    |
| 2  | 413938   | 78.3   | 984876   | 5.7    | 429062   | 84.0   | 570938    | 25938    | 96578    |
| 3  | 414408   | 78.2   | 984842   | 5.7    | 429566   | 83.9   | 570434    | 25966    | 96570    |
| 4  | 414878   | 78.1   | 984808   | 5.7    | 430070   | 83.8   | 569930    | 25994    | 96562    |
| 5  | 415347   | 78.0   | 984774   | 5.7    | 430573   | 83.7   | 569427    | 26022    | 96555    |
| 6  | 415815   | 77.9   | 984740   | 5.7    | 431075   | 83.6   | 568925    | 26050    | 96547    |
| 7  | 416283   | 77.8   | 984706   | 5.7    | 431577   | 83.5   | 568423    | 26079    | 96540    |
| 8  | 416751   | 77.7   | 984672   | 5.7    | 432079   | 83.4   | 567921    | 26107    | 96532    |
| 9  | 417217   | 77.6   | 984637   | 5.7    | 432580   | 83.3   | 567420    | 26135    | 96524    |
| 10 | 417684   | 77.5   | 984603   | 5.7    | 433080   | 83.2   | 566920    | 26163    | 96517    |
| 11 | 9.418150 | 77.4   | 9.984569 | 5.7    | 9.433580 | 83.1   | 10.566420 | 26191    | 96509    |
| 12 | 418615   | 77.3   | 984535   | 5.7    | 434080   | 83.0   | 566420    | 26219    | 96502    |
| 13 | 419079   | 77.2   | 984500   | 5.7    | 434579   | 82.9   | 565920    | 26247    | 96494    |
| 14 | 419544   | 77.1   | 984466   | 5.7    | 435078   | 82.8   | 565421    | 26275    | 96486    |
| 15 | 420007   | 77.0   | 984432   | 5.8    | 435576   | 82.7   | 564922    | 26303    | 96479    |
| 16 | 420470   | 76.9   | 984397   | 5.8    | 436073   | 82.6   | 564424    | 26331    | 96471    |
| 17 | 420933   | 76.8   | 984363   | 5.8    | 436570   | 82.5   | 563927    | 26359    | 96463    |
| 18 | 421395   | 76.7   | 984328   | 5.8    | 437067   | 82.4   | 563430    | 26387    | 96456    |
| 19 | 421857   | 76.6   | 984294   | 5.8    | 437563   | 82.3   | 562933    | 26415    | 96448    |
| 20 | 422318   | 76.5   | 984259   | 5.8    | 438059   | 82.2   | 562437    | 26443    | 96440    |
| 21 | 9.422778 | 76.4   | 9.984224 | 5.8    | 9.438554 | 82.1   | 10.561446 | 26471    | 96433    |
| 22 | 423238   | 76.3   | 984190   | 5.8    | 439048   | 82.0   | 561941    | 26500    | 96425    |
| 23 | 423697   | 76.2   | 984155   | 5.8    | 439543   | 81.9   | 561446    | 26528    | 96417    |
| 24 | 424156   | 76.1   | 984120   | 5.8    | 440036   | 81.8   | 560952    | 26556    | 96410    |
| 25 | 424615   | 76.0   | 984085   | 5.8    | 440529   | 81.7   | 560457    | 26584    | 96402    |
| 26 | 425073   | 75.9   | 984050   | 5.8    | 441022   | 81.6   | 559964    | 26612    | 96394    |
| 27 | 425530   | 75.8   | 984015   | 5.8    | 441514   | 81.5   | 559471    | 26640    | 96386    |
| 28 | 425987   | 75.7   | 983981   | 5.8    | 442006   | 81.4   | 558978    | 26668    | 96379    |
| 29 | 426443   | 75.6   | 983946   | 5.8    | 442497   | 81.3   | 558486    | 26696    | 96371    |
| 30 | 426899   | 75.5   | 983911   | 5.8    | 442988   | 81.2   | 557994    | 26724    | 96363    |
| 31 | 9.427354 | 75.4   | 9.983875 | 5.8    | 9.443479 | 81.1   | 10.556521 | 26752    | 96355    |
| 32 | 427800   | 75.3   | 983840   | 5.8    | 443968   | 81.0   | 556032    | 26780    | 96347    |
| 33 | 428263   | 75.2   | 983805   | 5.9    | 444458   | 80.9   | 555542    | 26808    | 96340    |
| 34 | 428717   | 75.1   | 983770   | 5.9    | 444947   | 80.8   | 555053    | 26836    | 96332    |
| 35 | 429170   | 75.0   | 983735   | 5.9    | 445435   | 80.7   | 554565    | 26864    | 96324    |
| 36 | 429623   | 74.9   | 983700   | 5.9    | 445923   | 80.6   | 554077    | 26892    | 96316    |
| 37 | 430075   | 74.8   | 983664   | 5.9    | 446411   | 80.5   | 553589    | 26920    | 96308    |
| 38 | 430527   | 74.7   | 983629   | 5.9    | 446898   | 80.4   | 553102    | 26948    | 96301    |
| 39 | 430978   | 74.6   | 983594   | 5.9    | 447384   | 80.3   | 552616    | 26976    | 96293    |
| 40 | 431429   | 74.5   | 983558   | 5.9    | 447870   | 80.2   | 552130    | 27004    | 96285    |
| 41 | 9.431879 | 74.4   | 9.983523 | 5.9    | 9.448356 | 80.1   | 10.551644 | 27032    | 96277    |
| 42 | 432329   | 74.3   | 983487   | 5.9    | 448841   | 80.0   | 551159    | 27060    | 96269    |
| 43 | 432778   | 74.2   | 983452   | 5.9    | 449326   | 79.9   | 550674    | 27088    | 96261    |
| 44 | 433226   | 74.1   | 983416   | 5.9    | 449810   | 79.8   | 550190    | 27116    | 96253    |
| 45 | 433675   | 74.0   | 983381   | 5.9    | 450294   | 79.7   | 549706    | 27144    | 96246    |
| 46 | 434122   | 73.9   | 983345   | 5.9    | 450777   | 79.6   | 549223    | 27172    | 96238    |
| 47 | 434569   | 73.8   | 983309   | 5.9    | 451260   | 79.5   | 548740    | 27200    | 96230    |
| 48 | 435016   | 73.7   | 983273   | 6.0    | 451743   | 79.4   | 548257    | 27228    | 96222    |
| 49 | 435462   | 73.6   | 983238   | 6.0    | 452225   | 79.3   | 547775    | 27256    | 96214    |
| 50 | 435908   | 73.5   | 983202   | 6.0    | 452706   | 79.2   | 547294    | 27284    | 96206    |
| 51 | 9.436353 | 73.4   | 9.983166 | 6.0    | 9.453187 | 79.1   | 10.546813 | 27312    | 96198    |
| 52 | 436798   | 73.3   | 983130   | 6.0    | 453668   | 79.0   | 546832    | 27340    | 96190    |
| 53 | 437242   | 73.2   | 983094   | 6.0    | 454148   | 78.9   | 546352    | 27368    | 96182    |
| 54 | 437686   | 73.1   | 983058   | 6.0    | 454628   | 78.8   | 545872    | 27396    | 96174    |
| 55 | 438129   | 73.0   | 983022   | 6.0    | 455107   | 78.7   | 545393    | 27424    | 96166    |
| 56 | 438572   | 72.9   | 982986   | 6.0    | 455586   | 78.6   | 544914    | 27452    | 96158    |
| 57 | 439014   | 72.8   | 982950   | 6.0    | 456064   | 78.5   | 544435    | 27480    | 96150    |
| 58 | 439456   | 72.7   | 982914   | 6.0    | 456542   | 78.4   | 543956    | 27508    | 96142    |
| 59 | 439897   | 72.6   | 982878   | 6.0    | 457019   | 78.3   | 543477    | 27536    | 96134    |
| 60 | 440338   | 72.5   | 982842   | 6.0    | 457496   | 78.2   | 542998    | 27564    | 96126    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (16°) Natural Sines.

97

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.410338 |        | 9.982842 |        | 9.457496 |        | 10.542504 | 27564    | 96126    | 60 |
| 1  | 440778   | 73.4   | 982805   | 6.0    | 457973   | 79.4   | 542027    | 27592    | 96118    | 59 |
| 2  | 441218   | 73.3   | 982769   | 6.0    | 458449   | 79.3   | 541551    | 27620    | 96110    | 58 |
| 3  | 441658   | 73.2   | 982733   | 6.1    | 458925   | 79.3   | 541075    | 27648    | 96102    | 57 |
| 4  | 442096   | 73.1   | 982696   | 6.1    | 459400   | 79.2   | 540600    | 27676    | 96094    | 56 |
| 5  | 442535   | 73.1   | 982660   | 6.1    | 459875   | 79.1   | 540125    | 27704    | 96086    | 55 |
| 6  | 442973   | 72.9   | 982624   | 6.1    | 460349   | 79.0   | 539651    | 27731    | 96078    | 54 |
| 7  | 443410   | 72.8   | 982587   | 6.1    | 460823   | 78.9   | 539177    | 27759    | 96070    | 53 |
| 8  | 443847   | 72.7   | 982551   | 6.1    | 461297   | 78.8   | 538703    | 27787    | 96062    | 52 |
| 9  | 444284   | 72.7   | 982514   | 6.1    | 461770   | 78.8   | 538229    | 27815    | 96054    | 51 |
| 10 | 444720   | 72.7   | 982477   | 6.1    | 462242   | 78.9   | 537758    | 27843    | 96046    | 50 |
| 11 | 9.445155 | 72.6   | 9.982441 | 6.1    | 9.462714 | 78.7   | 10.537286 | 27871    | 96037    | 49 |
| 12 | 445590   | 72.5   | 982404   | 6.1    | 463186   | 78.6   | 536814    | 27899    | 96029    | 48 |
| 13 | 446025   | 72.3   | 982367   | 6.1    | 463658   | 78.5   | 536342    | 27927    | 96021    | 47 |
| 14 | 446459   | 72.3   | 982331   | 6.1    | 464129   | 78.5   | 535871    | 27955    | 96013    | 46 |
| 15 | 446893   | 72.2   | 982294   | 6.1    | 464599   | 78.4   | 535401    | 27983    | 96005    | 45 |
| 16 | 447326   | 72.2   | 982257   | 6.1    | 465069   | 78.3   | 534931    | 28011    | 95997    | 44 |
| 17 | 447759   | 72.1   | 982220   | 6.1    | 465539   | 78.3   | 534461    | 28039    | 95989    | 43 |
| 18 | 448191   | 72.0   | 982183   | 6.2    | 466008   | 78.2   | 533992    | 28067    | 95981    | 42 |
| 19 | 448623   | 71.9   | 982146   | 6.2    | 466476   | 78.1   | 533524    | 28095    | 95972    | 41 |
| 20 | 449054   | 71.9   | 982109   | 6.2    | 466945   | 78.0   | 533055    | 28123    | 95964    | 40 |
| 21 | 9.449485 | 71.8   | 9.982072 | 6.2    | 9.467413 | 78.0   | 10.532587 | 28150    | 95956    | 39 |
| 22 | 449915   | 71.7   | 982035   | 6.2    | 467880   | 77.9   | 532120    | 28178    | 95948    | 38 |
| 23 | 450345   | 71.6   | 981998   | 6.2    | 468347   | 77.8   | 531653    | 28206    | 95940    | 37 |
| 24 | 450775   | 71.6   | 981961   | 6.2    | 468814   | 77.8   | 531186    | 28234    | 95931    | 36 |
| 25 | 451204   | 71.5   | 981924   | 6.2    | 469280   | 77.7   | 530720    | 28262    | 95923    | 35 |
| 26 | 451632   | 71.4   | 981886   | 6.2    | 469746   | 77.6   | 530254    | 28290    | 95915    | 34 |
| 27 | 452060   | 71.3   | 981849   | 6.2    | 470211   | 77.5   | 529789    | 28318    | 95907    | 33 |
| 28 | 452488   | 71.2   | 981812   | 6.2    | 470676   | 77.5   | 529324    | 28346    | 95899    | 32 |
| 29 | 452915   | 71.1   | 981774   | 6.2    | 471141   | 77.4   | 528859    | 28374    | 95890    | 31 |
| 30 | 453342   | 71.1   | 981737   | 6.2    | 471605   | 77.3   | 528395    | 28402    | 95882    | 30 |
| 31 | 9.453768 | 71.0   | 9.981699 | 6.3    | 9.472068 | 77.2   | 10.527932 | 28429    | 95874    | 29 |
| 32 | 454194   | 70.9   | 981662   | 6.3    | 472532   | 77.1   | 527468    | 28457    | 95866    | 28 |
| 33 | 454619   | 70.8   | 981625   | 6.3    | 472995   | 77.1   | 527005    | 28485    | 95857    | 27 |
| 34 | 455044   | 70.7   | 981587   | 6.3    | 473457   | 77.0   | 526543    | 28513    | 95849    | 26 |
| 35 | 455469   | 70.7   | 981549   | 6.3    | 473919   | 76.9   | 526081    | 28541    | 95841    | 25 |
| 36 | 455893   | 70.7   | 981512   | 6.3    | 474381   | 76.9   | 525619    | 28569    | 95832    | 24 |
| 37 | 456316   | 70.6   | 981474   | 6.3    | 474842   | 76.9   | 525158    | 28597    | 95824    | 23 |
| 38 | 456739   | 70.5   | 981436   | 6.3    | 475303   | 76.8   | 524697    | 28625    | 95816    | 22 |
| 39 | 457162   | 70.4   | 981399   | 6.3    | 475763   | 76.7   | 524237    | 28652    | 95807    | 21 |
| 40 | 457584   | 70.3   | 981361   | 6.3    | 476223   | 76.7   | 523777    | 28680    | 95799    | 20 |
| 41 | 9.458006 | 70.2   | 9.981323 | 6.3    | 9.476683 | 76.6   | 10.523317 | 28708    | 95791    | 19 |
| 42 | 458427   | 70.1   | 981285   | 6.3    | 477142   | 76.5   | 522858    | 28736    | 95782    | 18 |
| 43 | 458848   | 70.1   | 981247   | 6.3    | 477601   | 76.5   | 522399    | 28764    | 95774    | 17 |
| 44 | 459268   | 70.0   | 981209   | 6.3    | 478059   | 76.4   | 521941    | 28792    | 95766    | 16 |
| 45 | 459688   | 69.9   | 981171   | 6.3    | 478517   | 76.3   | 521483    | 28820    | 95757    | 15 |
| 46 | 460108   | 69.8   | 981133   | 6.4    | 478975   | 76.2   | 521025    | 28847    | 95749    | 14 |
| 47 | 460527   | 69.8   | 981095   | 6.4    | 479432   | 76.1   | 520568    | 28875    | 95740    | 13 |
| 48 | 460946   | 69.7   | 981057   | 6.4    | 479889   | 76.1   | 520111    | 28903    | 95732    | 12 |
| 49 | 461364   | 69.6   | 981019   | 6.4    | 480345   | 76.0   | 519655    | 28931    | 95724    | 11 |
| 50 | 461782   | 69.5   | 980981   | 6.4    | 480801   | 75.9   | 519199    | 28959    | 95715    | 10 |
| 51 | 9.462199 | 69.5   | 9.980942 | 6.4    | 9.481257 | 75.9   | 10.518743 | 28987    | 95707    | 9  |
| 52 | 462616   | 69.4   | 980904   | 6.4    | 481712   | 75.8   | 518288    | 29015    | 95698    | 8  |
| 53 | 463032   | 69.3   | 980866   | 6.4    | 482167   | 75.7   | 517833    | 29042    | 95690    | 7  |
| 54 | 463448   | 69.3   | 980827   | 6.4    | 482621   | 75.7   | 517379    | 29070    | 95681    | 6  |
| 55 | 463864   | 69.2   | 980789   | 6.4    | 483075   | 75.6   | 516925    | 29098    | 95673    | 5  |
| 56 | 464279   | 69.1   | 980750   | 6.4    | 483529   | 75.5   | 516471    | 29126    | 95664    | 4  |
| 57 | 464694   | 69.0   | 980712   | 6.4    | 483982   | 75.4   | 516018    | 29154    | 95656    | 3  |
| 58 | 465108   | 69.0   | 980673   | 6.4    | 484435   | 75.4   | 515565    | 29182    | 95647    | 2  |
| 59 | 465522   | 68.9   | 980635   | 6.4    | 484887   | 75.3   | 515113    | 29209    | 95639    | 1  |
| 60 | 465935   |        | 980596   | 6.4    | 485339   |        | 514661    | 29247    | 95630    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

73 Degrees.



|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.465935 | 68.8   | 9.980596 | 6.4    | 9.485339 | 75.3   | 10.514661 | 29237    | 95630    | 60 |
| 1  | 466348   | 68.8   | 980558   | 6.4    | 485791   | 75.2   | 514209    | 29265    | 95622    | 59 |
| 2  | 466761   | 68.7   | 980519   | 6.5    | 486242   | 75.1   | 513758    | 29293    | 95613    | 58 |
| 3  | 467173   | 68.6   | 980480   | 6.5    | 486693   | 75.1   | 513307    | 29321    | 95605    | 57 |
| 4  | 467585   | 68.5   | 980442   | 6.5    | 487143   | 75.0   | 512857    | 29348    | 95596    | 56 |
| 5  | 467996   | 68.5   | 980403   | 6.5    | 487593   | 74.9   | 512407    | 29376    | 95588    | 55 |
| 6  | 468407   | 68.4   | 980364   | 6.5    | 488043   | 74.9   | 511957    | 29404    | 95579    | 54 |
| 7  | 468817   | 68.3   | 980325   | 6.5    | 488492   | 74.8   | 511508    | 29432    | 95571    | 53 |
| 8  | 469227   | 68.3   | 980286   | 6.5    | 488941   | 74.7   | 511059    | 29460    | 95562    | 52 |
| 9  | 469637   | 68.2   | 980247   | 6.5    | 489390   | 74.7   | 510610    | 29487    | 95554    | 51 |
| 10 | 470046   | 68.1   | 980208   | 6.5    | 489838   | 74.6   | 510162    | 29515    | 95545    | 50 |
| 11 | 9.470455 | 68.0   | 9.980169 | 6.5    | 9.490286 | 74.6   | 10.509714 | 29543    | 95536    | 49 |
| 12 | 470863   | 68.0   | 980130   | 6.5    | 490733   | 74.5   | 509267    | 29571    | 95528    | 48 |
| 13 | 471271   | 67.9   | 980091   | 6.5    | 491180   | 74.4   | 508820    | 29599    | 95519    | 47 |
| 14 | 471679   | 67.8   | 980052   | 6.5    | 491627   | 74.4   | 508373    | 29626    | 95511    | 46 |
| 15 | 472086   | 67.8   | 980012   | 6.5    | 492073   | 74.3   | 507927    | 29654    | 95502    | 45 |
| 16 | 472492   | 67.7   | 979973   | 6.5    | 492519   | 74.3   | 507481    | 29682    | 95493    | 44 |
| 17 | 472898   | 67.6   | 979934   | 6.5    | 492965   | 74.2   | 507035    | 29710    | 95485    | 43 |
| 18 | 473304   | 67.6   | 979895   | 6.6    | 493410   | 74.1   | 506590    | 29737    | 95476    | 42 |
| 19 | 473710   | 67.5   | 979855   | 6.6    | 493854   | 74.0   | 506146    | 29765    | 95467    | 41 |
| 20 | 474115   | 67.4   | 979816   | 6.6    | 494299   | 74.0   | 505701    | 29793    | 95459    | 40 |
| 21 | 9.474519 | 67.4   | 9.979776 | 6.6    | 9.494743 | 74.0   | 10.505257 | 29821    | 95450    | 39 |
| 22 | 474923   | 67.3   | 979737   | 6.6    | 495186   | 73.9   | 504814    | 29849    | 95441    | 38 |
| 23 | 475327   | 67.2   | 979697   | 6.6    | 495630   | 73.8   | 504370    | 29876    | 95433    | 37 |
| 24 | 475730   | 67.2   | 979658   | 6.6    | 496073   | 73.7   | 503927    | 29904    | 95424    | 36 |
| 25 | 476133   | 67.1   | 979618   | 6.6    | 496515   | 73.7   | 503485    | 29932    | 95415    | 35 |
| 26 | 476536   | 67.0   | 979579   | 6.6    | 496957   | 73.6   | 503043    | 29960    | 95407    | 34 |
| 27 | 476938   | 66.9   | 979539   | 6.6    | 497399   | 73.6   | 502601    | 29987    | 95398    | 33 |
| 28 | 477340   | 66.9   | 979499   | 6.6    | 497841   | 73.5   | 502159    | 30015    | 95389    | 32 |
| 29 | 477741   | 66.8   | 979459   | 6.6    | 498282   | 73.4   | 501718    | 30043    | 95380    | 31 |
| 30 | 478142   | 66.7   | 979420   | 6.6    | 498722   | 73.4   | 501278    | 30071    | 95372    | 30 |
| 31 | 9.478542 | 66.7   | 9.979380 | 6.6    | 9.499163 | 73.3   | 10.500837 | 30098    | 95363    | 29 |
| 32 | 478942   | 66.6   | 979340   | 6.6    | 499603   | 73.3   | 500397    | 30126    | 95354    | 28 |
| 33 | 479342   | 66.5   | 979300   | 6.7    | 500042   | 73.3   | 499958    | 30154    | 95345    | 27 |
| 34 | 479741   | 66.5   | 979260   | 6.7    | 500481   | 73.2   | 499519    | 30182    | 95337    | 26 |
| 35 | 480140   | 66.4   | 979220   | 6.7    | 500920   | 73.1   | 499080    | 30209    | 95328    | 25 |
| 36 | 480539   | 66.3   | 979180   | 6.7    | 501359   | 73.0   | 498641    | 30237    | 95319    | 24 |
| 37 | 480937   | 66.3   | 979140   | 6.7    | 501797   | 73.0   | 498203    | 30265    | 95310    | 23 |
| 38 | 481334   | 66.2   | 979100   | 6.7    | 502235   | 72.9   | 497765    | 30292    | 95301    | 22 |
| 39 | 481731   | 66.1   | 979059   | 6.7    | 502672   | 72.8   | 497328    | 30320    | 95293    | 21 |
| 40 | 482128   | 66.1   | 979019   | 6.7    | 503109   | 72.8   | 496891    | 30348    | 95284    | 20 |
| 41 | 9.482525 | 66.0   | 9.978979 | 6.7    | 9.503546 | 72.7   | 10.496454 | 30376    | 95275    | 19 |
| 42 | 482921   | 65.9   | 978939   | 6.7    | 503982   | 72.7   | 496018    | 30403    | 95266    | 18 |
| 43 | 483316   | 65.9   | 978898   | 6.7    | 504418   | 72.6   | 495582    | 30431    | 95257    | 17 |
| 44 | 483712   | 65.8   | 978858   | 6.7    | 504854   | 72.5   | 495146    | 30459    | 95248    | 16 |
| 45 | 484107   | 65.7   | 978817   | 6.7    | 505290   | 72.4   | 494711    | 30486    | 95240    | 15 |
| 46 | 484501   | 65.7   | 978777   | 6.7    | 505724   | 72.4   | 494276    | 30514    | 95231    | 14 |
| 47 | 484895   | 65.6   | 978736   | 6.7    | 506159   | 72.3   | 493841    | 30542    | 95222    | 13 |
| 48 | 485289   | 65.5   | 978696   | 6.8    | 506593   | 72.3   | 493407    | 30570    | 95213    | 12 |
| 49 | 485682   | 65.5   | 978655   | 6.8    | 507027   | 72.2   | 492973    | 30597    | 95204    | 11 |
| 50 | 486075   | 65.4   | 978615   | 6.8    | 507460   | 72.1   | 492540    | 30625    | 95195    | 10 |
| 51 | 9.486467 | 65.3   | 9.978574 | 6.8    | 9.507893 | 72.1   | 10.492107 | 30653    | 95186    | 9  |
| 52 | 486860   | 65.3   | 978533   | 6.8    | 508326   | 72.1   | 491674    | 30680    | 95177    | 8  |
| 53 | 487251   | 65.2   | 978493   | 6.8    | 508759   | 72.0   | 491241    | 30708    | 95168    | 7  |
| 54 | 487643   | 65.1   | 978452   | 6.8    | 509191   | 71.9   | 490809    | 30736    | 95159    | 6  |
| 55 | 488034   | 65.1   | 978411   | 6.8    | 509622   | 71.8   | 490378    | 30763    | 95150    | 5  |
| 56 | 488424   | 65.0   | 978370   | 6.8    | 510054   | 71.8   | 489946    | 30791    | 95142    | 4  |
| 57 | 488814   | 64.9   | 978329   | 6.8    | 510485   | 71.7   | 489515    | 30819    | 95133    | 3  |
| 58 | 489204   | 64.8   | 978288   | 6.8    | 510916   | 71.6   | 489084    | 30846    | 95124    | 2  |
| 59 | 489593   | 64.8   | 978247   | 6.8    | 511346   | 71.6   | 488654    | 30874    | 95115    | 1  |
| 60 | 489982   |        | 978206   |        | 511776   |        | 488224    | 30902    | 95106    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (18°) Natural Sines.

39

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.489982 |        | 9.978206 | 6.8    | 9.511776 | 71.6   | 10.488224 | 30902    | 95106    |
| 1  | 490371   | 64.8   | 978165   | 6.8    | 512206   | 71.6   | 487794    | 30929    | 95097    |
| 2  | 490759   | 64.8   | 978124   | 6.8    | 512635   | 71.5   | 487365    | 30957    | 95088    |
| 3  | 491147   | 64.6   | 978083   | 6.9    | 513064   | 71.4   | 486936    | 30985    | 95079    |
| 4  | 491535   | 64.6   | 978042   | 6.9    | 513493   | 71.4   | 486507    | 31012    | 95070    |
| 5  | 491922   | 64.6   | 978001   | 6.9    | 513921   | 71.4   | 486079    | 31040    | 95061    |
| 6  | 492308   | 64.5   | 977959   | 6.9    | 514349   | 71.3   | 485651    | 31068    | 95052    |
| 7  | 492695   | 64.4   | 977918   | 6.9    | 514777   | 71.3   | 485223    | 31095    | 95043    |
| 8  | 493081   | 64.3   | 977877   | 6.9    | 515204   | 71.2   | 484796    | 31123    | 95033    |
| 9  | 493466   | 64.2   | 977835   | 6.9    | 515631   | 71.1   | 484369    | 31151    | 95024    |
| 10 | 493851   | 64.2   | 977794   | 6.9    | 516057   | 71.1   | 483943    | 31178    | 95015    |
| 11 | 9.494236 | 64.1   | 9.977752 | 6.9    | 9.516484 | 71.0   | 10.483516 | 31206    | 95006    |
| 12 | 494621   | 64.1   | 977711   | 6.9    | 516910   | 71.0   | 483090    | 31233    | 94997    |
| 13 | 495005   | 64.0   | 977669   | 6.9    | 517335   | 70.9   | 482665    | 31261    | 94988    |
| 14 | 495388   | 63.9   | 977628   | 6.9    | 517761   | 70.8   | 482239    | 31289    | 94979    |
| 15 | 495772   | 63.9   | 977586   | 6.9    | 518185   | 70.8   | 481815    | 31316    | 94970    |
| 16 | 496154   | 63.8   | 977544   | 7.0    | 518610   | 70.7   | 481390    | 31344    | 94961    |
| 17 | 496537   | 63.7   | 977503   | 7.0    | 519034   | 70.6   | 480966    | 31372    | 94952    |
| 18 | 496919   | 63.7   | 977461   | 7.0    | 519458   | 70.6   | 480542    | 31399    | 94943    |
| 19 | 497301   | 63.6   | 977419   | 7.0    | 519882   | 70.5   | 480118    | 31427    | 94933    |
| 20 | 497682   | 63.6   | 977377   | 7.0    | 520305   | 70.5   | 479695    | 31454    | 94924    |
| 21 | 9.498064 | 63.5   | 9.977335 | 7.0    | 9.520728 | 70.4   | 10.479272 | 31482    | 94915    |
| 22 | 498444   | 63.4   | 977293   | 7.0    | 521151   | 70.3   | 478849    | 31510    | 94906    |
| 23 | 498825   | 63.4   | 977251   | 7.0    | 521573   | 70.3   | 478427    | 31537    | 94897    |
| 24 | 499204   | 63.3   | 977209   | 7.0    | 521995   | 70.3   | 478005    | 31565    | 94888    |
| 25 | 499584   | 63.2   | 977167   | 7.0    | 522417   | 70.2   | 477583    | 31593    | 94879    |
| 26 | 499963   | 63.2   | 977125   | 7.0    | 522838   | 70.2   | 477162    | 31620    | 94869    |
| 27 | 500342   | 63.1   | 977083   | 7.0    | 523259   | 70.1   | 476741    | 31648    | 94860    |
| 28 | 500721   | 63.1   | 977041   | 7.0    | 523680   | 70.1   | 476320    | 31675    | 94851    |
| 29 | 501099   | 63.0   | 976999   | 7.0    | 524100   | 70.0   | 475900    | 31703    | 94842    |
| 30 | 501476   | 62.9   | 976957   | 7.0    | 524520   | 69.9   | 475480    | 31730    | 94833    |
| 31 | 9.501854 | 62.9   | 9.976914 | 7.0    | 9.524939 | 69.9   | 10.475061 | 31758    | 94823    |
| 32 | 502231   | 62.8   | 976872   | 7.1    | 525359   | 69.8   | 474641    | 31786    | 94814    |
| 33 | 502607   | 62.8   | 976830   | 7.1    | 525778   | 69.8   | 474222    | 31813    | 94805    |
| 34 | 502984   | 62.7   | 976787   | 7.1    | 526197   | 69.7   | 473803    | 31841    | 94795    |
| 35 | 503360   | 62.6   | 976745   | 7.1    | 526615   | 69.7   | 473385    | 31868    | 94786    |
| 36 | 503735   | 62.6   | 976702   | 7.1    | 527033   | 69.6   | 472967    | 31896    | 94777    |
| 37 | 504110   | 62.5   | 976660   | 7.1    | 527451   | 69.6   | 472549    | 31923    | 94768    |
| 38 | 504485   | 62.5   | 976617   | 7.1    | 527868   | 69.5   | 472132    | 31951    | 94758    |
| 39 | 504860   | 62.4   | 976574   | 7.1    | 528285   | 69.5   | 471715    | 31979    | 94749    |
| 40 | 505234   | 62.3   | 976532   | 7.1    | 528702   | 69.4   | 471298    | 32006    | 94740    |
| 41 | 9.505608 | 62.3   | 9.976489 | 7.1    | 9.529119 | 69.3   | 10.470881 | 32034    | 94730    |
| 42 | 505981   | 62.2   | 976446   | 7.1    | 529535   | 69.3   | 470465    | 32061    | 94721    |
| 43 | 506354   | 62.2   | 976404   | 7.1    | 529950   | 69.3   | 470050    | 32089    | 94712    |
| 44 | 506727   | 62.1   | 976361   | 7.1    | 530366   | 69.2   | 469634    | 32116    | 94702    |
| 45 | 507099   | 62.1   | 976318   | 7.1    | 530781   | 69.1   | 469219    | 32144    | 94693    |
| 46 | 507471   | 62.0   | 976275   | 7.1    | 531196   | 69.1   | 468804    | 32171    | 94684    |
| 47 | 507843   | 61.9   | 976232   | 7.2    | 531611   | 69.0   | 468389    | 32199    | 94674    |
| 48 | 508214   | 61.9   | 976189   | 7.2    | 532025   | 69.0   | 467975    | 32227    | 94665    |
| 49 | 508585   | 61.8   | 976146   | 7.2    | 532439   | 68.9   | 467561    | 32255    | 94656    |
| 50 | 508956   | 61.8   | 976103   | 7.2    | 532853   | 68.9   | 467147    | 32283    | 94646    |
| 51 | 9.509326 | 61.7   | 9.976060 | 7.2    | 9.533266 | 68.8   | 10.466734 | 32309    | 94637    |
| 52 | 509696   | 61.6   | 976017   | 7.2    | 533679   | 68.8   | 466321    | 32337    | 94627    |
| 53 | 510065   | 61.6   | 975974   | 7.2    | 534092   | 68.7   | 465908    | 32364    | 94618    |
| 54 | 510434   | 61.5   | 975930   | 7.2    | 534504   | 68.7   | 465494    | 32392    | 94609    |
| 55 | 510803   | 61.5   | 975887   | 7.2    | 534916   | 68.6   | 465080    | 32419    | 94599    |
| 56 | 511172   | 61.4   | 975844   | 7.2    | 535328   | 68.6   | 464672    | 32447    | 94590    |
| 57 | 511540   | 61.3   | 975800   | 7.2    | 535739   | 68.5   | 464261    | 32474    | 94580    |
| 58 | 511907   | 61.3   | 975757   | 7.2    | 536150   | 68.5   | 463850    | 32502    | 94571    |
| 59 | 512275   | 61.2   | 975714   | 7.2    | 536561   | 68.4   | 463439    | 32529    | 94561    |
| 60 | 512642   |        | 975670   | 7.2    | 536972   |        | 463028    | 32557    | 94552    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

71 Degrees.

| r  | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  | r  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.512642 | 61.2   | 9.975670 | 7.3    | 9.536972 | 68.4   | 10.463028 | 32557    | 94552    | 60 |
| 1  | 513009   | 61.1   | 975627   | 7.3    | 537382   | 68.3   | 462618    | 32584    | 94542    | 59 |
| 2  | 513375   | 61.1   | 975583   | 7.3    | 537792   | 68.3   | 462208    | 32612    | 94533    | 58 |
| 3  | 513741   | 61.0   | 975539   | 7.3    | 538202   | 68.2   | 461798    | 32639    | 94523    | 57 |
| 4  | 514107   | 60.9   | 975496   | 7.3    | 538611   | 68.2   | 461389    | 32667    | 94514    | 56 |
| 5  | 514472   | 60.9   | 975452   | 7.3    | 539020   | 68.1   | 460980    | 32694    | 94504    | 55 |
| 6  | 514837   | 60.8   | 975408   | 7.3    | 539429   | 68.1   | 460571    | 32722    | 94495    | 54 |
| 7  | 515202   | 60.8   | 975365   | 7.3    | 539837   | 68.1   | 460163    | 32749    | 94485    | 53 |
| 8  | 515566   | 60.7   | 975321   | 7.3    | 540245   | 68.0   | 459755    | 32777    | 94476    | 52 |
| 9  | 515930   | 60.7   | 975277   | 7.3    | 540653   | 67.9   | 459347    | 32804    | 94466    | 51 |
| 10 | 516294   | 60.6   | 975233   | 7.3    | 541061   | 67.9   | 458939    | 32832    | 94457    | 50 |
| 11 | 9.516657 | 60.5   | 9.975189 | 7.3    | 9.541468 | 67.8   | 10.458532 | 32859    | 94447    | 49 |
| 12 | 517020   | 60.5   | 975145   | 7.3    | 541875   | 67.8   | 458125    | 32887    | 94438    | 48 |
| 13 | 517382   | 60.4   | 975101   | 7.3    | 542281   | 67.7   | 457719    | 32914    | 94428    | 47 |
| 14 | 517745   | 60.4   | 975057   | 7.3    | 542688   | 67.7   | 457312    | 32942    | 94418    | 46 |
| 15 | 518107   | 60.3   | 975013   | 7.3    | 543094   | 67.6   | 456906    | 32969    | 94409    | 45 |
| 16 | 518468   | 60.3   | 974969   | 7.4    | 543499   | 67.6   | 456501    | 32997    | 94399    | 44 |
| 17 | 518829   | 60.2   | 974925   | 7.4    | 543905   | 67.5   | 456095    | 33024    | 94390    | 43 |
| 18 | 519190   | 60.1   | 974880   | 7.4    | 544310   | 67.5   | 455690    | 33051    | 94380    | 42 |
| 19 | 519551   | 60.1   | 974836   | 7.4    | 544715   | 67.4   | 455285    | 33079    | 94370    | 41 |
| 20 | 519911   | 60.1   | 974792   | 7.4    | 545119   | 67.4   | 454881    | 33106    | 94361    | 40 |
| 21 | 9.520271 | 60.0   | 9.974748 | 7.4    | 9.545524 | 67.3   | 10.454476 | 33134    | 94351    | 39 |
| 22 | 520631   | 59.9   | 974703   | 7.4    | 545928   | 67.3   | 454072    | 33161    | 94342    | 38 |
| 23 | 520990   | 59.9   | 974659   | 7.4    | 546331   | 67.2   | 453669    | 33189    | 94332    | 37 |
| 24 | 521349   | 59.8   | 974614   | 7.4    | 546735   | 67.2   | 453265    | 33216    | 94322    | 36 |
| 25 | 521707   | 59.8   | 974570   | 7.4    | 547138   | 67.1   | 452862    | 33244    | 94313    | 35 |
| 26 | 522066   | 59.7   | 974525   | 7.4    | 547540   | 67.1   | 452460    | 33271    | 94303    | 34 |
| 27 | 522424   | 59.6   | 974481   | 7.4    | 547943   | 67.0   | 452057    | 33298    | 94293    | 33 |
| 28 | 522781   | 59.6   | 974436   | 7.4    | 548345   | 67.0   | 451655    | 33326    | 94284    | 32 |
| 29 | 523138   | 59.5   | 974391   | 7.4    | 548747   | 66.9   | 451253    | 33353    | 94274    | 31 |
| 30 | 523495   | 59.5   | 974347   | 7.5    | 549149   | 66.9   | 450851    | 33381    | 94264    | 30 |
| 31 | 9.523852 | 59.4   | 9.974302 | 7.5    | 9.549550 | 66.8   | 10.450450 | 33408    | 94254    | 29 |
| 32 | 524208   | 59.4   | 974257   | 7.5    | 549951   | 66.8   | 450049    | 33436    | 94245    | 28 |
| 33 | 524564   | 59.3   | 974212   | 7.5    | 550352   | 66.7   | 449648    | 33463    | 94235    | 27 |
| 34 | 524920   | 59.3   | 974167   | 7.5    | 550752   | 66.7   | 449248    | 33490    | 94225    | 26 |
| 35 | 525275   | 59.2   | 974122   | 7.5    | 551152   | 66.6   | 448848    | 33518    | 94215    | 25 |
| 36 | 525630   | 59.1   | 974077   | 7.5    | 551552   | 66.6   | 448448    | 33545    | 94206    | 24 |
| 37 | 525984   | 59.1   | 974032   | 7.5    | 551952   | 66.5   | 448048    | 33573    | 94196    | 23 |
| 38 | 526339   | 59.0   | 973987   | 7.5    | 552351   | 66.5   | 447649    | 33600    | 94186    | 22 |
| 39 | 526693   | 59.0   | 973942   | 7.5    | 552750   | 66.5   | 447250    | 33627    | 94176    | 21 |
| 40 | 527046   | 58.9   | 973897   | 7.5    | 553149   | 66.4   | 446851    | 33655    | 94167    | 20 |
| 41 | 9.527400 | 58.9   | 9.973852 | 7.5    | 9.553548 | 66.4   | 10.446452 | 33682    | 94157    | 19 |
| 42 | 527753   | 58.8   | 973807   | 7.5    | 553946   | 66.3   | 446054    | 33710    | 94147    | 18 |
| 43 | 528105   | 58.8   | 973761   | 7.5    | 554344   | 66.3   | 445656    | 33737    | 94137    | 17 |
| 44 | 528458   | 58.7   | 973716   | 7.6    | 554741   | 66.2   | 445259    | 33764    | 94127    | 16 |
| 45 | 528810   | 58.7   | 973671   | 7.6    | 555139   | 66.2   | 444861    | 33792    | 94118    | 15 |
| 46 | 529161   | 58.6   | 973625   | 7.6    | 555536   | 66.1   | 444464    | 33819    | 94108    | 14 |
| 47 | 529513   | 58.6   | 973580   | 7.6    | 555933   | 66.1   | 444067    | 33846    | 94098    | 13 |
| 48 | 529864   | 58.5   | 973535   | 7.6    | 556329   | 66.0   | 443671    | 33874    | 94088    | 12 |
| 49 | 530216   | 58.5   | 973489   | 7.6    | 556725   | 66.0   | 443275    | 33901    | 94078    | 11 |
| 50 | 530565   | 58.4   | 973444   | 7.6    | 557121   | 65.9   | 442879    | 33929    | 94068    | 10 |
| 51 | 9.530915 | 58.4   | 9.973398 | 7.6    | 9.557517 | 65.9   | 10.442483 | 33956    | 94058    | 9  |
| 52 | 531265   | 58.3   | 973352   | 7.6    | 557913   | 65.9   | 442087    | 33983    | 94049    | 8  |
| 53 | 531614   | 58.2   | 973307   | 7.6    | 558308   | 65.8   | 441692    | 34011    | 94039    | 7  |
| 54 | 531963   | 58.2   | 973261   | 7.6    | 558702   | 65.8   | 441298    | 34038    | 94029    | 6  |
| 55 | 532312   | 58.1   | 973215   | 7.6    | 559097   | 65.7   | 440903    | 34065    | 94019    | 5  |
| 56 | 532661   | 58.1   | 973169   | 7.6    | 559491   | 65.7   | 440509    | 34093    | 94009    | 4  |
| 57 | 533009   | 58.0   | 973124   | 7.6    | 559885   | 65.6   | 440115    | 34120    | 93999    | 3  |
| 58 | 533357   | 58.0   | 973078   | 7.6    | 560279   | 65.6   | 439721    | 34147    | 93989    | 2  |
| 59 | 533704   | 57.9   | 973032   | 7.7    | 560673   | 65.5   | 439327    | 34175    | 93979    | 1  |
| 60 | 534052   |        | 972986   |        | 561066   |        | 438934    | 34202    | 93969    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (20°) Natural Sines.

41

|    | Sine.   | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|---------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 534052  | 57.8   | 9.972986 | 7.7    | 5.561066 | 65.5   | 10.438934 | 34202    | 93969    |
| 1  | 534399  | 57.7   | 972940   | 7.7    | 561459   | 65.4   | 438541    | 34229    | 93959    |
| 2  | 534745  | 57.7   | 972894   | 7.7    | 561851   | 65.4   | 438149    | 34257    | 93949    |
| 3  | 535092  | 57.7   | 972848   | 7.7    | 562244   | 65.4   | 437756    | 34284    | 93939    |
| 4  | 535438  | 57.7   | 972802   | 7.7    | 562636   | 65.3   | 437364    | 34311    | 93929    |
| 5  | 535783  | 57.6   | 972755   | 7.7    | 563028   | 65.3   | 436972    | 34339    | 93919    |
| 6  | 536129  | 57.6   | 972709   | 7.7    | 563419   | 65.3   | 436581    | 34366    | 93909    |
| 7  | 536474  | 57.5   | 972663   | 7.7    | 563811   | 65.2   | 436189    | 34393    | 93899    |
| 8  | 536818  | 57.4   | 972617   | 7.7    | 564202   | 65.2   | 435798    | 34421    | 93889    |
| 9  | 537163  | 57.3   | 972570   | 7.7    | 564592   | 65.1   | 435408    | 34448    | 93879    |
| 10 | 537507  | 57.3   | 972524   | 7.7    | 564983   | 65.1   | 435017    | 34475    | 93869    |
| 11 | 537851  | 57.2   | 9.972478 | 7.7    | 9.565373 | 65.0   | 10.434627 | 34503    | 93859    |
| 12 | 538194  | 57.2   | 972431   | 7.7    | 565763   | 64.9   | 434237    | 34530    | 93849    |
| 13 | 538538  | 57.1   | 972385   | 7.8    | 566153   | 64.9   | 433847    | 34557    | 93839    |
| 14 | 538880  | 57.1   | 972338   | 7.8    | 566542   | 64.9   | 433458    | 34584    | 93829    |
| 15 | 539223  | 57.1   | 972291   | 7.8    | 566932   | 64.8   | 433068    | 34612    | 93819    |
| 16 | 539565  | 57.0   | 972245   | 7.8    | 567320   | 64.8   | 432680    | 34639    | 93809    |
| 17 | 539907  | 57.0   | 972198   | 7.8    | 567709   | 64.8   | 432291    | 34666    | 93799    |
| 18 | 540249  | 56.9   | 972151   | 7.8    | 568098   | 64.7   | 431902    | 34694    | 93789    |
| 19 | 540590  | 56.8   | 972105   | 7.8    | 568486   | 64.7   | 431514    | 34721    | 93779    |
| 20 | 540931  | 56.8   | 972058   | 7.8    | 568873   | 64.6   | 431127    | 34748    | 93769    |
| 21 | 541272  | 56.7   | 9.972011 | 7.8    | 9.569261 | 64.5   | 10.430739 | 34775    | 93759    |
| 22 | 541613  | 56.7   | 971964   | 7.8    | 569648   | 64.5   | 430352    | 34803    | 93748    |
| 23 | 541953  | 56.6   | 971917   | 7.8    | 570035   | 64.5   | 429965    | 34830    | 93738    |
| 24 | 542293  | 56.6   | 971870   | 7.8    | 570422   | 64.4   | 429578    | 34857    | 93728    |
| 25 | 542633  | 56.5   | 971823   | 7.8    | 570809   | 64.4   | 429191    | 34884    | 93718    |
| 26 | 542971  | 56.5   | 971776   | 7.8    | 571195   | 64.3   | 428805    | 34912    | 93708    |
| 27 | 543310  | 56.4   | 971729   | 7.9    | 571581   | 64.3   | 428419    | 34939    | 93698    |
| 28 | 543649  | 56.4   | 971682   | 7.9    | 571967   | 64.2   | 428033    | 34966    | 93688    |
| 29 | 543987  | 56.3   | 971635   | 7.9    | 572352   | 64.2   | 427648    | 34993    | 93677    |
| 30 | 544325  | 56.3   | 971588   | 7.9    | 572738   | 64.2   | 427262    | 35021    | 93667    |
| 31 | 544663  | 56.2   | 9.971540 | 7.9    | 9.573123 | 64.1   | 10.426877 | 35048    | 93657    |
| 32 | 545000  | 56.2   | 971493   | 7.9    | 573507   | 64.1   | 426493    | 35075    | 93647    |
| 33 | 545338  | 56.1   | 971446   | 7.9    | 573892   | 64.0   | 426108    | 35102    | 93637    |
| 34 | 545674  | 56.1   | 971398   | 7.9    | 574276   | 64.0   | 425724    | 35130    | 93626    |
| 35 | 546011  | 56.0   | 971351   | 7.9    | 574660   | 63.9   | 425340    | 35157    | 93616    |
| 36 | 546347  | 56.0   | 971303   | 7.9    | 575044   | 63.9   | 424956    | 35184    | 93606    |
| 37 | 546683  | 55.9   | 971256   | 7.9    | 575427   | 63.9   | 424573    | 35211    | 93596    |
| 38 | 547019  | 55.9   | 971208   | 7.9    | 575810   | 63.8   | 424190    | 35239    | 93585    |
| 39 | 547354  | 55.8   | 971161   | 7.9    | 576193   | 63.8   | 423807    | 35266    | 93575    |
| 40 | 547689  | 55.8   | 971113   | 7.9    | 576576   | 63.7   | 423424    | 35293    | 93565    |
| 41 | 548024  | 55.7   | 9.971066 | 8.0    | 9.576958 | 63.7   | 10.423041 | 35320    | 93555    |
| 42 | 548359  | 55.7   | 971018   | 8.0    | 577341   | 63.6   | 422659    | 35347    | 93544    |
| 43 | 548693  | 55.6   | 970970   | 8.0    | 577723   | 63.6   | 422277    | 35375    | 93534    |
| 44 | 549027  | 55.6   | 970922   | 8.0    | 578104   | 63.6   | 421896    | 35402    | 93524    |
| 45 | 549360  | 55.5   | 970874   | 8.0    | 578486   | 63.5   | 421514    | 35429    | 93514    |
| 46 | 549693  | 55.5   | 970827   | 8.0    | 578867   | 63.5   | 421133    | 35456    | 93503    |
| 47 | 550026  | 55.4   | 970779   | 8.0    | 579248   | 63.4   | 420752    | 35484    | 93493    |
| 48 | 550359  | 55.4   | 970731   | 8.0    | 579629   | 63.4   | 420371    | 35511    | 93483    |
| 49 | 550692  | 55.3   | 970683   | 8.0    | 580009   | 63.4   | 419991    | 35538    | 93472    |
| 50 | 551024  | 55.3   | 970635   | 8.0    | 580389   | 63.3   | 419611    | 35565    | 93462    |
| 51 | 551356  | 55.2   | 9.970588 | 8.0    | 9.580769 | 63.3   | 10.419231 | 35592    | 93452    |
| 52 | 551687  | 55.2   | 970538   | 8.0    | 581149   | 63.2   | 418851    | 35619    | 93441    |
| 53 | 552018  | 55.2   | 970490   | 8.0    | 581528   | 63.2   | 418472    | 35647    | 93431    |
| 54 | 552349  | 55.1   | 970442   | 8.0    | 581907   | 63.2   | 418093    | 35674    | 93420    |
| 55 | 552680  | 55.1   | 970394   | 8.0    | 582286   | 63.1   | 417714    | 35701    | 93410    |
| 56 | 553010  | 55.0   | 970345   | 8.1    | 582665   | 63.1   | 417335    | 35728    | 93400    |
| 57 | 553341  | 55.0   | 970297   | 8.1    | 583043   | 63.0   | 416957    | 35755    | 93389    |
| 58 | 553670  | 54.9   | 970249   | 8.1    | 583422   | 63.0   | 416578    | 35782    | 93379    |
| 59 | 554000  | 54.9   | 970200   | 8.1    | 583800   | 62.9   | 416200    | 35810    | 93368    |
| 60 | 554329  | 54.9   | 970152   | 8.1    | 584177   | 62.9   | 415823    | 35837    | 93358    |
|    | Cosine. |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

69 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.554329 | 54.8   | 9.970152 | 8.1    | 9.584177 | 62.9   | 10.415823 | 35837    | 93358    | 60 |
| 1  | 554658   | 54.8   | 970103   | 8.1    | 584555   | 62.9   | 415445    | 35864    | 93348    | 59 |
| 2  | 554987   | 54.7   | 970055   | 8.1    | 584932   | 62.8   | 415068    | 35891    | 93337    | 58 |
| 3  | 555315   | 54.7   | 970006   | 8.1    | 585309   | 62.8   | 414691    | 35918    | 93327    | 57 |
| 4  | 555643   | 54.6   | 969957   | 8.1    | 585686   | 62.7   | 414314    | 35945    | 93316    | 56 |
| 5  | 555971   | 54.6   | 969909   | 8.1    | 586062   | 62.7   | 413938    | 35973    | 93306    | 55 |
| 6  | 556299   | 54.5   | 969860   | 8.1    | 586439   | 62.7   | 413561    | 36000    | 93295    | 54 |
| 7  | 556626   | 54.5   | 969811   | 8.1    | 586815   | 62.6   | 413185    | 36027    | 93285    | 53 |
| 8  | 556953   | 54.4   | 969762   | 8.1    | 587190   | 62.6   | 412810    | 36054    | 93274    | 52 |
| 9  | 557280   | 54.4   | 969714   | 8.1    | 587566   | 62.6   | 412434    | 36081    | 93264    | 51 |
| 10 | 557606   | 54.3   | 969665   | 8.1    | 587941   | 62.5   | 412059    | 36108    | 93253    | 50 |
| 11 | 9.557932 | 54.3   | 9.969616 | 8.2    | 9.588316 | 62.5   | 10.411684 | 36135    | 93243    | 49 |
| 12 | 558258   | 54.3   | 969567   | 8.2    | 588691   | 62.4   | 411309    | 36162    | 93232    | 48 |
| 13 | 558583   | 54.2   | 969518   | 8.2    | 589066   | 62.4   | 410934    | 36190    | 93222    | 47 |
| 14 | 558909   | 54.2   | 969469   | 8.2    | 589440   | 62.3   | 410560    | 36217    | 93211    | 46 |
| 15 | 559234   | 54.1   | 969420   | 8.2    | 589814   | 62.3   | 410186    | 36244    | 93201    | 45 |
| 16 | 559558   | 54.1   | 969370   | 8.2    | 590188   | 62.3   | 409812    | 36271    | 93190    | 44 |
| 17 | 559883   | 54.0   | 969321   | 8.2    | 590562   | 62.2   | 409438    | 36298    | 93180    | 43 |
| 18 | 560207   | 54.0   | 969272   | 8.2    | 590935   | 62.2   | 409065    | 36325    | 93169    | 42 |
| 19 | 560531   | 53.9   | 969223   | 8.2    | 591308   | 62.2   | 408692    | 36352    | 93159    | 41 |
| 20 | 560855   | 53.9   | 969173   | 8.2    | 591681   | 62.1   | 408319    | 36379    | 93148    | 40 |
| 21 | 9.561178 | 53.8   | 9.969124 | 8.2    | 9.592054 | 62.1   | 10.407946 | 36406    | 93137    | 39 |
| 22 | 561501   | 53.8   | 969075   | 8.2    | 592428   | 62.0   | 407574    | 36434    | 93127    | 38 |
| 23 | 561824   | 53.7   | 969025   | 8.2    | 592798   | 62.0   | 407202    | 36461    | 93116    | 37 |
| 24 | 562146   | 53.7   | 968976   | 8.2    | 593170   | 61.9   | 406829    | 36488    | 93106    | 36 |
| 25 | 562468   | 53.6   | 968926   | 8.3    | 593542   | 61.9   | 406458    | 36515    | 93095    | 35 |
| 26 | 562790   | 53.6   | 968877   | 8.3    | 593914   | 61.8   | 406086    | 36542    | 93084    | 34 |
| 27 | 563112   | 53.6   | 968827   | 8.3    | 594285   | 61.8   | 405715    | 36569    | 93073    | 33 |
| 28 | 563433   | 53.5   | 968777   | 8.3    | 594656   | 61.8   | 405344    | 36596    | 93063    | 32 |
| 29 | 563755   | 53.5   | 968728   | 8.3    | 595027   | 61.7   | 404973    | 36623    | 93052    | 31 |
| 30 | 564075   | 53.4   | 968678   | 8.3    | 595398   | 61.7   | 404602    | 36650    | 93042    | 30 |
| 31 | 9.564396 | 53.4   | 9.968628 | 8.3    | 9.595768 | 61.7   | 10.404232 | 36677    | 93031    | 29 |
| 32 | 564716   | 53.3   | 968578   | 8.3    | 596138   | 61.6   | 403862    | 36704    | 93020    | 28 |
| 33 | 565036   | 53.3   | 968528   | 8.3    | 596508   | 61.6   | 403492    | 36731    | 93010    | 27 |
| 34 | 565356   | 53.2   | 968479   | 8.3    | 596878   | 61.6   | 403122    | 36758    | 92999    | 26 |
| 35 | 565676   | 53.2   | 968429   | 8.3    | 597247   | 61.5   | 402753    | 36785    | 92988    | 25 |
| 36 | 565995   | 53.1   | 968379   | 8.3    | 597616   | 61.5   | 402384    | 36812    | 92978    | 24 |
| 37 | 566314   | 53.1   | 968329   | 8.3    | 597985   | 61.5   | 402015    | 36839    | 92967    | 23 |
| 38 | 566632   | 53.1   | 968278   | 8.3    | 598354   | 61.4   | 401646    | 36867    | 92956    | 22 |
| 39 | 566951   | 53.0   | 968228   | 8.4    | 598723   | 61.4   | 401278    | 36894    | 92945    | 21 |
| 40 | 567269   | 53.0   | 968178   | 8.4    | 599091   | 61.3   | 400909    | 36921    | 92935    | 20 |
| 41 | 9.567587 | 52.9   | 9.968128 | 8.4    | 9.599459 | 61.3   | 10.400541 | 36948    | 92924    | 19 |
| 42 | 567904   | 52.9   | 968078   | 8.4    | 599827   | 61.3   | 400173    | 36975    | 92913    | 18 |
| 43 | 568222   | 52.8   | 968027   | 8.4    | 600194   | 61.2   | 399806    | 37002    | 92902    | 17 |
| 44 | 568539   | 52.8   | 967977   | 8.4    | 600562   | 61.2   | 399438    | 37029    | 92892    | 16 |
| 45 | 568856   | 52.8   | 967927   | 8.4    | 600929   | 61.1   | 399071    | 37056    | 92881    | 15 |
| 46 | 569172   | 52.7   | 967876   | 8.4    | 601296   | 61.1   | 398704    | 37083    | 92870    | 14 |
| 47 | 569488   | 52.7   | 967826   | 8.4    | 601662   | 61.1   | 398338    | 37110    | 92859    | 13 |
| 48 | 569804   | 52.6   | 967775   | 8.4    | 602029   | 61.0   | 397971    | 37137    | 92849    | 12 |
| 49 | 570120   | 52.6   | 967725   | 8.4    | 602395   | 61.0   | 397605    | 37164    | 92838    | 11 |
| 50 | 570435   | 52.5   | 967674   | 8.4    | 602761   | 61.0   | 397239    | 37191    | 92827    | 10 |
| 51 | 9.570751 | 52.5   | 9.967624 | 8.4    | 9.603127 | 60.9   | 10.396873 | 37218    | 92816    | 9  |
| 52 | 571066   | 52.4   | 967573   | 8.4    | 603493   | 60.9   | 396507    | 37245    | 92805    | 8  |
| 53 | 571380   | 52.4   | 967522   | 8.5    | 603858   | 60.9   | 396142    | 37272    | 92794    | 7  |
| 54 | 571695   | 52.3   | 967471   | 8.5    | 604223   | 60.8   | 395777    | 37299    | 92784    | 6  |
| 55 | 572009   | 52.3   | 967421   | 8.5    | 604588   | 60.8   | 395412    | 37326    | 92773    | 5  |
| 56 | 572323   | 52.3   | 967370   | 8.5    | 604953   | 60.7   | 395047    | 37353    | 92762    | 4  |
| 57 | 572636   | 52.2   | 967319   | 8.5    | 605317   | 60.7   | 394683    | 37380    | 92751    | 3  |
| 58 | 572950   | 52.2   | 967268   | 8.5    | 605682   | 60.7   | 394318    | 37407    | 92740    | 2  |
| 59 | 573263   | 52.1   | 967217   | 8.5    | 606046   | 60.6   | 393954    | 37434    | 92729    | 1  |
| 60 | 573575   |        | 967166   |        | 606410   |        | 393590    | 37461    | 92718    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. Sine. |    |

TABLE II. Log. Sines and Tangents. (22°) Natural Sines.

43

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.573575 | 52.1   | 9.967166 | 8.5    | 9.606410 | 60.6   | 10.393590 | 37461    | 92718    |
| 1  | 573888   | 52.0   | 967115   | 8.5    | 606773   | 60.6   | 393227    | 37488    | 92707    |
| 2  | 574200   | 52.0   | 967064   | 8.5    | 607137   | 60.6   | 392863    | 37515    | 92697    |
| 3  | 574512   | 51.9   | 967013   | 8.5    | 607500   | 60.5   | 392500    | 37542    | 92686    |
| 4  | 574824   | 51.9   | 966961   | 8.5    | 607863   | 60.5   | 392137    | 37569    | 92675    |
| 5  | 575136   | 51.9   | 966910   | 8.5    | 608225   | 60.4   | 391775    | 37595    | 92664    |
| 6  | 575447   | 51.8   | 966859   | 8.5    | 608588   | 60.4   | 391412    | 37622    | 92653    |
| 7  | 575758   | 51.8   | 966808   | 8.5    | 608950   | 60.3   | 391050    | 37649    | 92642    |
| 8  | 576069   | 51.7   | 966756   | 8.6    | 609312   | 60.3   | 390688    | 37676    | 92631    |
| 9  | 576379   | 51.7   | 966705   | 8.6    | 609674   | 60.3   | 390326    | 37703    | 92620    |
| 10 | 576689   | 51.6   | 966653   | 8.6    | 610036   | 60.2   | 389964    | 37730    | 92609    |
| 11 | 9.576999 | 51.6   | 9.966602 | 8.6    | 9.610397 | 60.2   | 10.389603 | 37757    | 92598    |
| 12 | 577309   | 51.6   | 966550   | 8.6    | 610759   | 60.2   | 389241    | 37784    | 92587    |
| 13 | 577618   | 51.5   | 966499   | 8.6    | 611120   | 60.2   | 388880    | 37811    | 92576    |
| 14 | 577927   | 51.5   | 966447   | 8.6    | 611480   | 60.1   | 388520    | 37838    | 92565    |
| 15 | 578236   | 51.4   | 966395   | 8.6    | 611841   | 60.1   | 388159    | 37865    | 92554    |
| 16 | 578545   | 51.4   | 966344   | 8.6    | 612201   | 60.1   | 387799    | 37892    | 92543    |
| 17 | 578853   | 51.3   | 966292   | 8.6    | 612561   | 60.0   | 387439    | 37919    | 92532    |
| 18 | 579162   | 51.3   | 966240   | 8.6    | 612921   | 60.0   | 387079    | 37946    | 92521    |
| 19 | 579470   | 51.3   | 966188   | 8.6    | 613281   | 60.0   | 386719    | 37973    | 92510    |
| 20 | 579777   | 51.2   | 966136   | 8.6    | 613641   | 59.9   | 386359    | 37999    | 92499    |
| 21 | 9.580085 | 51.2   | 9.966085 | 8.7    | 9.614000 | 59.9   | 10.386000 | 38026    | 92488    |
| 22 | 580392   | 51.1   | 966033   | 8.7    | 614359   | 59.8   | 385641    | 38053    | 92477    |
| 23 | 580699   | 51.1   | 965981   | 8.7    | 614718   | 59.8   | 385282    | 38080    | 92466    |
| 24 | 581005   | 51.1   | 965928   | 8.7    | 615077   | 59.7   | 384923    | 38107    | 92455    |
| 25 | 581312   | 51.0   | 965876   | 8.7    | 615435   | 59.7   | 384565    | 38134    | 92444    |
| 26 | 581618   | 51.0   | 965824   | 8.7    | 615793   | 59.7   | 384207    | 38161    | 92433    |
| 27 | 581924   | 50.9   | 965772   | 8.7    | 616151   | 59.6   | 383849    | 38188    | 92421    |
| 28 | 582229   | 50.9   | 965720   | 8.7    | 616509   | 59.6   | 383491    | 38215    | 92410    |
| 29 | 582535   | 50.9   | 965668   | 8.7    | 616867   | 59.6   | 383133    | 38241    | 92399    |
| 30 | 582840   | 50.8   | 965615   | 8.7    | 617224   | 59.5   | 382776    | 38268    | 92388    |
| 31 | 9.583145 | 50.8   | 9.965563 | 8.7    | 9.617582 | 59.5   | 10.382418 | 38295    | 92377    |
| 32 | 583449   | 50.8   | 965511   | 8.7    | 617939   | 59.5   | 382461    | 38322    | 92366    |
| 33 | 583754   | 50.7   | 965458   | 8.7    | 618295   | 59.5   | 382105    | 38349    | 92355    |
| 34 | 584058   | 50.6   | 965406   | 8.7    | 618652   | 59.4   | 381748    | 38376    | 92344    |
| 35 | 584361   | 50.6   | 965353   | 8.8    | 619008   | 59.4   | 381392    | 38403    | 92332    |
| 36 | 584665   | 50.6   | 965301   | 8.8    | 619364   | 59.3   | 381036    | 38430    | 92321    |
| 37 | 584968   | 50.5   | 965248   | 8.8    | 619721   | 59.3   | 380679    | 38456    | 92310    |
| 38 | 585272   | 50.5   | 965195   | 8.8    | 620076   | 59.3   | 379924    | 38483    | 92299    |
| 39 | 585574   | 50.5   | 965143   | 8.8    | 620432   | 59.3   | 379568    | 38510    | 92287    |
| 40 | 585877   | 50.4   | 965090   | 8.8    | 620787   | 59.2   | 379213    | 38537    | 92276    |
| 41 | 9.586179 | 50.3   | 9.965037 | 8.8    | 9.621142 | 59.2   | 10.378858 | 38564    | 92265    |
| 42 | 586482   | 50.3   | 964984   | 8.8    | 621497   | 59.2   | 378503    | 38591    | 92254    |
| 43 | 586783   | 50.3   | 964931   | 8.8    | 621852   | 59.1   | 378148    | 38617    | 92243    |
| 44 | 587085   | 50.2   | 964879   | 8.8    | 622207   | 59.1   | 377793    | 38644    | 92231    |
| 45 | 587386   | 50.2   | 964826   | 8.8    | 622561   | 59.0   | 377439    | 38671    | 92220    |
| 46 | 587688   | 50.2   | 964773   | 8.8    | 622915   | 59.0   | 377085    | 38698    | 92209    |
| 47 | 587989   | 50.1   | 964719   | 8.8    | 623269   | 58.9   | 376731    | 38725    | 92198    |
| 48 | 588289   | 50.1   | 964666   | 8.9    | 623623   | 58.9   | 376377    | 38752    | 92186    |
| 49 | 588590   | 50.0   | 964613   | 8.9    | 623976   | 58.9   | 376024    | 38778    | 92175    |
| 50 | 588890   | 50.0   | 964560   | 8.9    | 624330   | 58.8   | 375670    | 38805    | 92164    |
| 51 | 9.589190 | 50.0   | 9.964507 | 8.9    | 9.624683 | 58.8   | 10.375817 | 38832    | 92152    |
| 52 | 589489   | 49.9   | 964454   | 8.9    | 625036   | 58.8   | 374964    | 38859    | 92141    |
| 53 | 589789   | 49.9   | 964400   | 8.9    | 625388   | 58.8   | 374612    | 38886    | 92130    |
| 54 | 590088   | 49.8   | 964347   | 8.9    | 625741   | 58.7   | 374259    | 38912    | 92119    |
| 55 | 590387   | 49.8   | 964294   | 8.9    | 626093   | 58.7   | 373907    | 38939    | 92107    |
| 56 | 590686   | 49.7   | 964240   | 8.9    | 626445   | 58.6   | 373555    | 38966    | 92096    |
| 57 | 590984   | 49.7   | 964187   | 8.9    | 626797   | 58.6   | 373203    | 38993    | 92085    |
| 58 | 591282   | 49.7   | 964133   | 8.9    | 627149   | 58.6   | 372851    | 39020    | 92073    |
| 59 | 591580   | 49.6   | 964080   | 8.9    | 627501   | 58.5   | 372499    | 39046    | 92062    |
| 60 | 591878   | 49.6   | 964026   | 8.9    | 627852   | 58.5   | 372148    | 39073    | 92050    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

67 Degrees.

|    | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.    | D. 10' | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.591878 | 49.6   | 9.964026 | 8.9    | 9.627852 | 58.5   | 10.372148 | 39078    | 92050    |
| 1  | 592176   | 49.5   | 963972   | 8.9    | 628203   | 58.5   | 371797    | 39100    | 92039    |
| 2  | 592473   | 49.5   | 963919   | 8.9    | 628554   | 58.5   | 371446    | 39127    | 92028    |
| 3  | 592770   | 49.5   | 963865   | 9.0    | 628905   | 58.4   | 371095    | 39153    | 92016    |
| 4  | 593067   | 49.4   | 963811   | 9.0    | 629255   | 58.4   | 370745    | 39180    | 92005    |
| 5  | 593363   | 49.4   | 963757   | 9.0    | 629606   | 58.3   | 370394    | 39207    | 91994    |
| 6  | 593659   | 49.3   | 963704   | 9.0    | 629956   | 58.3   | 370044    | 39234    | 91982    |
| 7  | 593955   | 49.3   | 963650   | 9.0    | 630306   | 58.3   | 369694    | 39260    | 91971    |
| 8  | 594251   | 49.3   | 963596   | 9.0    | 630656   | 58.3   | 369344    | 39287    | 91959    |
| 9  | 594547   | 49.2   | 963542   | 9.0    | 631005   | 58.2   | 368995    | 39314    | 91948    |
| 10 | 594842   | 49.2   | 963488   | 9.0    | 631355   | 58.2   | 368645    | 39341    | 91936    |
| 11 | 9.595137 | 49.1   | 9.963434 | 9.0    | 9.631704 | 58.2   | 10.368296 | 39367    | 91925    |
| 12 | 595432   | 49.1   | 963379   | 9.0    | 632053   | 58.1   | 367947    | 39394    | 91914    |
| 13 | 595727   | 49.1   | 963325   | 9.0    | 632401   | 58.1   | 367599    | 39421    | 91902    |
| 14 | 596021   | 49.0   | 963271   | 9.0    | 632750   | 58.1   | 367250    | 39448    | 91891    |
| 15 | 596315   | 49.0   | 963217   | 9.0    | 633098   | 58.0   | 366902    | 39474    | 91879    |
| 16 | 596609   | 48.9   | 963163   | 9.0    | 633447   | 58.0   | 366553    | 39501    | 91868    |
| 17 | 596903   | 48.9   | 963108   | 9.1    | 633795   | 58.0   | 366205    | 39528    | 91856    |
| 18 | 597196   | 48.9   | 963054   | 9.1    | 634143   | 57.9   | 365857    | 39555    | 91845    |
| 19 | 597490   | 48.8   | 962999   | 9.1    | 634490   | 57.9   | 365510    | 39581    | 91833    |
| 20 | 597783   | 48.8   | 962945   | 9.1    | 634838   | 57.9   | 365162    | 39608    | 91822    |
| 21 | 9.598075 | 48.7   | 9.962890 | 9.1    | 9.635185 | 57.8   | 10.364815 | 39635    | 91810    |
| 22 | 598368   | 48.7   | 962836   | 9.1    | 635532   | 57.8   | 364468    | 39661    | 91799    |
| 23 | 598660   | 48.7   | 962781   | 9.1    | 635879   | 57.8   | 364121    | 39688    | 91787    |
| 24 | 598952   | 48.6   | 962727   | 9.1    | 636226   | 57.7   | 363774    | 39715    | 91775    |
| 25 | 599244   | 48.6   | 962672   | 9.1    | 636572   | 57.7   | 363428    | 39741    | 91764    |
| 26 | 599536   | 48.6   | 962617   | 9.1    | 636919   | 57.7   | 363081    | 39768    | 91752    |
| 27 | 599827   | 48.5   | 962562   | 9.1    | 637265   | 57.7   | 362735    | 39795    | 91741    |
| 28 | 600118   | 48.5   | 962508   | 9.1    | 637611   | 57.6   | 362389    | 39822    | 91729    |
| 29 | 600409   | 48.4   | 962453   | 9.1    | 637956   | 57.6   | 362044    | 39848    | 91718    |
| 30 | 600700   | 48.4   | 962398   | 9.2    | 638302   | 57.6   | 361698    | 39875    | 91706    |
| 31 | 9.600990 | 48.4   | 9.962343 | 9.2    | 9.638647 | 57.6   | 10.361353 | 39902    | 91694    |
| 32 | 601280   | 48.3   | 962288   | 9.2    | 638992   | 57.5   | 361008    | 39928    | 91683    |
| 33 | 601570   | 48.3   | 962233   | 9.2    | 639337   | 57.5   | 360662    | 39955    | 91671    |
| 34 | 601860   | 48.2   | 962178   | 9.2    | 639682   | 57.4   | 360315    | 39982    | 91660    |
| 35 | 602150   | 48.2   | 962123   | 9.2    | 640027   | 57.4   | 359973    | 40008    | 91648    |
| 36 | 602439   | 48.2   | 962067   | 9.2    | 640371   | 57.4   | 359629    | 40035    | 91636    |
| 37 | 602728   | 48.1   | 962012   | 9.2    | 640716   | 57.3   | 359284    | 40062    | 91625    |
| 38 | 603017   | 48.1   | 961957   | 9.2    | 641060   | 57.3   | 358940    | 40088    | 91613    |
| 39 | 603305   | 48.1   | 961902   | 9.2    | 641404   | 57.3   | 358596    | 40115    | 91601    |
| 40 | 603594   | 48.0   | 961846   | 9.2    | 641747   | 57.2   | 358253    | 40141    | 91590    |
| 41 | 9.603882 | 48.0   | 9.961791 | 9.2    | 9.642091 | 57.2   | 10.357909 | 40168    | 91578    |
| 42 | 604170   | 47.9   | 961735   | 9.2    | 642434   | 57.2   | 357566    | 40195    | 91566    |
| 43 | 604457   | 47.9   | 961680   | 9.2    | 642777   | 57.2   | 357223    | 40221    | 91555    |
| 44 | 604745   | 47.9   | 961624   | 9.3    | 643120   | 57.1   | 356880    | 40248    | 91543    |
| 45 | 605032   | 47.8   | 961569   | 9.3    | 643463   | 57.1   | 356537    | 40275    | 91531    |
| 46 | 605319   | 47.8   | 961513   | 9.3    | 643806   | 57.1   | 356194    | 40301    | 91519    |
| 47 | 605606   | 47.8   | 961458   | 9.3    | 644148   | 57.0   | 355852    | 40328    | 91508    |
| 48 | 605892   | 47.7   | 961402   | 9.3    | 644490   | 57.0   | 355510    | 40355    | 91496    |
| 49 | 606179   | 47.7   | 961346   | 9.3    | 644832   | 57.0   | 355168    | 40381    | 91484    |
| 50 | 606465   | 47.6   | 961290   | 9.3    | 645174   | 56.9   | 354826    | 40408    | 91472    |
| 51 | 9.606751 | 47.6   | 9.961235 | 9.3    | 9.645516 | 56.9   | 10.354484 | 40434    | 91461    |
| 52 | 607036   | 47.6   | 961179   | 9.3    | 645857   | 56.9   | 354443    | 40461    | 91449    |
| 53 | 607322   | 47.5   | 961123   | 9.3    | 646199   | 56.9   | 353801    | 40488    | 91437    |
| 54 | 607607   | 47.5   | 961067   | 9.3    | 646540   | 56.8   | 353460    | 40514    | 91425    |
| 55 | 607892   | 47.4   | 961011   | 9.3    | 646881   | 56.8   | 353119    | 40541    | 91414    |
| 56 | 608177   | 47.4   | 960955   | 9.3    | 647222   | 56.8   | 352778    | 40567    | 91402    |
| 57 | 608461   | 47.4   | 960899   | 9.3    | 647562   | 56.7   | 352438    | 40594    | 91390    |
| 58 | 608745   | 47.3   | 960843   | 9.3    | 647903   | 56.7   | 352097    | 40621    | 91378    |
| 59 | 609029   | 47.3   | 960786   | 9.4    | 648243   | 56.7   | 351757    | 40647    | 91366    |
| 60 | 609313   | 47.3   | 960730   | 9.4    | 648583   | 56.7   | 351417    | 40674    | 91355    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (24°) Natural Sines.

45

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.609313 | 47.3   | 9.960730 | 9.4    | 9.648583 | 56.6   | 10.351417 | 40674    | 91355    |
| 1  | 609597   | 47.2   | 960674   | 9.4    | 648923   | 56.6   | 351077    | 40700    | 91343    |
| 2  | 609880   | 47.2   | 960618   | 9.4    | 649263   | 56.6   | 350737    | 40727    | 91331    |
| 3  | 610164   | 47.2   | 960561   | 9.4    | 649602   | 56.6   | 350398    | 40753    | 91319    |
| 4  | 610447   | 47.2   | 960505   | 9.4    | 649942   | 56.6   | 350058    | 40780    | 91307    |
| 5  | 610729   | 47.1   | 960448   | 9.4    | 650281   | 56.5   | 349719    | 40806    | 91295    |
| 6  | 611012   | 47.1   | 960392   | 9.4    | 650620   | 56.5   | 349380    | 40833    | 91283    |
| 7  | 611294   | 47.0   | 960335   | 9.4    | 650959   | 56.5   | 349041    | 40860    | 91272    |
| 8  | 611576   | 47.0   | 960279   | 9.4    | 651297   | 56.4   | 348703    | 40886    | 91260    |
| 9  | 611858   | 46.9   | 960222   | 9.4    | 651636   | 56.4   | 348364    | 40913    | 91248    |
| 10 | 612140   | 46.9   | 960165   | 9.4    | 651974   | 56.4   | 348026    | 40939    | 91236    |
| 11 | 9.612421 | 46.9   | 9.960109 | 9.5    | 9.652312 | 56.3   | 10.347688 | 40966    | 91224    |
| 12 | 612702   | 46.8   | 960052   | 9.5    | 652650   | 56.3   | 347350    | 40992    | 91212    |
| 13 | 612983   | 46.8   | 959995   | 9.5    | 652988   | 56.3   | 347012    | 41019    | 91200    |
| 14 | 613264   | 46.7   | 959938   | 9.5    | 653326   | 56.2   | 346674    | 41045    | 91188    |
| 15 | 613545   | 46.7   | 959882   | 9.5    | 653663   | 56.2   | 346337    | 41072    | 91176    |
| 16 | 613825   | 46.7   | 959825   | 9.5    | 654000   | 56.2   | 346000    | 41098    | 91164    |
| 17 | 614105   | 46.7   | 959768   | 9.5    | 654337   | 56.1   | 345663    | 41125    | 91152    |
| 18 | 614385   | 46.6   | 959711   | 9.5    | 654674   | 56.1   | 345326    | 41151    | 91140    |
| 19 | 614665   | 46.6   | 959654   | 9.5    | 655011   | 56.1   | 344989    | 41178    | 91128    |
| 20 | 614944   | 46.5   | 959596   | 9.5    | 655348   | 56.1   | 344652    | 41204    | 91116    |
| 21 | 9.615223 | 46.5   | 9.959539 | 9.5    | 9.655684 | 56.0   | 10.344316 | 41231    | 91104    |
| 22 | 615502   | 46.5   | 959482   | 9.5    | 656020   | 56.0   | 343980    | 41257    | 91092    |
| 23 | 615781   | 46.4   | 959425   | 9.5    | 656356   | 56.0   | 343644    | 41284    | 91080    |
| 24 | 616060   | 46.4   | 959368   | 9.5    | 656692   | 56.0   | 343308    | 41310    | 91068    |
| 25 | 616338   | 46.4   | 959310   | 9.5    | 657028   | 55.9   | 342972    | 41337    | 91056    |
| 26 | 616616   | 46.3   | 959253   | 9.6    | 657364   | 55.9   | 342636    | 41363    | 91044    |
| 27 | 616894   | 46.3   | 959195   | 9.6    | 657699   | 55.9   | 342301    | 41390    | 91032    |
| 28 | 617172   | 46.2   | 959138   | 9.6    | 658034   | 55.8   | 341966    | 41416    | 91020    |
| 29 | 617450   | 46.2   | 959081   | 9.6    | 658369   | 55.8   | 341631    | 41443    | 91008    |
| 30 | 617727   | 46.2   | 959023   | 9.6    | 658704   | 55.8   | 341296    | 41469    | 90996    |
| 31 | 9.618004 | 46.1   | 9.958965 | 9.6    | 9.659039 | 55.8   | 10.340961 | 41496    | 90984    |
| 32 | 618281   | 46.1   | 958908   | 9.6    | 659373   | 55.8   | 340627    | 41522    | 90972    |
| 33 | 618558   | 46.1   | 958850   | 9.6    | 659708   | 55.7   | 340292    | 41549    | 90960    |
| 34 | 618834   | 46.0   | 958792   | 9.6    | 660042   | 55.7   | 339958    | 41575    | 90948    |
| 35 | 619110   | 46.0   | 958734   | 9.6    | 660376   | 55.7   | 339624    | 41602    | 90936    |
| 36 | 619386   | 46.0   | 958677   | 9.6    | 660710   | 55.6   | 339290    | 41628    | 90924    |
| 37 | 619662   | 45.9   | 958619   | 9.6    | 661043   | 55.6   | 338957    | 41655    | 90911    |
| 38 | 619938   | 45.9   | 958561   | 9.6    | 661377   | 55.6   | 338623    | 41681    | 90899    |
| 39 | 620213   | 45.9   | 958503   | 9.6    | 661710   | 55.6   | 338290    | 41707    | 90887    |
| 40 | 620488   | 45.8   | 958445   | 9.7    | 662043   | 55.5   | 337957    | 41734    | 90875    |
| 41 | 9.620763 | 45.8   | 9.958387 | 9.7    | 9.662376 | 55.5   | 10.337624 | 41760    | 90863    |
| 42 | 621038   | 45.8   | 958329   | 9.7    | 662709   | 55.4   | 337291    | 41787    | 90851    |
| 43 | 621313   | 45.7   | 958271   | 9.7    | 663042   | 55.4   | 336958    | 41813    | 90839    |
| 44 | 621587   | 45.7   | 958213   | 9.7    | 663375   | 55.4   | 336625    | 41840    | 90826    |
| 45 | 621861   | 45.6   | 958154   | 9.7    | 663707   | 55.4   | 336293    | 41866    | 90814    |
| 46 | 622135   | 45.6   | 958096   | 9.7    | 664039   | 55.3   | 335961    | 41892    | 90802    |
| 47 | 622409   | 45.6   | 958038   | 9.7    | 664371   | 55.3   | 335629    | 41919    | 90790    |
| 48 | 622682   | 45.5   | 957979   | 9.7    | 664703   | 55.3   | 335297    | 41945    | 90778    |
| 49 | 622956   | 45.5   | 957921   | 9.7    | 665035   | 55.3   | 334965    | 41972    | 90766    |
| 50 | 623229   | 45.5   | 957863   | 9.7    | 665366   | 55.2   | 334634    | 41998    | 90753    |
| 51 | 9.623512 | 45.4   | 9.957804 | 9.7    | 9.665697 | 55.2   | 10.334303 | 42024    | 90741    |
| 52 | 623774   | 45.4   | 957746   | 9.7    | 666029   | 55.2   | 333971    | 42051    | 90729    |
| 53 | 624047   | 45.4   | 957687   | 9.8    | 666360   | 55.1   | 333620    | 42077    | 90717    |
| 54 | 624319   | 45.3   | 957628   | 9.8    | 666691   | 55.1   | 333309    | 42104    | 90704    |
| 55 | 624591   | 45.3   | 957570   | 9.8    | 667021   | 55.1   | 332979    | 42130    | 90692    |
| 56 | 624863   | 45.3   | 957511   | 9.8    | 667352   | 55.1   | 332648    | 42156    | 90680    |
| 57 | 625135   | 45.2   | 957452   | 9.8    | 667682   | 55.0   | 332318    | 42183    | 90668    |
| 58 | 625406   | 45.2   | 957393   | 9.8    | 668013   | 55.0   | 331987    | 42209    | 90655    |
| 59 | 625677   | 45.2   | 957335   | 9.8    | 668343   | 55.0   | 331657    | 42235    | 90643    |
| 60 | 625948   |        | 957276   | 9.8    | 668672   |        | 331328    | 42262    | 90631    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

65 Degrees.



|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.625948 | 45.1   | 9.957276 | 9.8    | 9.668673 | 55.0   | 10.331327 | 42262    | 90631    |
| 1  | 626219   | 45.1   | 957217   | 9.8    | 669002   | 54.9   | 330998    | 42288    | 90613    |
| 2  | 626490   | 45.1   | 957158   | 9.8    | 669332   | 54.9   | 330668    | 42315    | 90606    |
| 3  | 626760   | 45.0   | 957099   | 9.8    | 669661   | 54.9   | 330339    | 42341    | 90594    |
| 4  | 627030   | 45.0   | 957040   | 9.8    | 669991   | 54.8   | 330009    | 42367    | 90582    |
| 5  | 627300   | 45.0   | 956981   | 9.8    | 670320   | 54.8   | 329680    | 42394    | 90569    |
| 6  | 627570   | 44.9   | 956921   | 9.9    | 670649   | 54.8   | 329351    | 42420    | 90557    |
| 7  | 627840   | 44.9   | 956862   | 9.9    | 670977   | 54.8   | 329023    | 42446    | 90545    |
| 8  | 628109   | 44.9   | 956803   | 9.9    | 671306   | 54.7   | 328694    | 42473    | 90532    |
| 9  | 628378   | 44.8   | 956744   | 9.9    | 671634   | 54.7   | 328366    | 42499    | 90520    |
| 10 | 628647   | 44.8   | 956684   | 9.9    | 671963   | 54.7   | 328037    | 42525    | 90507    |
| 11 | 9.628916 | 44.7   | 9.956625 | 9.9    | 9.672291 | 54.7   | 10.327709 | 42552    | 90495    |
| 12 | 629185   | 44.7   | 956566   | 9.9    | 672619   | 54.6   | 327381    | 42578    | 90483    |
| 13 | 629453   | 44.7   | 956506   | 9.9    | 672947   | 54.6   | 327053    | 42604    | 90470    |
| 14 | 629721   | 44.6   | 956447   | 9.9    | 673274   | 54.6   | 326726    | 42631    | 90458    |
| 15 | 629989   | 44.6   | 956387   | 9.9    | 673602   | 54.6   | 326398    | 42657    | 90446    |
| 16 | 630257   | 44.6   | 956327   | 9.9    | 673929   | 54.5   | 326071    | 42683    | 90433    |
| 17 | 630524   | 44.6   | 956268   | 9.9    | 674257   | 54.5   | 325743    | 42709    | 90421    |
| 18 | 630792   | 44.5   | 956208   | 10.0   | 674584   | 54.5   | 325416    | 42736    | 90408    |
| 19 | 631059   | 44.5   | 956148   | 10.0   | 674910   | 54.4   | 325090    | 42762    | 90396    |
| 20 | 631326   | 44.5   | 956089   | 10.0   | 675237   | 54.4   | 324763    | 42788    | 90383    |
| 21 | 9.631593 | 44.4   | 9.956029 | 10.0   | 9.675564 | 54.4   | 10.324436 | 42815    | 90371    |
| 22 | 631859   | 44.4   | 955969   | 10.0   | 675890   | 54.4   | 324110    | 42841    | 90358    |
| 23 | 632125   | 44.4   | 955909   | 10.0   | 676216   | 54.3   | 323784    | 42867    | 90346    |
| 24 | 632392   | 44.3   | 955849   | 10.0   | 676543   | 54.3   | 323457    | 42894    | 90334    |
| 25 | 632658   | 44.3   | 955789   | 10.0   | 676869   | 54.3   | 323131    | 42920    | 90321    |
| 26 | 632923   | 44.3   | 955729   | 10.0   | 677194   | 54.3   | 322806    | 42946    | 90309    |
| 27 | 633189   | 44.2   | 955669   | 10.0   | 677520   | 54.2   | 322480    | 42972    | 90296    |
| 28 | 633454   | 44.2   | 955609   | 10.0   | 677846   | 54.2   | 322154    | 42999    | 90284    |
| 29 | 633719   | 44.2   | 955548   | 10.0   | 678171   | 54.2   | 321829    | 43025    | 90271    |
| 30 | 633984   | 44.1   | 955488   | 10.0   | 678496   | 54.2   | 321504    | 43051    | 90259    |
| 31 | 9.634249 | 44.1   | 9.955428 | 10.1   | 9.678821 | 54.1   | 10.321179 | 43077    | 90246    |
| 32 | 634514   | 44.0   | 955368   | 10.1   | 679146   | 54.1   | 320854    | 43104    | 90233    |
| 33 | 634778   | 44.0   | 955307   | 10.1   | 679471   | 54.1   | 320529    | 43130    | 90221    |
| 34 | 635042   | 44.0   | 955247   | 10.1   | 679795   | 54.1   | 320205    | 43156    | 90208    |
| 35 | 635306   | 43.9   | 955186   | 10.1   | 680120   | 54.0   | 319880    | 43182    | 90196    |
| 36 | 635570   | 43.9   | 955126   | 10.1   | 680444   | 54.0   | 319556    | 43209    | 90183    |
| 37 | 635834   | 43.9   | 955065   | 10.1   | 680768   | 54.0   | 319232    | 43235    | 90171    |
| 38 | 636097   | 43.8   | 955005   | 10.1   | 681092   | 54.0   | 318908    | 43261    | 90158    |
| 39 | 636360   | 43.8   | 954944   | 10.1   | 681416   | 53.9   | 318584    | 43287    | 90146    |
| 40 | 636623   | 43.8   | 954883   | 10.1   | 681740   | 53.9   | 318260    | 43313    | 90133    |
| 41 | 9.636886 | 43.7   | 9.954823 | 10.1   | 9.682063 | 53.9   | 10.317937 | 43340    | 90120    |
| 42 | 637148   | 43.7   | 954762   | 10.1   | 682387   | 53.9   | 317613    | 43366    | 90108    |
| 43 | 637411   | 43.7   | 954701   | 10.1   | 682710   | 53.8   | 317290    | 43392    | 90095    |
| 44 | 637673   | 43.7   | 954640   | 10.1   | 683033   | 53.8   | 316967    | 43418    | 90082    |
| 45 | 637935   | 43.6   | 954579   | 10.1   | 683356   | 53.8   | 316644    | 43445    | 90070    |
| 46 | 638197   | 43.6   | 954518   | 10.2   | 683679   | 53.8   | 316321    | 43471    | 90057    |
| 47 | 638458   | 43.6   | 954457   | 10.2   | 684001   | 53.7   | 315999    | 43497    | 90045    |
| 48 | 638720   | 43.5   | 954396   | 10.2   | 684324   | 53.7   | 315676    | 43523    | 90032    |
| 49 | 638981   | 43.5   | 954335   | 10.2   | 684646   | 53.7   | 315354    | 43549    | 90019    |
| 50 | 639242   | 43.5   | 954274   | 10.2   | 684968   | 53.7   | 315032    | 43575    | 90007    |
| 51 | 9.639503 | 43.4   | 9.954213 | 10.2   | 9.685290 | 53.7   | 10.314710 | 43602    | 89994    |
| 52 | 639764   | 43.4   | 954152   | 10.2   | 685612   | 53.6   | 314388    | 43628    | 89981    |
| 53 | 640024   | 43.4   | 954090   | 10.2   | 685934   | 53.6   | 314066    | 43654    | 89968    |
| 54 | 640284   | 43.3   | 954029   | 10.2   | 686255   | 53.6   | 313745    | 43680    | 89956    |
| 55 | 640544   | 43.3   | 953968   | 10.2   | 686577   | 53.5   | 313423    | 43706    | 89943    |
| 56 | 640804   | 43.3   | 953906   | 10.2   | 686898   | 53.5   | 313102    | 43733    | 89930    |
| 57 | 641064   | 43.2   | 953845   | 10.2   | 687219   | 53.5   | 312781    | 43759    | 89918    |
| 58 | 641324   | 43.2   | 953783   | 10.2   | 687540   | 53.5   | 312460    | 43785    | 89905    |
| 59 | 641584   | 43.2   | 953722   | 10.3   | 687861   | 53.4   | 312139    | 43811    | 89892    |
| 60 | 641842   |        | 953660   |        | 688182   |        | 311818    | 43837    | 89879    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

64 Degrees.

TABLE II. Log. Sines and Tangents. (26°) Natural Sines.

47

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.641842 |        | 9.953660 |        | 9.688182 |        | 10.311818 | 43837    | 89879    | 60 |
| 1  | 642101   | 43.1   | 953599   | 10.3   | 688502   | 53.4   | 311498    | 43863    | 89867    | 59 |
| 2  | 642360   | 43.1   | 953537   | 10.3   | 688823   | 53.4   | 311177    | 43889    | 89854    | 58 |
| 3  | 642618   | 43.0   | 953475   | 10.3   | 689143   | 53.3   | 310857    | 43916    | 89841    | 57 |
| 4  | 642877   | 43.0   | 953413   | 10.3   | 689463   | 53.3   | 310537    | 43942    | 89828    | 56 |
| 5  | 643135   | 43.0   | 953352   | 10.3   | 689783   | 53.3   | 310217    | 43968    | 89816    | 55 |
| 6  | 643393   | 43.0   | 953290   | 10.3   | 690103   | 53.3   | 309897    | 43994    | 89803    | 54 |
| 7  | 643650   | 42.9   | 953228   | 10.3   | 690423   | 53.3   | 309577    | 44020    | 89790    | 53 |
| 8  | 643908   | 42.9   | 953166   | 10.3   | 690742   | 53.2   | 309258    | 44046    | 89777    | 52 |
| 9  | 644165   | 42.9   | 953104   | 10.3   | 691062   | 53.2   | 308938    | 44072    | 89764    | 51 |
| 10 | 644423   | 42.8   | 953042   | 10.3   | 691381   | 53.2   | 308619    | 44098    | 89752    | 50 |
| 11 | 9.644680 | 42.8   | 9.952980 | 10.4   | 9.691700 | 53.1   | 10.308300 | 44124    | 89739    | 49 |
| 12 | 644936   | 42.8   | 952918   | 10.4   | 692019   | 53.1   | 307981    | 44151    | 89726    | 48 |
| 13 | 645193   | 42.7   | 952855   | 10.4   | 692338   | 53.1   | 307662    | 44177    | 89713    | 47 |
| 14 | 645450   | 42.7   | 952793   | 10.4   | 692656   | 53.1   | 307344    | 44203    | 89700    | 46 |
| 15 | 645706   | 42.7   | 952731   | 10.4   | 692975   | 53.1   | 307025    | 44229    | 89687    | 45 |
| 16 | 645962   | 42.6   | 952669   | 10.4   | 693293   | 53.1   | 306707    | 44255    | 89674    | 44 |
| 17 | 646218   | 42.6   | 952606   | 10.4   | 693612   | 53.0   | 306388    | 44281    | 89662    | 43 |
| 18 | 646474   | 42.6   | 952544   | 10.4   | 693930   | 53.0   | 306070    | 44307    | 89649    | 42 |
| 19 | 646729   | 42.5   | 952481   | 10.4   | 694248   | 53.0   | 305752    | 44333    | 89636    | 41 |
| 20 | 646984   | 42.5   | 952419   | 10.4   | 694566   | 52.9   | 305434    | 44359    | 89623    | 40 |
| 21 | 9.647240 | 42.5   | 9.952356 | 10.4   | 9.694883 | 52.9   | 10.305117 | 44385    | 89610    | 39 |
| 22 | 647494   | 42.4   | 952294   | 10.4   | 695201   | 52.9   | 304799    | 44411    | 89597    | 38 |
| 23 | 647749   | 42.4   | 952231   | 10.4   | 695518   | 52.9   | 304482    | 44437    | 89584    | 37 |
| 24 | 648004   | 42.4   | 952168   | 10.5   | 695836   | 52.9   | 304164    | 44464    | 89571    | 36 |
| 25 | 648258   | 42.4   | 952106   | 10.5   | 696153   | 52.8   | 303847    | 44490    | 89558    | 35 |
| 26 | 648512   | 42.3   | 952043   | 10.5   | 696470   | 52.8   | 303530    | 44516    | 89545    | 34 |
| 27 | 648766   | 42.3   | 951980   | 10.5   | 696787   | 52.8   | 303213    | 44542    | 89532    | 33 |
| 28 | 649020   | 42.3   | 951917   | 10.5   | 697103   | 52.8   | 302897    | 44568    | 89519    | 32 |
| 29 | 649274   | 42.2   | 951854   | 10.5   | 697420   | 52.7   | 302580    | 44594    | 89506    | 31 |
| 30 | 649527   | 42.2   | 951791   | 10.5   | 697736   | 52.7   | 302264    | 44620    | 89493    | 30 |
| 31 | 9.649781 | 42.2   | 9.951728 | 10.5   | 9.698053 | 52.7   | 10.301947 | 44646    | 89480    | 29 |
| 32 | 650034   | 42.2   | 951665   | 10.5   | 698369   | 52.7   | 301631    | 44672    | 89467    | 28 |
| 33 | 650287   | 42.1   | 951602   | 10.5   | 698686   | 52.6   | 301315    | 44698    | 89454    | 27 |
| 34 | 650539   | 42.1   | 951539   | 10.5   | 699001   | 52.6   | 300999    | 44724    | 89441    | 26 |
| 35 | 650792   | 42.1   | 951476   | 10.5   | 699316   | 52.6   | 300684    | 44750    | 89428    | 25 |
| 36 | 651044   | 42.0   | 951412   | 10.5   | 699632   | 52.6   | 300368    | 44776    | 89415    | 24 |
| 37 | 651297   | 42.0   | 951349   | 10.5   | 699947   | 52.6   | 300053    | 44802    | 89402    | 23 |
| 38 | 651549   | 42.0   | 951286   | 10.6   | 700263   | 52.6   | 299737    | 44828    | 89389    | 22 |
| 39 | 651800   | 41.9   | 951222   | 10.6   | 700578   | 52.5   | 299422    | 44854    | 89376    | 21 |
| 40 | 652052   | 41.9   | 951159   | 10.6   | 700893   | 52.5   | 299107    | 44880    | 89363    | 20 |
| 41 | 9.652304 | 41.9   | 9.951096 | 10.6   | 9.701208 | 52.4   | 10.298792 | 44906    | 89350    | 19 |
| 42 | 652555   | 41.8   | 951032   | 10.6   | 701523   | 52.4   | 298477    | 44932    | 89337    | 18 |
| 43 | 652806   | 41.8   | 950968   | 10.6   | 701837   | 52.4   | 298163    | 44958    | 89324    | 17 |
| 44 | 653057   | 41.8   | 950905   | 10.6   | 702152   | 52.4   | 297848    | 44984    | 89311    | 16 |
| 45 | 653308   | 41.8   | 950841   | 10.6   | 702466   | 52.4   | 297534    | 45010    | 89298    | 15 |
| 46 | 653558   | 41.7   | 950778   | 10.6   | 702780   | 52.3   | 297220    | 45036    | 89285    | 14 |
| 47 | 653808   | 41.7   | 950714   | 10.6   | 703095   | 52.3   | 296905    | 45062    | 89272    | 13 |
| 48 | 654059   | 41.7   | 950650   | 10.6   | 703409   | 52.3   | 296591    | 45088    | 89259    | 12 |
| 49 | 654309   | 41.6   | 950586   | 10.6   | 703723   | 52.3   | 296277    | 45114    | 89245    | 11 |
| 50 | 654558   | 41.6   | 950522   | 10.6   | 704036   | 52.3   | 295964    | 45140    | 89232    | 10 |
| 51 | 9.654808 | 41.6   | 9.950458 | 10.7   | 9.704350 | 52.2   | 10.295650 | 45166    | 89219    | 9  |
| 52 | 655058   | 41.6   | 950394   | 10.7   | 704663   | 52.2   | 295337    | 45192    | 89206    | 8  |
| 53 | 655307   | 41.5   | 950330   | 10.7   | 704977   | 52.2   | 295023    | 45218    | 89193    | 7  |
| 54 | 655556   | 41.5   | 950366   | 10.7   | 705290   | 52.2   | 294710    | 45244    | 89180    | 6  |
| 55 | 655805   | 41.5   | 950202   | 10.7   | 705603   | 52.1   | 294397    | 45269    | 89167    | 5  |
| 56 | 656054   | 41.4   | 950138   | 10.7   | 705916   | 52.1   | 294084    | 45295    | 89153    | 4  |
| 57 | 656302   | 41.4   | 950074   | 10.7   | 706228   | 52.1   | 293772    | 45321    | 89140    | 3  |
| 58 | 656551   | 41.4   | 950010   | 10.7   | 706541   | 52.1   | 293459    | 45347    | 89127    | 2  |
| 59 | 656799   | 41.3   | 949945   | 10.7   | 706854   | 52.1   | 293146    | 45373    | 89114    | 1  |
| 60 | 657047   |        | 949881   | 10.7   | 707166   |        | 292834    | 45399    | 89101    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

63 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.657047 | 41.3   | 9.949881 | 10.7   | 9.707166 | 52.0   | 10.292834 | 45399    | 89101    | 60 |
| 1  | 657295   | 41.3   | 949816   | 10.7   | 707478   | 52.0   | 292522    | 45425    | 89087    | 59 |
| 2  | 657542   | 41.2   | 949752   | 10.7   | 707790   | 52.0   | 292210    | 45451    | 89074    | 58 |
| 3  | 657790   | 41.2   | 949688   | 10.8   | 708102   | 52.0   | 291898    | 45477    | 89061    | 57 |
| 4  | 658037   | 41.2   | 949623   | 10.8   | 708414   | 51.9   | 291586    | 45503    | 89048    | 56 |
| 5  | 658284   | 41.2   | 949558   | 10.8   | 708726   | 51.9   | 291274    | 45529    | 89035    | 55 |
| 6  | 658531   | 41.1   | 949494   | 10.8   | 709037   | 51.9   | 290963    | 45554    | 89021    | 54 |
| 7  | 658778   | 41.1   | 949429   | 10.8   | 709349   | 51.9   | 290651    | 45580    | 89008    | 53 |
| 8  | 659025   | 41.1   | 949364   | 10.8   | 709660   | 51.9   | 290340    | 45606    | 88995    | 52 |
| 9  | 659271   | 41.0   | 949300   | 10.8   | 709971   | 51.8   | 290029    | 45632    | 88981    | 51 |
| 10 | 659517   | 41.0   | 949235   | 10.8   | 710282   | 51.8   | 289718    | 45658    | 88968    | 50 |
| 11 | 9.659763 | 41.0   | 9.949170 | 10.8   | 9.710593 | 51.8   | 10.289407 | 45684    | 88955    | 49 |
| 12 | 660009   | 40.9   | 949105   | 10.8   | 710904   | 51.8   | 289096    | 45710    | 88942    | 48 |
| 13 | 660255   | 40.9   | 949040   | 10.8   | 711215   | 51.8   | 288785    | 45736    | 88928    | 47 |
| 14 | 660501   | 40.9   | 948975   | 10.8   | 711525   | 51.7   | 288475    | 45762    | 88915    | 46 |
| 15 | 660746   | 40.9   | 948910   | 10.8   | 711836   | 51.7   | 288164    | 45787    | 88902    | 45 |
| 16 | 660991   | 40.8   | 948845   | 10.8   | 712146   | 51.7   | 287854    | 45813    | 88888    | 44 |
| 17 | 661236   | 40.8   | 948780   | 10.9   | 712456   | 51.7   | 287544    | 45839    | 88875    | 43 |
| 18 | 661481   | 40.8   | 948715   | 10.9   | 712766   | 51.6   | 287234    | 45865    | 88862    | 42 |
| 19 | 661726   | 40.7   | 948650   | 10.9   | 713076   | 51.6   | 286924    | 45891    | 88848    | 41 |
| 20 | 661970   | 40.7   | 948584   | 10.9   | 713386   | 51.6   | 286614    | 45917    | 88835    | 40 |
| 21 | 9.662214 | 40.7   | 9.948519 | 10.9   | 9.713696 | 51.6   | 10.286304 | 45942    | 88822    | 39 |
| 22 | 662459   | 40.7   | 948519   | 10.9   | 714005   | 51.6   | 286395    | 45968    | 88808    | 38 |
| 23 | 662703   | 40.6   | 948388   | 10.9   | 714314   | 51.5   | 286086    | 45994    | 88795    | 37 |
| 24 | 662946   | 40.6   | 948323   | 10.9   | 714624   | 51.5   | 285776    | 46020    | 88782    | 36 |
| 25 | 663190   | 40.6   | 948257   | 10.9   | 714933   | 51.5   | 285467    | 46046    | 88768    | 35 |
| 26 | 663433   | 40.5   | 948192   | 10.9   | 715242   | 51.5   | 285158    | 46072    | 88755    | 34 |
| 27 | 663677   | 40.5   | 948126   | 10.9   | 715551   | 51.4   | 284849    | 46097    | 88741    | 33 |
| 28 | 663920   | 40.5   | 948060   | 10.9   | 715860   | 51.4   | 284540    | 46123    | 88728    | 32 |
| 29 | 664163   | 40.5   | 947995   | 11.0   | 716168   | 51.4   | 284232    | 46149    | 88715    | 31 |
| 30 | 664406   | 40.4   | 947929   | 11.0   | 716477   | 51.4   | 283923    | 46175    | 88701    | 30 |
| 31 | 9.664648 | 40.4   | 9.947863 | 11.0   | 9.716785 | 51.4   | 10.283215 | 46201    | 88688    | 29 |
| 32 | 664891   | 40.4   | 947797   | 11.0   | 717093   | 51.4   | 283614    | 46226    | 88674    | 28 |
| 33 | 665133   | 40.3   | 947731   | 11.0   | 717401   | 51.3   | 283305    | 46252    | 88661    | 27 |
| 34 | 665375   | 40.3   | 947665   | 11.0   | 717709   | 51.3   | 283000    | 46278    | 88647    | 26 |
| 35 | 665617   | 40.3   | 947600   | 11.0   | 718017   | 51.3   | 282695    | 46304    | 88634    | 25 |
| 36 | 665859   | 40.2   | 947533   | 11.0   | 718325   | 51.3   | 282390    | 46330    | 88620    | 24 |
| 37 | 666100   | 40.2   | 947467   | 11.0   | 718633   | 51.2   | 282086    | 46355    | 88607    | 23 |
| 38 | 666342   | 40.2   | 947401   | 11.0   | 718940   | 51.2   | 281781    | 46381    | 88593    | 22 |
| 39 | 666583   | 40.2   | 947335   | 11.0   | 719248   | 51.2   | 281476    | 46407    | 88580    | 21 |
| 40 | 666824   | 40.1   | 947269   | 11.0   | 719555   | 51.2   | 281171    | 46433    | 88566    | 20 |
| 41 | 9.667065 | 40.1   | 9.947203 | 11.0   | 9.719862 | 51.2   | 10.280138 | 46458    | 88553    | 19 |
| 42 | 667305   | 40.1   | 947136   | 11.1   | 720169   | 51.1   | 279831    | 46484    | 88539    | 18 |
| 43 | 667546   | 40.1   | 947070   | 11.1   | 720476   | 51.1   | 279524    | 46510    | 88526    | 17 |
| 44 | 667786   | 40.0   | 947004   | 11.1   | 720783   | 51.1   | 279217    | 46536    | 88512    | 16 |
| 45 | 668027   | 40.0   | 946937   | 11.1   | 721089   | 51.1   | 278911    | 46561    | 88499    | 15 |
| 46 | 668267   | 40.0   | 946871   | 11.1   | 721396   | 51.1   | 278604    | 46587    | 88485    | 14 |
| 47 | 668506   | 39.9   | 946804   | 11.1   | 721702   | 51.0   | 278298    | 46613    | 88472    | 13 |
| 48 | 668746   | 39.9   | 946738   | 11.1   | 722009   | 51.0   | 277991    | 46639    | 88458    | 12 |
| 49 | 668986   | 39.9   | 946671   | 11.1   | 722315   | 51.0   | 277685    | 46664    | 88445    | 11 |
| 50 | 669225   | 39.9   | 946604   | 11.1   | 722621   | 51.0   | 277379    | 46690    | 88431    | 10 |
| 51 | 9.669464 | 39.8   | 9.946538 | 11.1   | 9.722927 | 51.0   | 10.277073 | 46716    | 88417    | 9  |
| 52 | 669703   | 39.8   | 946471   | 11.1   | 723232   | 51.0   | 277073    | 46742    | 88404    | 8  |
| 53 | 669942   | 39.8   | 946404   | 11.1   | 723538   | 50.9   | 276768    | 46767    | 88390    | 7  |
| 54 | 670181   | 39.7   | 946337   | 11.1   | 723844   | 50.9   | 276462    | 46793    | 88377    | 6  |
| 55 | 670419   | 39.7   | 946270   | 11.2   | 724149   | 50.9   | 276156    | 46819    | 88363    | 5  |
| 56 | 670658   | 39.7   | 946203   | 11.2   | 724454   | 50.9   | 275851    | 46844    | 88349    | 4  |
| 57 | 670896   | 39.6   | 946136   | 11.2   | 724759   | 50.8   | 275546    | 46870    | 88336    | 3  |
| 58 | 671134   | 39.6   | 946069   | 11.2   | 725065   | 50.8   | 275241    | 46896    | 88322    | 2  |
| 59 | 671372   | 39.6   | 946002   | 11.2   | 725369   | 50.8   | 274935    | 46921    | 88308    | 1  |
| 60 | 671609   | 39.6   | 945935   | 11.2   | 725674   | 50.8   | 274630    | 46947    | 88295    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (28°) Natural Sines.

49

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.671609 | 39.6   | 9.945935 | 11.2   | 9.725674 | 50.8   | 10.274326 | 46947    | 88295    |
| 1  | 671847   | 39.5   | 945865   | 11.2   | 725979   | 50.8   | 274021    | 46973    | 88281    |
| 2  | 672084   | 39.5   | 945800   | 11.2   | 726284   | 50.7   | 273716    | 46999    | 88267    |
| 3  | 672321   | 39.5   | 945733   | 11.2   | 726588   | 50.7   | 273412    | 47024    | 88254    |
| 4  | 672558   | 39.5   | 945666   | 11.2   | 726892   | 50.7   | 273108    | 47050    | 88240    |
| 5  | 672795   | 39.4   | 945598   | 11.2   | 727197   | 50.7   | 272803    | 47076    | 88226    |
| 6  | 673032   | 39.4   | 945531   | 11.2   | 727501   | 50.7   | 272499    | 47101    | 88213    |
| 7  | 673268   | 39.4   | 945464   | 11.2   | 727805   | 50.7   | 272195    | 47127    | 88199    |
| 8  | 673505   | 39.4   | 945396   | 11.3   | 728109   | 50.6   | 271891    | 47153    | 88185    |
| 9  | 673741   | 39.4   | 945328   | 11.3   | 728412   | 50.6   | 271588    | 47178    | 88172    |
| 10 | 673977   | 39.3   | 945261   | 11.3   | 728716   | 50.6   | 271284    | 47204    | 88158    |
| 11 | 9.674213 | 39.3   | 9.945193 | 11.3   | 9.729020 | 50.6   | 10.270980 | 47229    | 88144    |
| 12 | 674448   | 39.3   | 945125   | 11.3   | 729323   | 50.5   | 270677    | 47255    | 88130    |
| 13 | 674684   | 39.2   | 945058   | 11.3   | 729626   | 50.5   | 270374    | 47281    | 88117    |
| 14 | 674919   | 39.2   | 944990   | 11.3   | 729929   | 50.5   | 270071    | 47306    | 88103    |
| 15 | 675155   | 39.2   | 944922   | 11.3   | 730233   | 50.5   | 269767    | 47332    | 88089    |
| 16 | 675390   | 39.1   | 944854   | 11.3   | 730535   | 50.5   | 269465    | 47358    | 88075    |
| 17 | 675624   | 39.1   | 944786   | 11.3   | 730838   | 50.4   | 269162    | 47383    | 88062    |
| 18 | 675859   | 39.1   | 944718   | 11.3   | 731141   | 50.4   | 268859    | 47409    | 88048    |
| 19 | 676094   | 39.0   | 944650   | 11.3   | 731444   | 50.4   | 268556    | 47434    | 88034    |
| 20 | 9.676328 | 39.0   | 9.944582 | 11.4   | 9.731746 | 50.4   | 10.268254 | 47460    | 88020    |
| 21 | 676562   | 39.0   | 944514   | 11.4   | 732048   | 50.4   | 268254    | 47486    | 88006    |
| 22 | 676796   | 39.0   | 944446   | 11.4   | 732351   | 50.3   | 267949    | 47511    | 87993    |
| 23 | 677030   | 39.0   | 944377   | 11.4   | 732653   | 50.3   | 267647    | 47537    | 87979    |
| 24 | 677264   | 38.9   | 944309   | 11.4   | 732955   | 50.3   | 267345    | 47562    | 87965    |
| 25 | 677498   | 38.9   | 944241   | 11.4   | 733257   | 50.3   | 267043    | 47588    | 87951    |
| 26 | 677731   | 38.9   | 944172   | 11.4   | 733558   | 50.3   | 266742    | 47614    | 87937    |
| 27 | 677964   | 38.8   | 944104   | 11.4   | 733860   | 50.2   | 266440    | 47639    | 87923    |
| 28 | 678197   | 38.8   | 944036   | 11.4   | 734162   | 50.2   | 266140    | 47665    | 87909    |
| 29 | 678430   | 38.8   | 943967   | 11.4   | 734463   | 50.2   | 265838    | 47690    | 87896    |
| 30 | 9.678663 | 38.8   | 9.943899 | 11.4   | 9.734764 | 50.2   | 10.265537 | 47716    | 87882    |
| 31 | 678896   | 38.7   | 943830   | 11.4   | 735066   | 50.2   | 265236    | 47741    | 87868    |
| 32 | 679128   | 38.7   | 943761   | 11.4   | 735367   | 50.2   | 264934    | 47767    | 87854    |
| 33 | 679360   | 38.7   | 943693   | 11.5   | 735668   | 50.1   | 264633    | 47793    | 87840    |
| 34 | 679592   | 38.7   | 943624   | 11.5   | 735969   | 50.1   | 264332    | 47818    | 87826    |
| 35 | 679824   | 38.6   | 943555   | 11.5   | 736269   | 50.1   | 264031    | 47844    | 87812    |
| 36 | 680056   | 38.6   | 943486   | 11.5   | 736570   | 50.1   | 263731    | 47869    | 87798    |
| 37 | 680288   | 38.6   | 943417   | 11.5   | 736871   | 50.1   | 263430    | 47895    | 87784    |
| 38 | 680519   | 38.5   | 943348   | 11.5   | 737171   | 50.0   | 263129    | 47920    | 87770    |
| 39 | 680750   | 38.5   | 943279   | 11.5   | 737471   | 50.0   | 262829    | 47946    | 87756    |
| 40 | 680982   | 38.5   | 943210   | 11.5   | 737771   | 50.0   | 262529    | 47971    | 87742    |
| 41 | 9.681213 | 38.5   | 9.943141 | 11.5   | 9.738071 | 50.0   | 10.262229 | 47997    | 87729    |
| 42 | 681443   | 38.4   | 943072   | 11.5   | 738371   | 50.0   | 262229    | 48022    | 87715    |
| 43 | 681674   | 38.4   | 943003   | 11.5   | 738671   | 50.0   | 261929    | 48048    | 87701    |
| 44 | 681905   | 38.4   | 942934   | 11.5   | 738971   | 49.9   | 261629    | 48073    | 87687    |
| 45 | 682135   | 38.4   | 942864   | 11.5   | 739271   | 49.9   | 261329    | 48099    | 87673    |
| 46 | 682365   | 38.3   | 942795   | 11.6   | 739570   | 49.9   | 261029    | 48124    | 87659    |
| 47 | 682595   | 38.3   | 942726   | 11.6   | 739870   | 49.9   | 260729    | 48149    | 87645    |
| 48 | 682825   | 38.3   | 942656   | 11.6   | 740169   | 49.9   | 260430    | 48175    | 87631    |
| 49 | 683055   | 38.3   | 942587   | 11.6   | 740468   | 49.8   | 260130    | 48201    | 87617    |
| 50 | 683284   | 38.2   | 942517   | 11.6   | 740767   | 49.8   | 259831    | 48226    | 87603    |
| 51 | 9.683514 | 38.2   | 9.942448 | 11.6   | 9.741066 | 49.8   | 10.259532 | 48252    | 87589    |
| 52 | 683743   | 38.2   | 942378   | 11.6   | 741365   | 49.8   | 259233    | 48277    | 87575    |
| 53 | 683972   | 38.2   | 942306   | 11.6   | 741664   | 49.8   | 258934    | 48303    | 87561    |
| 54 | 684201   | 38.1   | 942239   | 11.6   | 741962   | 49.7   | 258635    | 48328    | 87546    |
| 55 | 684430   | 38.1   | 942169   | 11.6   | 742261   | 49.7   | 258336    | 48354    | 87532    |
| 56 | 684658   | 38.1   | 942099   | 11.6   | 742559   | 49.7   | 258038    | 48379    | 87518    |
| 57 | 684887   | 38.0   | 942029   | 11.6   | 742858   | 49.7   | 257739    | 48405    | 87504    |
| 58 | 685115   | 38.0   | 941959   | 11.6   | 743156   | 49.7   | 257441    | 48430    | 87490    |
| 59 | 685343   | 38.0   | 941889   | 11.6   | 743454   | 49.7   | 257142    | 48456    | 87476    |
| 60 | 685571   | 38.0   | 941819   | 11.7   | 743752   | 49.7   | 256844    | 48481    | 87462    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

61 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.685571 | 38.0   | 9.941819 | 11.7   | 9.743752 | 49.6   | 10.256248 | 48481    | 87462    |
| 1  | 685799   | 37.9   | 941749   | 11.7   | 744050   | 49.6   | 255950    | 48506    | 87448    |
| 2  | 686027   | 37.9   | 941679   | 11.7   | 744348   | 49.6   | 255652    | 48532    | 87434    |
| 3  | 686254   | 37.9   | 941609   | 11.7   | 744645   | 49.6   | 255355    | 48557    | 87420    |
| 4  | 686482   | 37.9   | 941539   | 11.7   | 744943   | 49.6   | 255057    | 48583    | 87406    |
| 5  | 686709   | 37.8   | 941469   | 11.7   | 745240   | 49.6   | 254760    | 48608    | 87391    |
| 6  | 686936   | 37.8   | 941398   | 11.7   | 745538   | 49.5   | 254462    | 48634    | 87377    |
| 7  | 687163   | 37.8   | 941328   | 11.7   | 745835   | 49.5   | 254165    | 48659    | 87363    |
| 8  | 687389   | 37.8   | 941258   | 11.7   | 746132   | 49.5   | 253868    | 48684    | 87349    |
| 9  | 687616   | 37.8   | 941187   | 11.7   | 746429   | 49.5   | 253571    | 48710    | 87335    |
| 10 | 687843   | 37.7   | 941117   | 11.7   | 746726   | 49.5   | 253274    | 48735    | 87321    |
| 11 | 9.688069 | 37.7   | 9.941046 | 11.7   | 9.747023 | 49.5   | 10.252977 | 48761    | 87306    |
| 12 | 688295   | 37.7   | 940975   | 11.8   | 747319   | 49.4   | 252981    | 48786    | 87292    |
| 13 | 688521   | 37.6   | 940905   | 11.8   | 747616   | 49.4   | 252684    | 48811    | 87278    |
| 14 | 688747   | 37.6   | 940834   | 11.8   | 747913   | 49.4   | 252387    | 48837    | 87264    |
| 15 | 688972   | 37.6   | 940763   | 11.8   | 748209   | 49.4   | 252090    | 48862    | 87250    |
| 16 | 689198   | 37.6   | 940693   | 11.8   | 748505   | 49.4   | 251793    | 48888    | 87235    |
| 17 | 689423   | 37.5   | 940622   | 11.8   | 748801   | 49.3   | 251495    | 48913    | 87221    |
| 18 | 689648   | 37.5   | 940551   | 11.8   | 749097   | 49.3   | 251199    | 48938    | 87207    |
| 19 | 689873   | 37.5   | 940480   | 11.8   | 749393   | 49.3   | 250903    | 48964    | 87193    |
| 20 | 690098   | 37.5   | 940409   | 11.8   | 749689   | 49.3   | 250607    | 48989    | 87178    |
| 21 | 9.690323 | 37.4   | 9.940338 | 11.8   | 9.749985 | 49.3   | 10.250015 | 49014    | 87164    |
| 22 | 690548   | 37.4   | 940267   | 11.8   | 750281   | 49.2   | 249719    | 49040    | 87150    |
| 23 | 690772   | 37.4   | 940196   | 11.8   | 750576   | 49.2   | 249424    | 49065    | 87136    |
| 24 | 690996   | 37.4   | 940125   | 11.8   | 750872   | 49.2   | 249128    | 49090    | 87122    |
| 25 | 691220   | 37.3   | 940054   | 11.9   | 751167   | 49.2   | 248833    | 49116    | 87107    |
| 26 | 691444   | 37.3   | 939982   | 11.9   | 751462   | 49.2   | 248538    | 49141    | 87093    |
| 27 | 691668   | 37.3   | 939911   | 11.9   | 751757   | 49.2   | 248243    | 49166    | 87079    |
| 28 | 691892   | 37.3   | 939840   | 11.9   | 752052   | 49.1   | 247948    | 49192    | 87064    |
| 29 | 692115   | 37.2   | 939768   | 11.9   | 752347   | 49.1   | 247653    | 49217    | 87050    |
| 30 | 692339   | 37.2   | 939697   | 11.9   | 752642   | 49.1   | 247358    | 49242    | 87036    |
| 31 | 9.692562 | 37.2   | 9.939625 | 11.9   | 9.752937 | 49.1   | 10.247063 | 49268    | 87021    |
| 32 | 692785   | 37.1   | 939554   | 11.9   | 753231   | 49.1   | 246769    | 49293    | 87007    |
| 33 | 693008   | 37.1   | 939482   | 11.9   | 753526   | 49.1   | 246474    | 49318    | 86993    |
| 34 | 693231   | 37.1   | 939410   | 11.9   | 753820   | 49.0   | 246180    | 49344    | 86978    |
| 35 | 693453   | 37.1   | 939339   | 11.9   | 754115   | 49.0   | 245885    | 49369    | 86964    |
| 36 | 693676   | 37.0   | 939267   | 12.0   | 754409   | 49.0   | 245591    | 49394    | 86949    |
| 37 | 693898   | 37.0   | 939195   | 12.0   | 754703   | 49.0   | 245297    | 49419    | 86935    |
| 38 | 694120   | 37.0   | 939123   | 12.0   | 754997   | 49.0   | 245003    | 49445    | 86921    |
| 39 | 694342   | 37.0   | 939052   | 12.0   | 755291   | 49.0   | 244709    | 49470    | 86906    |
| 40 | 694564   | 36.9   | 938980   | 12.0   | 755585   | 48.9   | 244415    | 49495    | 86892    |
| 41 | 9.694786 | 36.9   | 9.938908 | 12.0   | 9.755878 | 48.9   | 10.244122 | 49521    | 86878    |
| 42 | 695007   | 36.9   | 938836   | 12.0   | 756172   | 48.9   | 244122    | 49546    | 86863    |
| 43 | 695229   | 36.9   | 938763   | 12.0   | 756465   | 48.9   | 243828    | 49571    | 86849    |
| 44 | 695450   | 36.9   | 938691   | 12.0   | 756759   | 48.9   | 243535    | 49596    | 86834    |
| 45 | 695671   | 36.8   | 938619   | 12.0   | 757052   | 48.9   | 243241    | 49622    | 86820    |
| 46 | 695892   | 36.8   | 938547   | 12.0   | 757345   | 48.8   | 242948    | 49647    | 86806    |
| 47 | 696113   | 36.8   | 938475   | 12.0   | 757638   | 48.8   | 242655    | 49672    | 86791    |
| 48 | 696334   | 36.7   | 938402   | 12.1   | 757931   | 48.8   | 242362    | 49697    | 86777    |
| 49 | 696554   | 36.7   | 938330   | 12.1   | 758224   | 48.8   | 242069    | 49723    | 86762    |
| 50 | 696775   | 36.7   | 938258   | 12.1   | 758517   | 48.8   | 241776    | 49748    | 86748    |
| 51 | 9.696995 | 36.7   | 9.938185 | 12.1   | 9.758810 | 48.8   | 10.241190 | 49773    | 86733    |
| 52 | 697215   | 36.6   | 938113   | 12.1   | 759102   | 48.7   | 241483    | 49798    | 86719    |
| 53 | 697435   | 36.6   | 938040   | 12.1   | 759395   | 48.7   | 241190    | 49824    | 86704    |
| 54 | 697654   | 36.6   | 937967   | 12.1   | 759687   | 48.7   | 240898    | 49849    | 86690    |
| 55 | 697874   | 36.6   | 937895   | 12.1   | 759979   | 48.7   | 240605    | 49874    | 86675    |
| 56 | 698094   | 36.5   | 937822   | 12.1   | 760272   | 48.7   | 240313    | 49899    | 86661    |
| 57 | 698313   | 36.5   | 937749   | 12.1   | 760564   | 48.7   | 240021    | 49924    | 86646    |
| 58 | 698532   | 36.5   | 937676   | 12.1   | 760856   | 48.6   | 239728    | 49949    | 86632    |
| 59 | 698751   | 36.5   | 937604   | 12.1   | 761148   | 48.6   | 239436    | 49974    | 86617    |
| 60 | 698970   | 36.5   | 937531   | 12.1   | 761439   | 48.6   | 239144    | 49999    | 86603    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

60 Degrees.

TABLE II. Log. Sines and Tangents. (30°) Natural Sines.

51

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.698970 | 36.4   | 9.937531 | 12.1   | 9.761439 | 48.6   | 10.238561 | 50000    | 86603    |
| 1  | 699189   | 36.4   | 937458   | 12.2   | 761731   | 48.6   | 238269    | 50025    | 86588    |
| 2  | 699407   | 36.4   | 937385   | 12.2   | 762023   | 48.6   | 237977    | 50050    | 86573    |
| 3  | 699626   | 36.4   | 937312   | 12.2   | 762314   | 48.6   | 237686    | 50076    | 86559    |
| 4  | 699844   | 36.3   | 937238   | 12.2   | 762606   | 48.5   | 237394    | 50101    | 86544    |
| 5  | 700062   | 36.3   | 937165   | 12.2   | 762897   | 48.5   | 237103    | 50126    | 86530    |
| 6  | 700280   | 36.3   | 937092   | 12.2   | 763188   | 48.5   | 236812    | 50151    | 86515    |
| 7  | 700498   | 36.3   | 937019   | 12.2   | 763479   | 48.5   | 236521    | 50176    | 86501    |
| 8  | 700716   | 36.3   | 936946   | 12.2   | 763770   | 48.5   | 236230    | 50201    | 86486    |
| 9  | 700933   | 36.2   | 936872   | 12.2   | 764061   | 48.5   | 235939    | 50227    | 86471    |
| 10 | 701151   | 36.2   | 936799   | 12.2   | 764352   | 48.4   | 235648    | 50252    | 86457    |
| 11 | 9.701368 | 36.2   | 9.936725 | 12.2   | 9.764643 | 48.4   | 10.235357 | 50277    | 86442    |
| 12 | 701585   | 36.2   | 936652   | 12.2   | 764933   | 48.4   | 235067    | 50302    | 86427    |
| 13 | 701802   | 36.2   | 936578   | 12.3   | 765224   | 48.4   | 234776    | 50327    | 86413    |
| 14 | 702019   | 36.1   | 936505   | 12.3   | 765514   | 48.4   | 234486    | 50352    | 86398    |
| 15 | 702236   | 36.1   | 936431   | 12.3   | 765805   | 48.4   | 234195    | 50377    | 86384    |
| 16 | 702452   | 36.1   | 936357   | 12.3   | 766095   | 48.4   | 233905    | 50403    | 86369    |
| 17 | 702669   | 36.0   | 936284   | 12.3   | 766385   | 48.3   | 233615    | 50428    | 86354    |
| 18 | 702885   | 36.0   | 936210   | 12.3   | 766675   | 48.3   | 233325    | 50453    | 86340    |
| 19 | 703101   | 36.0   | 936136   | 12.3   | 766965   | 48.3   | 233035    | 50478    | 86325    |
| 20 | 703317   | 36.0   | 936062   | 12.3   | 767255   | 48.3   | 232745    | 50503    | 86310    |
| 21 | 9.703533 | 36.0   | 9.935988 | 12.3   | 9.767545 | 48.3   | 10.232455 | 50528    | 86295    |
| 22 | 703749   | 35.9   | 935914   | 12.3   | 767834   | 48.3   | 232166    | 50553    | 86281    |
| 23 | 703964   | 35.9   | 935840   | 12.3   | 768124   | 48.2   | 231876    | 50578    | 86266    |
| 24 | 704179   | 35.9   | 935766   | 12.3   | 768413   | 48.2   | 231587    | 50603    | 86251    |
| 25 | 704395   | 35.9   | 935692   | 12.4   | 768703   | 48.2   | 231297    | 50628    | 86237    |
| 26 | 704610   | 35.8   | 935618   | 12.4   | 768992   | 48.2   | 231008    | 50654    | 86222    |
| 27 | 704825   | 35.8   | 935543   | 12.4   | 769281   | 48.2   | 230719    | 50679    | 86207    |
| 28 | 705040   | 35.8   | 935469   | 12.4   | 769570   | 48.2   | 230430    | 50704    | 86192    |
| 29 | 705254   | 35.8   | 935395   | 12.4   | 769860   | 48.1   | 230140    | 50729    | 86178    |
| 30 | 705469   | 35.7   | 935320   | 12.4   | 770148   | 48.1   | 229852    | 50754    | 86163    |
| 31 | 9.705683 | 35.7   | 9.935246 | 12.4   | 9.770437 | 48.1   | 10.229563 | 50779    | 86148    |
| 32 | 705898   | 35.7   | 935171   | 12.4   | 770726   | 48.1   | 229274    | 50804    | 86133    |
| 33 | 706112   | 35.7   | 935097   | 12.4   | 771015   | 48.1   | 228985    | 50829    | 86119    |
| 34 | 706326   | 35.6   | 935022   | 12.4   | 771303   | 48.1   | 228697    | 50854    | 86104    |
| 35 | 706539   | 35.6   | 934948   | 12.4   | 771592   | 48.1   | 228408    | 50879    | 86089    |
| 36 | 706753   | 35.6   | 934873   | 12.4   | 771880   | 48.0   | 228120    | 50904    | 86074    |
| 37 | 706967   | 35.6   | 934798   | 12.5   | 772168   | 48.0   | 227832    | 50929    | 86059    |
| 38 | 707180   | 35.5   | 934723   | 12.5   | 772457   | 48.0   | 227543    | 50954    | 86045    |
| 39 | 707393   | 35.5   | 934649   | 12.5   | 772745   | 48.0   | 227255    | 50979    | 86030    |
| 40 | 707606   | 35.5   | 934574   | 12.5   | 773033   | 48.0   | 226967    | 51004    | 86015    |
| 41 | 9.707819 | 35.5   | 9.934499 | 12.5   | 9.773321 | 48.0   | 10.226679 | 51029    | 86000    |
| 42 | 708032   | 35.5   | 934424   | 12.5   | 773608   | 48.0   | 226392    | 51054    | 85985    |
| 43 | 708245   | 35.4   | 934349   | 12.5   | 773896   | 47.9   | 226104    | 51079    | 85970    |
| 44 | 708458   | 35.4   | 934274   | 12.5   | 774184   | 47.9   | 225816    | 51104    | 85956    |
| 45 | 708670   | 35.4   | 934199   | 12.5   | 774471   | 47.9   | 225529    | 51129    | 85941    |
| 46 | 708882   | 35.3   | 934123   | 12.5   | 774759   | 47.9   | 225241    | 51154    | 85926    |
| 47 | 709094   | 35.3   | 934048   | 12.5   | 775046   | 47.9   | 224954    | 51179    | 85911    |
| 48 | 709306   | 35.3   | 933973   | 12.5   | 775333   | 47.9   | 224667    | 51204    | 85896    |
| 49 | 709518   | 35.3   | 933898   | 12.5   | 775621   | 47.9   | 224379    | 51229    | 85881    |
| 50 | 709730   | 35.3   | 933822   | 12.6   | 775908   | 47.8   | 224092    | 51254    | 85866    |
| 51 | 9.709941 | 35.2   | 9.933747 | 12.6   | 9.776196 | 47.8   | 10.223805 | 51279    | 85851    |
| 52 | 710153   | 35.2   | 933671   | 12.6   | 776482   | 47.8   | 223518    | 51304    | 85836    |
| 53 | 710364   | 35.2   | 933596   | 12.6   | 776769   | 47.8   | 223231    | 51329    | 85821    |
| 54 | 710575   | 35.2   | 933520   | 12.6   | 777055   | 47.8   | 222945    | 51354    | 85806    |
| 55 | 710786   | 35.1   | 933445   | 12.6   | 777342   | 47.8   | 222658    | 51379    | 85792    |
| 56 | 710997   | 35.1   | 933369   | 12.6   | 777628   | 47.7   | 222372    | 51404    | 85777    |
| 57 | 711208   | 35.1   | 933293   | 12.6   | 777915   | 47.7   | 222085    | 51429    | 85762    |
| 58 | 711419   | 35.1   | 933217   | 12.6   | 778201   | 47.7   | 221799    | 51454    | 85747    |
| 59 | 711629   | 35.0   | 933141   | 12.6   | 778487   | 47.7   | 221512    | 51479    | 85732    |
| 60 | 711839   | 35.0   | 933066   | 12.6   | 778774   | 47.7   | 221226    | 51504    | 85717    |
|    | Cosine.  |        | Sine     |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

59 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N.sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|---------|----------|----|
| 0  | 9.711839 | 35.0   | 9.933066 | 12.6   | 9.778774 | 47.7   | 10.221226 | 51504   | 85717    | 60 |
| 1  | 712050   | 35.0   | 932990   | 12.7   | 779060   | 47.7   | 220940    | 51529   | 85702    | 59 |
| 2  | 712260   | 35.0   | 932914   | 12.7   | 779346   | 47.6   | 220654    | 51554   | 85687    | 58 |
| 3  | 712469   | 34.9   | 932838   | 12.7   | 779632   | 47.6   | 220368    | 51579   | 85672    | 57 |
| 4  | 712679   | 34.9   | 932762   | 12.7   | 779918   | 47.6   | 220082    | 51604   | 85657    | 56 |
| 5  | 712889   | 34.9   | 932685   | 12.7   | 780203   | 47.6   | 219797    | 51628   | 85642    | 55 |
| 6  | 713098   | 34.9   | 932609   | 12.7   | 780489   | 47.6   | 219511    | 51653   | 85627    | 54 |
| 7  | 713308   | 34.9   | 932533   | 12.7   | 780775   | 47.6   | 219225    | 51678   | 85612    | 53 |
| 8  | 713517   | 34.8   | 932457   | 12.7   | 781060   | 47.6   | 218940    | 51703   | 85597    | 52 |
| 9  | 713726   | 34.8   | 932380   | 12.7   | 781346   | 47.5   | 218654    | 51728   | 85582    | 51 |
| 10 | 713935   | 34.8   | 932304   | 12.7   | 781631   | 47.5   | 218369    | 51753   | 85567    | 50 |
| 11 | 9.714144 | 34.8   | 9.932228 | 12.7   | 9.781916 | 47.5   | 10.218084 | 51778   | 85551    | 49 |
| 12 | 714352   | 34.7   | 932151   | 12.7   | 782201   | 47.5   | 217799    | 51803   | 85536    | 48 |
| 13 | 714561   | 34.7   | 932075   | 12.8   | 782486   | 47.5   | 217514    | 51828   | 85521    | 47 |
| 14 | 714769   | 34.7   | 931998   | 12.8   | 782771   | 47.5   | 217229    | 51852   | 85506    | 46 |
| 15 | 714978   | 34.7   | 931921   | 12.8   | 783056   | 47.5   | 216944    | 51877   | 85491    | 45 |
| 16 | 715186   | 34.7   | 931845   | 12.8   | 783341   | 47.5   | 216659    | 51902   | 85476    | 44 |
| 17 | 715394   | 34.6   | 931768   | 12.8   | 783626   | 47.4   | 216374    | 51927   | 85461    | 43 |
| 18 | 715602   | 34.6   | 931691   | 12.8   | 783910   | 47.4   | 216090    | 51952   | 85446    | 42 |
| 19 | 715809   | 34.6   | 931614   | 12.8   | 784195   | 47.4   | 215805    | 51977   | 85431    | 41 |
| 20 | 716017   | 34.6   | 931537   | 12.8   | 784479   | 47.4   | 215521    | 52002   | 85416    | 40 |
| 21 | 9.716224 | 34.5   | 9.931460 | 12.8   | 9.784764 | 47.4   | 10.215236 | 52026   | 85401    | 39 |
| 22 | 716432   | 34.5   | 931383   | 12.8   | 785048   | 47.4   | 214952    | 52051   | 85385    | 38 |
| 23 | 716639   | 34.5   | 931306   | 12.8   | 785332   | 47.3   | 214668    | 52076   | 85370    | 37 |
| 24 | 716846   | 34.5   | 931229   | 12.9   | 785616   | 47.3   | 214384    | 52101   | 85355    | 36 |
| 25 | 717053   | 34.5   | 931152   | 12.9   | 785900   | 47.3   | 214100    | 52126   | 85340    | 35 |
| 26 | 717259   | 34.5   | 931075   | 12.9   | 786184   | 47.3   | 213816    | 52151   | 85325    | 34 |
| 27 | 717466   | 34.4   | 930998   | 12.9   | 786468   | 47.3   | 213532    | 52175   | 85310    | 33 |
| 28 | 717673   | 34.4   | 930921   | 12.9   | 786752   | 47.3   | 213248    | 52200   | 85294    | 32 |
| 29 | 717879   | 34.4   | 930843   | 12.9   | 787036   | 47.3   | 212964    | 52225   | 85279    | 31 |
| 30 | 718085   | 34.3   | 930766   | 12.9   | 787319   | 47.2   | 212681    | 52250   | 85264    | 30 |
| 31 | 9.718291 | 34.3   | 9.930688 | 12.9   | 9.787603 | 47.2   | 10.212397 | 52275   | 85249    | 29 |
| 32 | 718497   | 34.3   | 930611   | 12.9   | 787886   | 47.2   | 212114    | 52299   | 85234    | 28 |
| 33 | 718703   | 34.3   | 930533   | 12.9   | 788170   | 47.2   | 211830    | 52324   | 85218    | 27 |
| 34 | 718909   | 34.3   | 930456   | 12.9   | 788453   | 47.2   | 211547    | 52349   | 85203    | 26 |
| 35 | 719114   | 34.2   | 930378   | 12.9   | 788736   | 47.2   | 211264    | 52374   | 85188    | 25 |
| 36 | 719320   | 34.2   | 930300   | 13.0   | 789019   | 47.2   | 210981    | 52399   | 85173    | 24 |
| 37 | 719525   | 34.2   | 930223   | 13.0   | 789302   | 47.1   | 210698    | 52423   | 85157    | 23 |
| 38 | 719730   | 34.2   | 930145   | 13.0   | 789585   | 47.1   | 210415    | 52448   | 85142    | 22 |
| 39 | 719935   | 34.2   | 930067   | 13.0   | 789868   | 47.1   | 210132    | 52473   | 85127    | 21 |
| 40 | 720140   | 34.1   | 929989   | 13.0   | 790151   | 47.1   | 209849    | 52498   | 85112    | 20 |
| 41 | 9.720345 | 34.1   | 9.929911 | 13.0   | 9.790433 | 47.1   | 10.209567 | 52522   | 85096    | 19 |
| 42 | 720549   | 34.1   | 929833   | 13.0   | 790716   | 47.1   | 209284    | 52547   | 85081    | 18 |
| 43 | 720754   | 34.0   | 929755   | 13.0   | 790999   | 47.1   | 209001    | 52572   | 85066    | 17 |
| 44 | 720958   | 34.0   | 929677   | 13.0   | 791281   | 47.1   | 208719    | 52597   | 85051    | 16 |
| 45 | 721162   | 34.0   | 929599   | 13.0   | 791563   | 47.0   | 208437    | 52621   | 85036    | 15 |
| 46 | 721366   | 34.0   | 929521   | 13.0   | 791846   | 47.0   | 208154    | 52646   | 85020    | 14 |
| 47 | 721570   | 34.0   | 929442   | 13.0   | 792128   | 47.0   | 207872    | 52671   | 85005    | 13 |
| 48 | 721774   | 33.9   | 929364   | 13.1   | 792410   | 47.0   | 207590    | 52696   | 84989    | 12 |
| 49 | 721978   | 33.9   | 929286   | 13.1   | 792692   | 47.0   | 207308    | 52720   | 84974    | 11 |
| 50 | 722181   | 33.9   | 929207   | 13.1   | 792974   | 47.0   | 207026    | 52745   | 84959    | 10 |
| 51 | 9.722385 | 33.9   | 9.929129 | 13.1   | 9.793256 | 47.0   | 10.206744 | 52770   | 84943    | 9  |
| 52 | 722588   | 33.9   | 929050   | 13.1   | 793538   | 46.9   | 206462    | 52794   | 84928    | 8  |
| 53 | 722791   | 33.8   | 928972   | 13.1   | 793819   | 46.9   | 206181    | 52819   | 84913    | 7  |
| 54 | 722994   | 33.8   | 928893   | 13.1   | 794101   | 46.9   | 205899    | 52844   | 84897    | 6  |
| 55 | 723197   | 33.8   | 928815   | 13.1   | 794383   | 46.9   | 205617    | 52869   | 84882    | 5  |
| 56 | 723400   | 33.8   | 928736   | 13.1   | 794664   | 46.9   | 205336    | 52893   | 84866    | 4  |
| 57 | 723603   | 33.7   | 928657   | 13.1   | 794945   | 46.9   | 205055    | 52918   | 84851    | 3  |
| 58 | 723805   | 33.7   | 928578   | 13.1   | 795227   | 46.9   | 204773    | 52943   | 84836    | 2  |
| 59 | 724007   | 33.7   | 928499   | 13.1   | 795508   | 46.8   | 204492    | 52967   | 84820    | 1  |
| 60 | 724210   | 33.7   | 928420   | 13.1   | 795789   |        | 204211    | 52992   | 84805    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos. | N. sine. |    |

TABLE II. Log. Sines and Tangents. (32°) Natural Sines.

53

| $\angle$ | Sine.    | D. 10'' | Cosine.  | D. 10'' | Tang.    | D. 10'' | Cotang.   | N. sine. | N. cos.  |
|----------|----------|---------|----------|---------|----------|---------|-----------|----------|----------|
| 0        | 9.724210 |         | 9.928420 |         | 9.795789 |         | 10.204211 | 52992    | 84805    |
| 1        | 724412   | 33.7    | 928342   | 13.2    | 796070   | 46.8    | 203930    | 53017    | 84789    |
| 2        | 724614   | 33.6    | 928263   | 13.2    | 796351   | 46.8    | 203649    | 53041    | 84774    |
| 3        | 724816   | 33.6    | 928183   | 13.2    | 796632   | 46.8    | 203368    | 53066    | 84759    |
| 4        | 725017   | 33.6    | 928104   | 13.2    | 796913   | 46.8    | 203087    | 53091    | 84743    |
| 5        | 725219   | 33.6    | 928025   | 13.2    | 797194   | 46.8    | 202806    | 53115    | 84728    |
| 6        | 725420   | 33.6    | 927946   | 13.2    | 797475   | 46.8    | 202525    | 53140    | 84712    |
| 7        | 725622   | 33.5    | 927867   | 13.2    | 797755   | 46.8    | 202245    | 53164    | 84697    |
| 8        | 725823   | 33.5    | 927787   | 13.2    | 798036   | 46.7    | 201964    | 53189    | 84681    |
| 9        | 726024   | 33.5    | 927708   | 13.2    | 798316   | 46.7    | 201684    | 53214    | 84666    |
| 10       | 726225   | 33.5    | 927629   | 13.2    | 798596   | 46.7    | 201404    | 53238    | 84650    |
| 11       | 9.726426 |         | 9.927549 |         | 9.798877 |         | 10.201123 | 53263    | 84635    |
| 12       | 726626   | 33.4    | 927470   | 13.3    | 799157   | 46.7    | 200843    | 53288    | 84619    |
| 13       | 726827   | 33.4    | 927390   | 13.3    | 799437   | 46.7    | 200563    | 53312    | 84604    |
| 14       | 727027   | 33.4    | 927310   | 13.3    | 799717   | 46.7    | 200283    | 53337    | 84588    |
| 15       | 727228   | 33.4    | 927231   | 13.3    | 799997   | 46.6    | 200003    | 53361    | 84573    |
| 16       | 727428   | 33.3    | 927151   | 13.3    | 800277   | 46.6    | 199723    | 53386    | 84557    |
| 17       | 727628   | 33.3    | 927071   | 13.3    | 800557   | 46.6    | 199443    | 53411    | 84542    |
| 18       | 727828   | 33.3    | 926991   | 13.3    | 800836   | 46.6    | 199164    | 53435    | 84526    |
| 19       | 728027   | 33.3    | 926911   | 13.3    | 801116   | 46.6    | 198884    | 53460    | 84511    |
| 20       | 728227   | 33.3    | 926831   | 13.3    | 801396   | 46.6    | 198604    | 53484    | 84495    |
| 21       | 9.728427 |         | 9.926751 |         | 9.801675 |         | 10.198325 | 53509    | 84480    |
| 22       | 728626   | 33.2    | 926671   | 13.3    | 801955   | 46.6    | 198045    | 53534    | 84464    |
| 23       | 728825   | 33.2    | 926591   | 13.3    | 802234   | 46.5    | 197766    | 53558    | 84448    |
| 24       | 729024   | 33.2    | 926511   | 13.4    | 802513   | 46.5    | 197487    | 53583    | 84433    |
| 25       | 729223   | 33.1    | 926431   | 13.4    | 802792   | 46.5    | 197208    | 53607    | 84417    |
| 26       | 729422   | 33.1    | 926351   | 13.4    | 803072   | 46.5    | 196928    | 53632    | 84402    |
| 27       | 729621   | 33.1    | 926270   | 13.4    | 803351   | 46.5    | 196649    | 53656    | 84386    |
| 28       | 729820   | 33.1    | 926190   | 13.4    | 803630   | 46.5    | 196370    | 53681    | 84370    |
| 29       | 730018   | 33.0    | 926110   | 13.4    | 803908   | 46.5    | 196092    | 53705    | 84355    |
| 30       | 730216   | 33.0    | 926029   | 13.4    | 804187   | 46.5    | 195813    | 53730    | 84339    |
| 31       | 9.730415 |         | 9.925949 |         | 9.804466 |         | 10.195534 | 53754    | 84324    |
| 32       | 730613   | 33.0    | 925868   | 13.4    | 804745   | 46.4    | 195555    | 53779    | 84308    |
| 33       | 730811   | 33.0    | 925788   | 13.4    | 805023   | 46.4    | 194977    | 53804    | 84292    |
| 34       | 731009   | 32.9    | 925707   | 13.4    | 805302   | 46.4    | 194698    | 53828    | 84277    |
| 35       | 731206   | 32.9    | 925626   | 13.4    | 805580   | 46.4    | 194420    | 53853    | 84261    |
| 36       | 731404   | 32.9    | 925545   | 13.5    | 805859   | 46.4    | 194141    | 53877    | 84245    |
| 37       | 731602   | 32.9    | 925465   | 13.5    | 806137   | 46.4    | 193863    | 53902    | 84230    |
| 38       | 731799   | 32.9    | 925384   | 13.5    | 806415   | 46.3    | 193585    | 53926    | 84214    |
| 39       | 731996   | 32.8    | 925303   | 13.5    | 806693   | 46.3    | 193307    | 53951    | 84198    |
| 40       | 732193   | 32.8    | 925222   | 13.5    | 806971   | 46.3    | 193029    | 53975    | 84182    |
| 41       | 9.732390 |         | 9.925141 |         | 9.807249 |         | 10.192751 | 54000    | 84167    |
| 42       | 732587   | 32.8    | 925060   | 13.5    | 807527   | 46.3    | 192743    | 54024    | 84151    |
| 43       | 732784   | 32.8    | 924979   | 13.5    | 807805   | 46.3    | 192195    | 54049    | 84135    |
| 44       | 732980   | 32.7    | 924897   | 13.5    | 808083   | 46.3    | 191917    | 54073    | 84120    |
| 45       | 733177   | 32.7    | 924816   | 13.5    | 808361   | 46.3    | 191639    | 54097    | 84104    |
| 46       | 733373   | 32.7    | 924735   | 13.6    | 808638   | 46.2    | 191362    | 54122    | 84088    |
| 47       | 733569   | 32.7    | 924654   | 13.6    | 808916   | 46.2    | 191084    | 54146    | 84072    |
| 48       | 733765   | 32.7    | 924572   | 13.6    | 809193   | 46.2    | 190807    | 54171    | 84057    |
| 49       | 733961   | 32.6    | 924491   | 13.6    | 809471   | 46.2    | 190529    | 54195    | 84041    |
| 50       | 734157   | 32.6    | 924409   | 13.6    | 809748   | 46.2    | 190252    | 54220    | 84025    |
| 51       | 9.734353 |         | 9.924328 |         | 9.810025 |         | 10.189975 | 54244    | 84009    |
| 52       | 734549   | 32.6    | 924246   | 13.6    | 810302   | 46.2    | 189698    | 54269    | 83994    |
| 53       | 734744   | 32.5    | 924164   | 13.6    | 810580   | 46.2    | 189420    | 54293    | 83978    |
| 54       | 734939   | 32.5    | 924083   | 13.6    | 810857   | 46.2    | 189143    | 54317    | 83962    |
| 55       | 735135   | 32.5    | 924001   | 13.6    | 811134   | 46.1    | 188866    | 54342    | 83946    |
| 56       | 735330   | 32.5    | 923919   | 13.6    | 811410   | 46.1    | 188590    | 54366    | 83930    |
| 57       | 735525   | 32.5    | 923837   | 13.6    | 811687   | 46.1    | 188313    | 54391    | 83915    |
| 58       | 735719   | 32.4    | 923755   | 13.7    | 811964   | 46.1    | 188036    | 54415    | 83899    |
| 59       | 735914   | 32.4    | 923673   | 13.7    | 812241   | 46.1    | 187759    | 54440    | 83883    |
| 60       | 736109   |         | 923591   |         | 812517   |         | 187483    | 54464    | 83867    |
|          | Cosine.  |         | Sine.    |         | Cotang.  |         | Tang.     | N. cos.  | N. sine. |

57 Degrees.



|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.736109 | 32.4   | 9.923591 | 13.7   | 9.812517 | 46.1   | 10.187482 | 54464    | 83867    |
| 1  | 736303   | 32.4   | 923509   | 13.7   | 812794   | 46.1   | 187206    | 54488    | 83851    |
| 2  | 736498   | 32.4   | 923427   | 13.7   | 813070   | 46.1   | 186930    | 54513    | 83835    |
| 3  | 736692   | 32.3   | 923345   | 13.7   | 813347   | 46.0   | 186653    | 54537    | 83819    |
| 4  | 736886   | 32.3   | 923263   | 13.7   | 813623   | 46.0   | 186377    | 54561    | 83804    |
| 5  | 737080   | 32.3   | 923181   | 13.7   | 813899   | 46.0   | 186101    | 54586    | 83788    |
| 6  | 737274   | 32.3   | 923098   | 13.7   | 814175   | 46.0   | 185825    | 54610    | 83772    |
| 7  | 737467   | 32.3   | 923016   | 13.7   | 814452   | 46.0   | 185548    | 54635    | 83756    |
| 8  | 737661   | 32.3   | 922933   | 13.7   | 814728   | 46.0   | 185272    | 54659    | 83740    |
| 9  | 737855   | 32.2   | 922851   | 13.7   | 815004   | 46.0   | 184996    | 54683    | 83724    |
| 10 | 738048   | 32.2   | 922768   | 13.7   | 815279   | 46.0   | 184721    | 54708    | 83708    |
| 11 | 9.738241 | 32.2   | 9.922686 | 13.8   | 9.815555 | 45.9   | 10.184445 | 54732    | 83692    |
| 12 | 738434   | 32.2   | 922603   | 13.8   | 815831   | 45.9   | 184169    | 54756    | 83676    |
| 13 | 738627   | 32.1   | 922520   | 13.8   | 816107   | 45.9   | 183893    | 54781    | 83660    |
| 14 | 738820   | 32.1   | 922438   | 13.8   | 816382   | 45.9   | 183618    | 54805    | 83645    |
| 15 | 739013   | 32.1   | 922355   | 13.8   | 816658   | 45.9   | 183342    | 54829    | 83629    |
| 16 | 739206   | 32.1   | 922272   | 13.8   | 816933   | 45.9   | 183067    | 54854    | 83613    |
| 17 | 739398   | 32.1   | 922189   | 13.8   | 817209   | 45.9   | 182791    | 54878    | 83597    |
| 18 | 739590   | 32.0   | 922106   | 13.8   | 817484   | 45.9   | 182516    | 54902    | 83581    |
| 19 | 739783   | 32.0   | 922023   | 13.8   | 817759   | 45.9   | 182241    | 54927    | 83565    |
| 20 | 739975   | 32.0   | 921940   | 13.8   | 818035   | 45.9   | 181965    | 54951    | 83549    |
| 21 | 9.740167 | 32.0   | 9.921857 | 13.9   | 9.818310 | 45.8   | 10.181690 | 54975    | 83533    |
| 22 | 740359   | 32.0   | 921774   | 13.9   | 818585   | 45.8   | 181415    | 54999    | 83517    |
| 23 | 740550   | 31.9   | 921691   | 13.9   | 818860   | 45.8   | 181140    | 55024    | 83501    |
| 24 | 740742   | 31.9   | 921607   | 13.9   | 819135   | 45.8   | 180865    | 55048    | 83485    |
| 25 | 740934   | 31.9   | 921524   | 13.9   | 819410   | 45.8   | 180590    | 55072    | 83469    |
| 26 | 741125   | 31.9   | 921441   | 13.9   | 819684   | 45.8   | 180316    | 55097    | 83453    |
| 27 | 741316   | 31.9   | 921357   | 13.9   | 819959   | 45.8   | 180041    | 55121    | 83437    |
| 28 | 741508   | 31.8   | 921274   | 13.9   | 820234   | 45.8   | 179766    | 55145    | 83421    |
| 29 | 741699   | 31.8   | 921190   | 13.9   | 820508   | 45.7   | 179492    | 55169    | 83405    |
| 30 | 741889   | 31.8   | 921107   | 13.9   | 820783   | 45.7   | 179217    | 55194    | 83389    |
| 31 | 9.742080 | 31.8   | 9.921023 | 13.9   | 9.821057 | 45.7   | 10.178943 | 55218    | 83373    |
| 32 | 742271   | 31.8   | 920939   | 14.0   | 821332   | 45.7   | 178668    | 55242    | 83356    |
| 33 | 742462   | 31.7   | 920856   | 14.0   | 821606   | 45.7   | 178394    | 55266    | 83340    |
| 34 | 742652   | 31.7   | 920772   | 14.0   | 821880   | 45.7   | 178120    | 55291    | 83324    |
| 35 | 742842   | 31.7   | 920688   | 14.0   | 822154   | 45.7   | 177846    | 55315    | 83308    |
| 36 | 743033   | 31.7   | 920604   | 14.0   | 822429   | 45.7   | 177571    | 55339    | 83292    |
| 37 | 743223   | 31.7   | 920520   | 14.0   | 822703   | 45.7   | 177297    | 55363    | 83276    |
| 38 | 743413   | 31.6   | 920436   | 14.0   | 822977   | 45.6   | 177023    | 55388    | 83260    |
| 39 | 743602   | 31.6   | 920352   | 14.0   | 823250   | 45.6   | 176750    | 55412    | 83244    |
| 40 | 743792   | 31.6   | 920268   | 14.0   | 823524   | 45.6   | 176476    | 55436    | 83228    |
| 41 | 9.743982 | 31.6   | 9.920184 | 14.0   | 9.823798 | 45.6   | 10.176202 | 55460    | 83212    |
| 42 | 744171   | 31.6   | 920099   | 14.0   | 824072   | 45.6   | 175928    | 55484    | 83196    |
| 43 | 744361   | 31.5   | 920015   | 14.0   | 824345   | 45.6   | 175655    | 55509    | 83179    |
| 44 | 744550   | 31.5   | 919931   | 14.1   | 824619   | 45.6   | 175381    | 55533    | 83163    |
| 45 | 744739   | 31.5   | 919846   | 14.1   | 824893   | 45.6   | 175107    | 55557    | 83147    |
| 46 | 744928   | 31.5   | 919762   | 14.1   | 825166   | 45.6   | 174834    | 55581    | 83131    |
| 47 | 745117   | 31.5   | 919677   | 14.1   | 825439   | 45.6   | 174561    | 55605    | 83115    |
| 48 | 745306   | 31.4   | 919593   | 14.1   | 825713   | 45.5   | 174287    | 55630    | 83098    |
| 49 | 745494   | 31.4   | 919508   | 14.1   | 825986   | 45.5   | 174014    | 55654    | 83082    |
| 50 | 745683   | 31.4   | 919424   | 14.1   | 826259   | 45.5   | 173741    | 55678    | 83066    |
| 51 | 9.745871 | 31.4   | 9.919339 | 14.1   | 9.826532 | 45.5   | 10.173468 | 55702    | 83050    |
| 52 | 746059   | 31.4   | 919254   | 14.1   | 826805   | 45.5   | 173195    | 55726    | 83034    |
| 53 | 746248   | 31.3   | 919169   | 14.1   | 827078   | 45.5   | 172922    | 55750    | 83017    |
| 54 | 746436   | 31.3   | 919085   | 14.1   | 827351   | 45.5   | 172649    | 55775    | 83001    |
| 55 | 746624   | 31.3   | 919000   | 14.1   | 827624   | 45.5   | 172376    | 55799    | 82985    |
| 56 | 746812   | 31.3   | 918915   | 14.2   | 827897   | 45.4   | 172103    | 55823    | 82969    |
| 57 | 746999   | 31.3   | 918830   | 14.2   | 828170   | 45.4   | 171830    | 55847    | 82953    |
| 58 | 747187   | 31.2   | 918745   | 14.2   | 828442   | 45.4   | 171558    | 55871    | 82936    |
| 59 | 747374   | 31.2   | 918659   | 14.2   | 828715   | 45.4   | 171285    | 55895    | 82920    |
| 60 | 747562   |        | 918574   |        | 828987   |        | 171013    | 55919    | 82904    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (34°)' Natural Sines.

55

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N.sine  | N.cos.   |
|----|----------|--------|----------|--------|----------|--------|-----------|---------|----------|
| 0  | 9.747562 |        | 9.918574 |        | 9.828987 |        | 10.171013 | 55919   | 82904    |
| 1  | 747749   | 31.2   | 918489   | 14.2   | 829260   | 45.4   | 170740    | 55943   | 82887    |
| 2  | 747936   | 31.2   | 918404   | 14.2   | 829532   | 45.4   | 170468    | 55968   | 82871    |
| 3  | 748123   | 31.1   | 918318   | 14.3   | 829805   | 45.4   | 170195    | 55992   | 82855    |
| 4  | 748310   | 31.1   | 918233   | 14.2   | 830077   | 45.4   | 169923    | 56016   | 82839    |
| 5  | 748497   | 31.1   | 918147   | 14.2   | 830349   | 45.3   | 169651    | 56040   | 82822    |
| 6  | 748683   | 31.1   | 918062   | 14.2   | 830621   | 45.3   | 169379    | 56064   | 82806    |
| 7  | 748870   | 31.1   | 917976   | 14.2   | 830893   | 45.3   | 169107    | 56088   | 82790    |
| 8  | 749056   | 31.0   | 917891   | 14.3   | 831165   | 45.3   | 168835    | 56112   | 82773    |
| 9  | 749243   | 31.0   | 917805   | 14.3   | 831437   | 45.3   | 168563    | 56136   | 82757    |
| 10 | 749426   | 31.0   | 917719   | 14.3   | 831709   | 45.3   | 168291    | 56160   | 82741    |
| 11 | 9.749615 |        | 9.917634 |        | 9.831981 |        | 10.168019 | 56184   | 82724    |
| 12 | 749801   | 31.0   | 917548   | 14.3   | 832253   | 45.3   | 167747    | 56208   | 82708    |
| 13 | 749987   | 30.9   | 917462   | 14.3   | 832525   | 45.3   | 167475    | 56232   | 82692    |
| 14 | 750172   | 30.9   | 917376   | 14.3   | 832796   | 45.3   | 167204    | 56256   | 82675    |
| 15 | 750358   | 30.9   | 917290   | 14.3   | 833068   | 45.2   | 166932    | 56280   | 82659    |
| 16 | 750543   | 30.9   | 917204   | 14.3   | 833339   | 45.2   | 166661    | 56305   | 82643    |
| 17 | 750729   | 30.9   | 917118   | 14.4   | 833611   | 45.2   | 166389    | 56329   | 82626    |
| 18 | 750914   | 30.8   | 917032   | 14.4   | 833882   | 45.2   | 166118    | 56353   | 82610    |
| 19 | 751099   | 30.8   | 916946   | 14.4   | 834154   | 45.2   | 165846    | 56377   | 82593    |
| 20 | 751284   | 30.8   | 916859   | 14.4   | 834425   | 45.2   | 165575    | 56401   | 82577    |
| 21 | 9.751469 |        | 9.916773 |        | 9.834696 |        | 10.165304 | 56425   | 82561    |
| 22 | 751654   | 30.8   | 916687   | 14.4   | 834967   | 45.2   | 165303    | 56449   | 82544    |
| 23 | 751839   | 30.8   | 916600   | 14.4   | 835238   | 45.2   | 164762    | 56473   | 82528    |
| 24 | 752023   | 30.7   | 916514   | 14.4   | 835509   | 45.2   | 164491    | 56497   | 82511    |
| 25 | 752208   | 30.7   | 916427   | 14.4   | 835780   | 45.1   | 164220    | 56521   | 82495    |
| 26 | 752392   | 30.7   | 916341   | 14.4   | 836051   | 45.1   | 163949    | 56545   | 82478    |
| 27 | 752576   | 30.7   | 916254   | 14.4   | 836322   | 45.1   | 163678    | 56569   | 82462    |
| 28 | 752760   | 30.7   | 916167   | 14.5   | 836593   | 45.1   | 163407    | 56593   | 82446    |
| 29 | 752944   | 30.6   | 916081   | 14.5   | 836864   | 45.1   | 163136    | 56617   | 82429    |
| 30 | 753128   | 30.6   | 915994   | 14.5   | 837134   | 45.1   | 162866    | 56641   | 82413    |
| 31 | 9.753312 |        | 9.915907 |        | 9.837405 |        | 10.162595 | 56665   | 82396    |
| 32 | 753495   | 30.6   | 915820   | 14.5   | 837675   | 45.1   | 162325    | 56689   | 82380    |
| 33 | 753679   | 30.6   | 915733   | 14.5   | 837946   | 45.1   | 162054    | 56713   | 82363    |
| 34 | 753862   | 30.5   | 915646   | 14.5   | 838216   | 45.1   | 161784    | 56736   | 82347    |
| 35 | 754046   | 30.5   | 915559   | 14.5   | 838487   | 45.0   | 161513    | 56760   | 82330    |
| 36 | 754229   | 30.5   | 915472   | 14.5   | 838757   | 45.0   | 161243    | 56784   | 82314    |
| 37 | 754412   | 30.5   | 915385   | 14.5   | 839027   | 45.0   | 160973    | 56808   | 82297    |
| 38 | 754595   | 30.5   | 915297   | 14.5   | 839297   | 45.0   | 160703    | 56832   | 82281    |
| 39 | 754778   | 30.4   | 915210   | 14.5   | 839568   | 45.0   | 160432    | 56856   | 82264    |
| 40 | 754960   | 30.4   | 915123   | 14.6   | 839838   | 45.0   | 160162    | 56880   | 82248    |
| 41 | 9.755143 |        | 9.915035 |        | 9.840108 |        | 10.159892 | 56904   | 82231    |
| 42 | 755326   | 30.4   | 914948   | 14.6   | 840378   | 45.0   | 159622    | 56928   | 82214    |
| 43 | 755508   | 30.4   | 914860   | 14.6   | 840647   | 45.0   | 159353    | 56952   | 82198    |
| 44 | 755690   | 30.4   | 914773   | 14.6   | 840917   | 44.9   | 159083    | 56976   | 82181    |
| 45 | 755872   | 30.3   | 914685   | 14.6   | 841187   | 44.9   | 158813    | 57000   | 82165    |
| 46 | 756054   | 30.3   | 914598   | 14.6   | 841457   | 44.9   | 158543    | 57024   | 82148    |
| 47 | 756236   | 30.3   | 914510   | 14.6   | 841726   | 44.9   | 158274    | 57047   | 82132    |
| 48 | 756418   | 30.3   | 914422   | 14.6   | 841996   | 44.9   | 158004    | 57071   | 82115    |
| 49 | 756600   | 30.3   | 914334   | 14.6   | 842266   | 44.9   | 157734    | 57095   | 82098    |
| 50 | 756782   | 30.2   | 914246   | 14.7   | 842535   | 44.9   | 157465    | 57119   | 82082    |
| 51 | 9.756963 |        | 9.914158 |        | 9.842805 |        | 10.157195 | 57143   | 82065    |
| 52 | 757144   | 30.2   | 914070   | 14.7   | 843074   | 44.9   | 156926    | 57167   | 82048    |
| 53 | 757326   | 30.2   | 913982   | 14.7   | 843343   | 44.9   | 156657    | 57191   | 82032    |
| 54 | 757507   | 30.2   | 913894   | 14.7   | 843612   | 44.9   | 156388    | 57215   | 82015    |
| 55 | 757688   | 30.1   | 913806   | 14.7   | 843882   | 44.8   | 156118    | 57238   | 81999    |
| 56 | 757869   | 30.1   | 913718   | 14.7   | 844151   | 44.8   | 155849    | 57262   | 81982    |
| 57 | 758050   | 30.1   | 913630   | 14.7   | 844420   | 44.8   | 155580    | 57286   | 81965    |
| 58 | 758230   | 30.1   | 913541   | 14.7   | 844689   | 44.8   | 155311    | 57310   | 81949    |
| 59 | 758411   | 30.1   | 913453   | 14.7   | 844958   | 44.8   | 155042    | 57334   | 81932    |
| 60 | 758591   | 30.1   | 913365   | 14.7   | 845227   | 44.8   | 154773    | 57358   | 81915    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos. | N. sine. |

55 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.758591 | 30.1   | 9.913365 | 14.7   | 9.845227 | 44.8   | 10.154773 | 57358    | 81915    |
| 1  | 758772   | 30.0   | 913276   | 14.7   | 845496   | 44.8   | 154504    | 57381    | 81899    |
| 2  | 758952   | 30.0   | 913187   | 14.7   | 845764   | 44.8   | 154236    | 57405    | 81882    |
| 3  | 759132   | 30.0   | 913099   | 14.8   | 846033   | 44.8   | 153967    | 57429    | 81865    |
| 4  | 759312   | 30.0   | 913010   | 14.8   | 846302   | 44.8   | 153698    | 57453    | 81848    |
| 5  | 759492   | 30.0   | 912922   | 14.8   | 846570   | 44.8   | 153430    | 57477    | 81832    |
| 6  | 759672   | 29.9   | 912833   | 14.8   | 846839   | 44.7   | 153161    | 57501    | 81815    |
| 7  | 759852   | 29.9   | 912744   | 14.8   | 847107   | 44.7   | 152893    | 57524    | 81798    |
| 8  | 760031   | 29.9   | 912655   | 14.8   | 847376   | 44.7   | 152624    | 57548    | 81782    |
| 9  | 760211   | 29.9   | 912566   | 14.8   | 847644   | 44.7   | 152356    | 57572    | 81765    |
| 10 | 760390   | 29.9   | 912477   | 14.8   | 847913   | 44.7   | 152087    | 57596    | 81748    |
| 11 | 9.760569 | 29.8   | 9.912388 | 14.8   | 9.848181 | 44.7   | 10.151819 | 57619    | 81731    |
| 12 | 760748   | 29.8   | 912299   | 14.9   | 848449   | 44.7   | 151561    | 57643    | 81714    |
| 13 | 760927   | 29.8   | 912210   | 14.9   | 848717   | 44.7   | 151283    | 57667    | 81698    |
| 14 | 761106   | 29.8   | 912121   | 14.9   | 848986   | 44.7   | 151014    | 57691    | 81681    |
| 15 | 761285   | 29.8   | 912031   | 14.9   | 849254   | 44.7   | 150746    | 57715    | 81664    |
| 16 | 761464   | 29.8   | 911942   | 14.9   | 849522   | 44.7   | 150478    | 57738    | 81647    |
| 17 | 761642   | 29.7   | 911853   | 14.9   | 849790   | 44.6   | 150210    | 57762    | 81631    |
| 18 | 761821   | 29.7   | 911763   | 14.9   | 850058   | 44.6   | 149942    | 57786    | 81614    |
| 19 | 761999   | 29.7   | 911674   | 14.9   | 850325   | 44.6   | 149675    | 57810    | 81597    |
| 20 | 762177   | 29.7   | 911584   | 14.9   | 850593   | 44.6   | 149407    | 57833    | 81580    |
| 21 | 9.762356 | 29.6   | 9.911495 | 14.9   | 9.850861 | 44.6   | 10.149139 | 57857    | 81563    |
| 22 | 762534   | 29.6   | 911405   | 14.9   | 851129   | 44.6   | 148871    | 57881    | 81546    |
| 23 | 762712   | 29.6   | 911315   | 14.9   | 851396   | 44.6   | 148604    | 57904    | 81530    |
| 24 | 762889   | 29.6   | 911226   | 15.0   | 851664   | 44.6   | 148336    | 57928    | 81513    |
| 25 | 763067   | 29.6   | 911136   | 15.0   | 851931   | 44.6   | 148069    | 57952    | 81496    |
| 26 | 763245   | 29.6   | 911046   | 15.0   | 852199   | 44.6   | 147801    | 57976    | 81479    |
| 27 | 763422   | 29.6   | 910956   | 15.0   | 852466   | 44.6   | 147534    | 57999    | 81462    |
| 28 | 763600   | 29.6   | 910866   | 15.0   | 852733   | 44.6   | 147267    | 58023    | 81445    |
| 29 | 763777   | 29.5   | 910776   | 15.0   | 853001   | 44.5   | 146999    | 58047    | 81428    |
| 30 | 763954   | 29.5   | 910686   | 15.0   | 853268   | 44.5   | 146732    | 58070    | 81412    |
| 31 | 9.764131 | 29.5   | 9.910596 | 15.0   | 9.853535 | 44.5   | 10.146465 | 58094    | 81395    |
| 32 | 764308   | 29.5   | 910506   | 15.0   | 853802   | 44.5   | 146198    | 58118    | 81378    |
| 33 | 764485   | 29.4   | 910415   | 15.0   | 854069   | 44.5   | 145931    | 58141    | 81361    |
| 34 | 764662   | 29.4   | 910325   | 15.1   | 854336   | 44.5   | 145664    | 58165    | 81344    |
| 35 | 764838   | 29.4   | 910235   | 15.1   | 854603   | 44.5   | 145397    | 58189    | 81327    |
| 36 | 765015   | 29.4   | 910144   | 15.1   | 854870   | 44.5   | 145130    | 58212    | 81310    |
| 37 | 765191   | 29.4   | 910054   | 15.1   | 855137   | 44.5   | 144863    | 58236    | 81293    |
| 38 | 765367   | 29.4   | 909963   | 15.1   | 855404   | 44.5   | 144596    | 58260    | 81276    |
| 39 | 765544   | 29.3   | 909873   | 15.1   | 855671   | 44.4   | 144329    | 58283    | 81259    |
| 40 | 765720   | 29.3   | 909782   | 15.1   | 855938   | 44.4   | 144062    | 58307    | 81242    |
| 41 | 9.765896 | 29.3   | 9.909691 | 15.1   | 9.856204 | 44.4   | 10.143796 | 58330    | 81225    |
| 42 | 766072   | 29.3   | 909601   | 15.1   | 856471   | 44.4   | 143529    | 58354    | 81208    |
| 43 | 766247   | 29.3   | 909510   | 15.1   | 856737   | 44.4   | 143263    | 58378    | 81191    |
| 44 | 766423   | 29.3   | 909419   | 15.1   | 857004   | 44.4   | 142996    | 58401    | 81174    |
| 45 | 766598   | 29.2   | 909328   | 15.2   | 857270   | 44.4   | 142730    | 58425    | 81157    |
| 46 | 766774   | 29.2   | 909237   | 15.2   | 857537   | 44.4   | 142463    | 58449    | 81140    |
| 47 | 766949   | 29.2   | 909146   | 15.2   | 857803   | 44.4   | 142197    | 58472    | 81123    |
| 48 | 767124   | 29.2   | 909055   | 15.2   | 858069   | 44.4   | 141931    | 58496    | 81106    |
| 49 | 767300   | 29.2   | 908964   | 15.2   | 858336   | 44.4   | 141664    | 58519    | 81089    |
| 50 | 767475   | 29.2   | 908873   | 15.2   | 858602   | 44.4   | 141398    | 58543    | 81072    |
| 51 | 9.767649 | 29.1   | 9.908781 | 15.2   | 9.858868 | 44.3   | 10.141132 | 58567    | 81055    |
| 52 | 767824   | 29.1   | 908690   | 15.2   | 859134   | 44.3   | 140866    | 58590    | 81038    |
| 53 | 767999   | 29.1   | 908599   | 15.2   | 859400   | 44.3   | 140600    | 58614    | 81021    |
| 54 | 768173   | 29.1   | 908507   | 15.2   | 859666   | 44.3   | 140334    | 58637    | 81004    |
| 55 | 768348   | 29.0   | 908416   | 15.3   | 859932   | 44.3   | 140068    | 58661    | 80987    |
| 56 | 768522   | 29.0   | 908324   | 15.3   | 860198   | 44.3   | 139802    | 58684    | 80970    |
| 57 | 768697   | 29.0   | 908233   | 15.3   | 860464   | 44.3   | 139536    | 58708    | 80953    |
| 58 | 768871   | 29.0   | 908141   | 15.3   | 860730   | 44.3   | 139270    | 58731    | 80936    |
| 59 | 769045   | 29.0   | 908049   | 15.3   | 860995   | 44.3   | 139005    | 58755    | 80919    |
| 60 | 769219   | 29.0   | 907958   | 15.3   | 861261   | 44.3   | 138739    | 58779    | 80902    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (36°) Natural Sines.

57

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.769219 |        | 9.907958 |        | 9.861261 |        | 10.138739 | 58779    | 80902    | 60 |
| 1  | 769393   | 29.0   | 907866   | 15.3   | 861527   | 44.3   | 138473    | 58802    | 80885    | 59 |
| 2  | 769566   | 28.9   | 907774   | 15.3   | 861792   | 44.3   | 138208    | 58826    | 80867    | 58 |
| 3  | 769740   | 28.9   | 907682   | 15.3   | 862058   | 44.2   | 137942    | 58849    | 80850    | 57 |
| 4  | 769913   | 28.9   | 907590   | 15.3   | 862323   | 44.2   | 137677    | 58873    | 80833    | 56 |
| 5  | 770087   | 28.9   | 907498   | 15.3   | 862589   | 44.2   | 137411    | 58896    | 80816    | 55 |
| 6  | 770260   | 28.8   | 907406   | 15.3   | 862854   | 44.2   | 137146    | 58920    | 80799    | 54 |
| 7  | 770433   | 28.8   | 907314   | 15.3   | 863119   | 44.2   | 136881    | 58943    | 80782    | 53 |
| 8  | 770606   | 28.8   | 907222   | 15.4   | 863385   | 44.2   | 136615    | 58967    | 80765    | 52 |
| 9  | 770779   | 28.8   | 907129   | 15.4   | 863650   | 44.2   | 136350    | 58990    | 80748    | 51 |
| 10 | 770952   | 28.8   | 907037   | 15.4   | 863915   | 44.2   | 136085    | 59014    | 80730    | 50 |
| 11 | 9.771125 |        | 9.906945 |        | 9.864180 |        | 10.135820 | 59037    | 80713    | 49 |
| 12 | 771298   | 28.8   | 906852   | 15.4   | 864445   | 44.2   | 135555    | 59061    | 80696    | 48 |
| 13 | 771470   | 28.7   | 906760   | 15.4   | 864710   | 44.2   | 135290    | 59084    | 80679    | 47 |
| 14 | 771643   | 28.7   | 906667   | 15.4   | 864975   | 44.2   | 135025    | 59108    | 80662    | 46 |
| 15 | 771815   | 28.7   | 906575   | 15.4   | 865240   | 44.1   | 134760    | 59131    | 80644    | 45 |
| 16 | 771987   | 28.7   | 906482   | 15.4   | 865505   | 44.1   | 134495    | 59154    | 80627    | 44 |
| 17 | 772159   | 23.7   | 906389   | 15.5   | 865770   | 44.1   | 134230    | 59178    | 80610    | 43 |
| 18 | 772331   | 28.6   | 906296   | 15.5   | 866035   | 44.1   | 133965    | 59201    | 80593    | 42 |
| 19 | 772503   | 28.6   | 906204   | 15.5   | 866300   | 44.1   | 133700    | 59225    | 80576    | 41 |
| 20 | 772675   | 28.6   | 906111   | 15.5   | 866564   | 44.1   | 133436    | 59248    | 80558    | 40 |
| 21 | 9.772847 |        | 9.906018 |        | 9.866829 |        | 10.133171 | 59272    | 80541    | 39 |
| 22 | 773018   | 28.6   | 905925   | 15.5   | 867094   | 44.1   | 132906    | 59295    | 80524    | 38 |
| 23 | 773190   | 28.6   | 905832   | 15.5   | 867358   | 44.1   | 132642    | 59318    | 80507    | 37 |
| 24 | 773361   | 28.5   | 905739   | 15.5   | 867623   | 44.1   | 132377    | 59342    | 80489    | 36 |
| 25 | 773533   | 28.5   | 905645   | 15.5   | 867887   | 44.1   | 132113    | 59365    | 80472    | 35 |
| 26 | 773704   | 28.5   | 905552   | 15.5   | 868152   | 44.0   | 131848    | 59389    | 80455    | 34 |
| 27 | 773875   | 28.5   | 905459   | 15.5   | 868416   | 44.0   | 131584    | 59412    | 80438    | 33 |
| 28 | 774046   | 28.5   | 905366   | 15.6   | 868680   | 44.0   | 131320    | 59436    | 80422    | 32 |
| 29 | 774217   | 28.5   | 905272   | 15.6   | 868945   | 44.0   | 131055    | 59459    | 80403    | 31 |
| 30 | 774388   | 28.4   | 905179   | 15.6   | 869209   | 44.0   | 130791    | 59482    | 80386    | 30 |
| 31 | 9.774558 |        | 9.905085 |        | 9.869473 |        | 10.130527 | 59506    | 80368    | 29 |
| 32 | 774729   | 28.4   | 904992   | 15.6   | 869737   | 44.0   | 130263    | 59529    | 80351    | 28 |
| 33 | 774899   | 28.4   | 904898   | 15.6   | 870001   | 44.0   | 129999    | 59552    | 80334    | 27 |
| 34 | 775070   | 28.4   | 904804   | 15.6   | 870265   | 44.0   | 129735    | 59576    | 80316    | 26 |
| 35 | 775240   | 28.4   | 904711   | 15.6   | 870529   | 44.0   | 129471    | 59599    | 80299    | 25 |
| 36 | 775410   | 28.3   | 904617   | 15.6   | 870793   | 44.0   | 129207    | 59622    | 80282    | 24 |
| 37 | 775580   | 28.3   | 904523   | 15.6   | 871057   | 44.0   | 128943    | 59646    | 80264    | 23 |
| 38 | 775750   | 28.3   | 904429   | 15.7   | 871321   | 44.0   | 128679    | 59669    | 80247    | 22 |
| 39 | 775920   | 28.3   | 904335   | 15.7   | 871585   | 44.0   | 128415    | 59693    | 80230    | 21 |
| 40 | 776090   | 28.3   | 904241   | 15.7   | 871849   | 43.9   | 128151    | 59716    | 80212    | 20 |
| 41 | 9.776259 |        | 9.904147 |        | 9.872112 |        | 10.127888 | 59739    | 80195    | 19 |
| 42 | 776429   | 28.2   | 904053   | 15.7   | 872376   | 43.9   | 127624    | 59763    | 80178    | 18 |
| 43 | 776598   | 28.2   | 903959   | 15.7   | 872640   | 43.9   | 127360    | 59786    | 80160    | 17 |
| 44 | 776768   | 28.2   | 903864   | 15.7   | 872903   | 43.9   | 127097    | 59809    | 80143    | 16 |
| 45 | 776937   | 28.2   | 903770   | 15.7   | 873167   | 43.9   | 126833    | 59832    | 80125    | 15 |
| 46 | 777106   | 28.2   | 903676   | 15.7   | 873430   | 43.9   | 126570    | 59856    | 80108    | 14 |
| 47 | 777275   | 28.1   | 903581   | 15.7   | 873694   | 43.9   | 126306    | 59879    | 80091    | 13 |
| 48 | 777444   | 28.1   | 903487   | 15.7   | 873957   | 43.9   | 126043    | 59902    | 80073    | 12 |
| 49 | 777613   | 28.1   | 903392   | 15.8   | 874220   | 43.9   | 125780    | 59926    | 80056    | 11 |
| 50 | 777781   | 28.1   | 903298   | 15.8   | 874484   | 43.9   | 125516    | 59949    | 80038    | 10 |
| 51 | 9.777950 |        | 9.903202 |        | 9.874747 |        | 10.125253 | 59972    | 80021    | 9  |
| 52 | 778119   | 28.1   | 903108   | 15.8   | 875010   | 43.9   | 124990    | 59995    | 80003    | 8  |
| 53 | 778287   | 28.0   | 903014   | 15.8   | 875273   | 43.8   | 124727    | 60019    | 79986    | 7  |
| 54 | 778455   | 28.0   | 902919   | 15.8   | 875536   | 43.8   | 124464    | 60042    | 79968    | 6  |
| 55 | 778624   | 28.0   | 902824   | 15.8   | 875800   | 43.8   | 124200    | 60065    | 79951    | 5  |
| 56 | 778792   | 28.0   | 902729   | 15.8   | 876063   | 43.8   | 123937    | 60089    | 79934    | 4  |
| 57 | 778960   | 28.0   | 902634   | 15.8   | 876326   | 43.8   | 123674    | 60112    | 79916    | 3  |
| 58 | 779128   | 28.0   | 902539   | 15.9   | 876589   | 43.8   | 123411    | 60135    | 79899    | 2  |
| 59 | 779295   | 27.9   | 902444   | 15.9   | 876851   | 43.8   | 123149    | 60158    | 79881    | 1  |
| 60 | 779463   |        | 902349   |        | 877114   |        | 122886    | 60182    | 79864    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

53 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.779463 | 27.9   | 9.902349 | 15.9   | 9.877114 | 43.8   | 10.122886 | 60182    | 79864    | 60 |
| 1  | 779631   | 27.9   | 902253   | 15.9   | 877377   | 43.8   | 122623    | 60205    | 79846    | 59 |
| 2  | 779798   | 27.9   | 902158   | 15.9   | 877640   | 43.8   | 122360    | 60228    | 79829    | 58 |
| 3  | 779966   | 27.9   | 902063   | 15.9   | 877903   | 43.8   | 122097    | 60251    | 79811    | 57 |
| 4  | 780133   | 27.9   | 901967   | 15.9   | 878165   | 43.8   | 121835    | 60274    | 79793    | 56 |
| 5  | 780300   | 27.9   | 901872   | 15.9   | 878428   | 43.8   | 121572    | 60298    | 79776    | 55 |
| 6  | 780467   | 27.8   | 901776   | 15.9   | 878691   | 43.8   | 121309    | 60321    | 79758    | 54 |
| 7  | 780634   | 27.8   | 901681   | 15.9   | 878953   | 43.7   | 121047    | 60344    | 79741    | 53 |
| 8  | 780801   | 27.8   | 901585   | 15.9   | 879216   | 43.7   | 120784    | 60367    | 79723    | 52 |
| 9  | 780968   | 27.8   | 901490   | 15.9   | 879478   | 43.7   | 120522    | 60390    | 79706    | 51 |
| 10 | 781134   | 27.8   | 901394   | 16.0   | 879741   | 43.7   | 120259    | 60414    | 79688    | 50 |
| 11 | 9.781301 | 27.8   | 9.901298 | 16.0   | 9.880003 | 43.7   | 10.119997 | 60437    | 79671    | 49 |
| 12 | 781468   | 27.7   | 901202   | 16.0   | 880265   | 43.7   | 119735    | 60460    | 79658    | 48 |
| 13 | 781634   | 27.7   | 901106   | 16.0   | 880528   | 43.7   | 119472    | 60483    | 79635    | 47 |
| 14 | 781800   | 27.7   | 901010   | 16.0   | 880790   | 43.7   | 119210    | 60506    | 79618    | 46 |
| 15 | 781966   | 27.7   | 900914   | 16.0   | 881052   | 43.7   | 118948    | 60529    | 79600    | 45 |
| 16 | 782132   | 27.7   | 900818   | 16.0   | 881314   | 43.7   | 118686    | 60553    | 79583    | 44 |
| 17 | 782298   | 27.6   | 900722   | 16.0   | 881576   | 43.7   | 118424    | 60576    | 79565    | 43 |
| 18 | 782464   | 27.6   | 900626   | 16.0   | 881839   | 43.7   | 118161    | 60599    | 79547    | 42 |
| 19 | 782630   | 27.6   | 900529   | 16.0   | 882101   | 43.7   | 117899    | 60622    | 79530    | 41 |
| 20 | 782796   | 27.6   | 900433   | 16.1   | 882363   | 43.6   | 117637    | 60645    | 79512    | 40 |
| 21 | 9.782961 | 27.6   | 9.900337 | 16.1   | 9.882625 | 43.6   | 10.117375 | 60668    | 79494    | 39 |
| 22 | 783127   | 27.6   | 900242   | 16.1   | 882887   | 43.6   | 117113    | 60691    | 79477    | 38 |
| 23 | 783292   | 27.5   | 900144   | 16.1   | 883148   | 43.6   | 116852    | 60714    | 79459    | 37 |
| 24 | 783458   | 27.5   | 900047   | 16.1   | 883410   | 43.6   | 116590    | 60738    | 79441    | 36 |
| 25 | 783623   | 27.5   | 899951   | 16.1   | 883672   | 43.6   | 116328    | 60761    | 79424    | 35 |
| 26 | 783788   | 27.5   | 899854   | 16.1   | 883934   | 43.6   | 116066    | 60784    | 79406    | 34 |
| 27 | 783953   | 27.5   | 899757   | 16.1   | 884196   | 43.6   | 115804    | 60807    | 79388    | 33 |
| 28 | 784118   | 27.5   | 899660   | 16.1   | 884457   | 43.6   | 115543    | 60830    | 79371    | 32 |
| 29 | 784282   | 27.5   | 899564   | 16.1   | 884719   | 43.6   | 115281    | 60853    | 79353    | 31 |
| 30 | 784447   | 27.4   | 899467   | 16.1   | 884980   | 43.6   | 115020    | 60876    | 79335    | 30 |
| 31 | 9.784612 | 27.4   | 9.899370 | 16.2   | 9.885242 | 43.6   | 10.114758 | 60899    | 79318    | 29 |
| 32 | 784776   | 27.4   | 899273   | 16.2   | 885503   | 43.6   | 114497    | 60922    | 79300    | 28 |
| 33 | 784941   | 27.4   | 899176   | 16.2   | 885765   | 43.6   | 114235    | 60945    | 79282    | 27 |
| 34 | 785105   | 27.4   | 899078   | 16.2   | 886026   | 43.6   | 113974    | 60968    | 79264    | 26 |
| 35 | 785269   | 27.3   | 898981   | 16.2   | 886288   | 43.6   | 113712    | 60991    | 79247    | 25 |
| 36 | 785433   | 27.3   | 898884   | 16.2   | 886549   | 43.5   | 113451    | 61015    | 79229    | 24 |
| 37 | 785597   | 27.3   | 898787   | 16.2   | 886810   | 43.5   | 113190    | 61038    | 79211    | 23 |
| 38 | 785761   | 27.3   | 898689   | 16.2   | 887072   | 43.5   | 112928    | 61061    | 79193    | 22 |
| 39 | 785925   | 27.3   | 898592   | 16.2   | 887333   | 43.5   | 112667    | 61084    | 79176    | 21 |
| 40 | 786089   | 27.3   | 898494   | 16.2   | 887594   | 43.5   | 112406    | 61107    | 79158    | 20 |
| 41 | 9.786252 | 27.2   | 9.898397 | 16.3   | 9.887855 | 43.5   | 10.112145 | 61130    | 79140    | 19 |
| 42 | 786416   | 27.2   | 898299   | 16.3   | 888116   | 43.5   | 111884    | 61153    | 79122    | 18 |
| 43 | 786579   | 27.2   | 898202   | 16.3   | 888377   | 43.5   | 111623    | 61176    | 79105    | 17 |
| 44 | 786742   | 27.2   | 898104   | 16.3   | 888639   | 43.5   | 111361    | 61199    | 79087    | 16 |
| 45 | 786906   | 27.2   | 898006   | 16.3   | 888900   | 43.5   | 111100    | 61223    | 79069    | 15 |
| 46 | 787069   | 27.2   | 897908   | 16.3   | 889160   | 43.5   | 110840    | 61245    | 79051    | 14 |
| 47 | 787232   | 27.1   | 897810   | 16.3   | 889421   | 43.5   | 110579    | 61268    | 79033    | 13 |
| 48 | 787395   | 27.1   | 897712   | 16.3   | 889682   | 43.5   | 110318    | 61291    | 79016    | 12 |
| 49 | 787557   | 27.1   | 897614   | 16.3   | 889943   | 43.5   | 110057    | 61314    | 78998    | 11 |
| 50 | 787720   | 27.1   | 897516   | 16.3   | 890204   | 43.4   | 109796    | 61337    | 78980    | 10 |
| 51 | 9.787883 | 27.1   | 9.897418 | 16.4   | 9.890465 | 43.4   | 10.109535 | 61360    | 78962    | 9  |
| 52 | 788045   | 27.1   | 897320   | 16.4   | 890725   | 43.4   | 109275    | 61383    | 78944    | 8  |
| 53 | 788208   | 27.1   | 897222   | 16.4   | 890986   | 43.4   | 109014    | 61406    | 78926    | 7  |
| 54 | 788370   | 27.0   | 897123   | 16.4   | 891247   | 43.4   | 108753    | 61429    | 78908    | 6  |
| 55 | 788532   | 27.0   | 897025   | 16.4   | 891507   | 43.4   | 108493    | 61451    | 78891    | 5  |
| 56 | 788694   | 27.0   | 896926   | 16.4   | 891768   | 43.4   | 108232    | 61474    | 78873    | 4  |
| 57 | 788856   | 27.0   | 896828   | 16.4   | 892028   | 43.4   | 107972    | 61497    | 78855    | 3  |
| 58 | 789018   | 27.0   | 896729   | 16.4   | 892289   | 43.4   | 107711    | 61520    | 78837    | 2  |
| 59 | 789180   | 27.0   | 896631   | 16.4   | 892549   | 43.4   | 107451    | 61543    | 78819    | 1  |
| 60 | 789342   | 27.0   | 896532   | 16.4   | 892810   | 43.4   | 107190    | 61566    | 78801    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (38°) Natural Sines.

59

|    | Sine.   | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|---------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 789342  | 26.9   | 9.896532 | 16.4   | 9.892810 | 43.4   | 10.107190 | 61566    | 78801    |
| 1  | 789504  | 26.9   | 896433   | 16.5   | 893070   | 43.4   | 106930    | 61589    | 78783    |
| 2  | 789665  | 26.9   | 896335   | 16.5   | 893331   | 43.4   | 106669    | 61612    | 78765    |
| 3  | 789827  | 26.9   | 896236   | 16.5   | 893591   | 43.4   | 106409    | 61635    | 78747    |
| 4  | 789988  | 26.9   | 896137   | 16.5   | 893851   | 43.4   | 106149    | 61658    | 78729    |
| 5  | 790149  | 26.9   | 896038   | 16.5   | 894111   | 43.4   | 105889    | 61681    | 78711    |
| 6  | 790310  | 26.9   | 895939   | 16.5   | 894371   | 43.4   | 105629    | 61704    | 78694    |
| 7  | 790471  | 26.8   | 895840   | 16.5   | 894632   | 43.4   | 105368    | 61726    | 78676    |
| 8  | 790632  | 26.8   | 895741   | 16.5   | 894892   | 43.3   | 105108    | 61749    | 78658    |
| 9  | 790793  | 26.8   | 895641   | 16.5   | 895152   | 43.3   | 104848    | 61772    | 78640    |
| 10 | 790954  | 26.8   | 895542   | 16.5   | 895412   | 43.3   | 104588    | 61795    | 78622    |
| 11 | 791115  | 26.8   | 9.895443 | 16.5   | 9.895672 | 43.3   | 10.104328 | 61818    | 78604    |
| 12 | 791276  | 26.7   | 895343   | 16.6   | 895932   | 43.3   | 104068    | 61841    | 78586    |
| 13 | 791436  | 26.7   | 895244   | 16.6   | 896192   | 43.3   | 103808    | 61864    | 78568    |
| 14 | 791596  | 26.7   | 895145   | 16.6   | 896452   | 43.3   | 103548    | 61887    | 78550    |
| 15 | 791757  | 26.7   | 895045   | 16.6   | 896712   | 43.3   | 103288    | 61909    | 78532    |
| 16 | 791917  | 26.7   | 894945   | 16.6   | 896971   | 43.3   | 103029    | 61932    | 78514    |
| 17 | 792077  | 26.7   | 894846   | 16.6   | 897231   | 43.3   | 102769    | 61955    | 78496    |
| 18 | 792237  | 26.6   | 894746   | 16.6   | 897491   | 43.3   | 102509    | 61978    | 78478    |
| 19 | 792397  | 26.6   | 894646   | 16.6   | 897751   | 43.3   | 102249    | 62001    | 78460    |
| 20 | 792557  | 26.6   | 894546   | 16.6   | 898010   | 43.3   | 101990    | 62024    | 78442    |
| 21 | 792716  | 26.6   | 9.894446 | 16.6   | 9.898270 | 43.3   | 10.101730 | 62046    | 78424    |
| 22 | 792876  | 26.6   | 894346   | 16.7   | 898530   | 43.3   | 101470    | 62069    | 78405    |
| 23 | 793035  | 26.6   | 894246   | 16.7   | 898789   | 43.3   | 101211    | 62092    | 78387    |
| 24 | 793195  | 26.5   | 894146   | 16.7   | 899049   | 43.2   | 100951    | 62115    | 78369    |
| 25 | 793354  | 26.5   | 894046   | 16.7   | 899308   | 43.2   | 100692    | 62138    | 78351    |
| 26 | 793514  | 26.5   | 893946   | 16.7   | 899568   | 43.2   | 100432    | 62160    | 78333    |
| 27 | 793673  | 26.5   | 893846   | 16.7   | 899827   | 43.2   | 100173    | 62183    | 78315    |
| 28 | 793832  | 26.5   | 893745   | 16.7   | 900086   | 43.2   | 999914    | 62206    | 78297    |
| 29 | 793991  | 26.5   | 893645   | 16.7   | 900346   | 43.2   | 999654    | 62229    | 78279    |
| 30 | 794150  | 26.4   | 893544   | 16.7   | 900605   | 43.2   | 999395    | 62251    | 78261    |
| 31 | 794308  | 26.4   | 9.893444 | 16.8   | 9.900864 | 43.2   | 10.099136 | 62274    | 78243    |
| 32 | 794467  | 26.4   | 893343   | 16.8   | 901124   | 43.2   | 998876    | 62297    | 78225    |
| 33 | 794626  | 26.4   | 893243   | 16.8   | 901383   | 43.2   | 998617    | 62320    | 78206    |
| 34 | 794784  | 26.4   | 893142   | 16.8   | 901642   | 43.2   | 998358    | 62342    | 78188    |
| 35 | 794942  | 26.4   | 893041   | 16.8   | 901901   | 43.2   | 998099    | 62365    | 78170    |
| 36 | 795101  | 26.4   | 892940   | 16.8   | 902160   | 43.2   | 997840    | 62388    | 78152    |
| 37 | 795259  | 26.3   | 892839   | 16.8   | 902419   | 43.2   | 997581    | 62411    | 78134    |
| 38 | 795417  | 26.3   | 892739   | 16.8   | 902679   | 43.2   | 997321    | 62433    | 78116    |
| 39 | 795575  | 26.3   | 892638   | 16.8   | 902938   | 43.2   | 997062    | 62456    | 78098    |
| 40 | 795733  | 26.3   | 892536   | 16.8   | 903197   | 43.1   | 996803    | 62479    | 78079    |
| 41 | 795891  | 26.3   | 9.892435 | 16.9   | 9.903455 | 43.1   | 10.096545 | 62502    | 78061    |
| 42 | 796049  | 26.3   | 892334   | 16.9   | 903714   | 43.1   | 996286    | 62524    | 78043    |
| 43 | 796206  | 26.3   | 892233   | 16.9   | 903973   | 43.1   | 996027    | 62547    | 78025    |
| 44 | 796364  | 26.2   | 892132   | 16.9   | 904232   | 43.1   | 995768    | 62570    | 78007    |
| 45 | 796521  | 26.2   | 892030   | 16.9   | 904491   | 43.1   | 995509    | 62592    | 77988    |
| 46 | 796679  | 26.2   | 891929   | 16.9   | 904750   | 43.1   | 995250    | 62615    | 77970    |
| 47 | 796836  | 26.2   | 891827   | 16.9   | 905008   | 43.1   | 994992    | 62638    | 77952    |
| 48 | 796993  | 26.2   | 891726   | 16.9   | 905267   | 43.1   | 994733    | 62660    | 77934    |
| 49 | 797150  | 26.1   | 891624   | 16.9   | 905526   | 43.1   | 994474    | 62683    | 77916    |
| 50 | 797307  | 26.1   | 891523   | 16.9   | 905784   | 43.1   | 994216    | 62706    | 77897    |
| 51 | 797464  | 26.1   | 9.891421 | 17.0   | 9.906043 | 43.1   | 10.093957 | 62728    | 77879    |
| 52 | 797621  | 26.1   | 891319   | 17.0   | 906302   | 43.1   | 993698    | 62751    | 77861    |
| 53 | 797777  | 26.1   | 891217   | 17.0   | 906560   | 43.1   | 993440    | 62774    | 77843    |
| 54 | 797934  | 26.1   | 891115   | 17.0   | 906819   | 43.1   | 993181    | 62796    | 77824    |
| 55 | 798091  | 26.1   | 891013   | 17.0   | 907077   | 43.1   | 992923    | 62819    | 77806    |
| 56 | 798247  | 26.1   | 890911   | 17.0   | 907336   | 43.1   | 992664    | 62842    | 77788    |
| 57 | 798403  | 26.1   | 890809   | 17.0   | 907594   | 43.1   | 992406    | 62864    | 77769    |
| 58 | 798560  | 26.0   | 890707   | 17.0   | 907852   | 43.1   | 992148    | 62887    | 77751    |
| 59 | 798716  | 26.0   | 890605   | 17.0   | 908111   | 43.0   | 991889    | 62909    | 77733    |
| 60 | 798872  | 26.0   | 890503   | 17.0   | 908369   | 43.0   | 991631    | 62932    | 77715    |
|    | Cosine. |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

51 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.798772 | 26.0   | 9.890503 | 17.0   | 9.908369 | 43.0   | 10.091631 | 62932    | 77715    |
| 1  | 799028   | 26.0   | 890400   | 17.1   | 908628   | 43.0   | 091372    | 62955    | 77696    |
| 2  | 799184   | 26.0   | 890298   | 17.1   | 908886   | 43.0   | 091114    | 62977    | 77678    |
| 3  | 799339   | 25.9   | 890195   | 17.1   | 909144   | 43.0   | 090856    | 63000    | 77660    |
| 4  | 799495   | 25.9   | 890093   | 17.1   | 909402   | 43.0   | 090598    | 63022    | 77641    |
| 5  | 799651   | 25.9   | 889990   | 17.1   | 909660   | 43.0   | 090340    | 63045    | 77623    |
| 6  | 799806   | 25.9   | 889888   | 17.1   | 909918   | 43.0   | 090082    | 63068    | 77605    |
| 7  | 799962   | 25.9   | 889785   | 17.1   | 910177   | 43.0   | 089823    | 63090    | 77586    |
| 8  | 800117   | 25.9   | 889682   | 17.1   | 910435   | 43.0   | 089565    | 63113    | 77568    |
| 9  | 800272   | 25.8   | 889579   | 17.1   | 910693   | 43.0   | 089307    | 63135    | 77550    |
| 10 | 800427   | 25.8   | 889477   | 17.1   | 910951   | 43.0   | 089049    | 63158    | 77531    |
| 11 | 9.800582 | 25.8   | 9.889374 | 17.2   | 9.911209 | 43.0   | 10.088791 | 93180    | 77513    |
| 12 | 800737   | 25.8   | 889271   | 17.2   | 911467   | 43.0   | 088553    | 63203    | 77494    |
| 13 | 800892   | 25.8   | 889168   | 17.2   | 911724   | 43.0   | 088276    | 63225    | 77476    |
| 14 | 801047   | 25.8   | 889064   | 17.2   | 911982   | 43.0   | 088018    | 63248    | 77458    |
| 15 | 801201   | 25.8   | 888961   | 17.2   | 912240   | 43.0   | 087760    | 63271    | 77439    |
| 16 | 801356   | 25.7   | 888858   | 17.2   | 912498   | 43.0   | 087502    | 63293    | 77421    |
| 17 | 801511   | 25.7   | 888755   | 17.2   | 912756   | 43.0   | 087244    | 63316    | 77402    |
| 18 | 801665   | 25.7   | 888651   | 17.2   | 913014   | 42.9   | 086986    | 63338    | 77384    |
| 19 | 801819   | 25.7   | 888548   | 17.2   | 913271   | 42.9   | 086729    | 63361    | 77366    |
| 20 | 801973   | 25.7   | 888444   | 17.3   | 913529   | 42.9   | 086471    | 63383    | 77347    |
| 21 | 9.802128 | 25.7   | 9.888341 | 17.3   | 9.913787 | 42.9   | 10.086213 | 63406    | 77329    |
| 22 | 802282   | 25.6   | 888237   | 17.3   | 914044   | 42.9   | 085956    | 63428    | 77310    |
| 23 | 802436   | 25.6   | 888134   | 17.3   | 914302   | 42.9   | 085698    | 63451    | 77292    |
| 24 | 802589   | 25.6   | 888030   | 17.3   | 914560   | 42.9   | 085440    | 63473    | 77273    |
| 25 | 802743   | 25.6   | 887926   | 17.3   | 914817   | 42.9   | 085183    | 63496    | 77255    |
| 26 | 802897   | 25.6   | 887822   | 17.3   | 915075   | 42.9   | 084925    | 63518    | 77236    |
| 27 | 803050   | 25.6   | 887718   | 17.3   | 915332   | 42.9   | 084668    | 63540    | 77218    |
| 28 | 803204   | 25.6   | 887614   | 17.3   | 915590   | 42.9   | 084410    | 63563    | 77199    |
| 29 | 803357   | 25.5   | 887510   | 17.3   | 915847   | 42.9   | 084153    | 63585    | 77181    |
| 30 | 803511   | 25.5   | 887406   | 17.4   | 916104   | 42.9   | 083896    | 63608    | 77162    |
| 31 | 9.803664 | 25.5   | 9.887302 | 17.4   | 9.916362 | 42.9   | 10.083638 | 63630    | 77144    |
| 32 | 803817   | 25.5   | 887198   | 17.4   | 916619   | 42.9   | 083381    | 63653    | 77125    |
| 33 | 803970   | 25.5   | 887093   | 17.4   | 916877   | 42.9   | 083123    | 63675    | 77107    |
| 34 | 804123   | 25.5   | 886989   | 17.4   | 917134   | 42.9   | 082866    | 63698    | 77088    |
| 35 | 804276   | 25.4   | 886885   | 17.4   | 917391   | 42.9   | 082609    | 63720    | 77070    |
| 36 | 804428   | 25.4   | 886780   | 17.4   | 917648   | 42.9   | 082352    | 63742    | 77051    |
| 37 | 804581   | 25.4   | 886676   | 17.4   | 917905   | 42.9   | 082095    | 63765    | 77033    |
| 38 | 804734   | 25.4   | 886571   | 17.4   | 918163   | 42.8   | 081837    | 63787    | 77014    |
| 39 | 804886   | 25.4   | 886466   | 17.4   | 918420   | 42.8   | 081580    | 63810    | 76996    |
| 40 | 805039   | 25.4   | 886362   | 17.5   | 918677   | 42.8   | 081323    | 63832    | 76977    |
| 41 | 9.805191 | 25.4   | 9.886257 | 17.5   | 9.918934 | 42.8   | 10.081066 | 63854    | 76959    |
| 42 | 805343   | 25.3   | 886152   | 17.5   | 919191   | 42.8   | 080809    | 63877    | 76940    |
| 43 | 805495   | 25.3   | 886047   | 17.5   | 919448   | 42.8   | 080552    | 63899    | 76921    |
| 44 | 805647   | 25.3   | 885942   | 17.5   | 919705   | 42.8   | 080295    | 63922    | 76903    |
| 45 | 805799   | 25.3   | 885837   | 17.5   | 919962   | 42.8   | 080038    | 63944    | 76884    |
| 46 | 805951   | 25.3   | 885732   | 17.5   | 920219   | 42.8   | 079781    | 63966    | 76866    |
| 47 | 806103   | 25.3   | 885627   | 17.5   | 920476   | 42.8   | 079524    | 63989    | 76847    |
| 48 | 806254   | 25.3   | 885522   | 17.5   | 920733   | 42.8   | 079267    | 64011    | 76828    |
| 49 | 806406   | 25.2   | 885416   | 17.5   | 920990   | 42.8   | 079010    | 64033    | 76810    |
| 50 | 806557   | 25.2   | 885311   | 17.6   | 921247   | 42.8   | 078753    | 64056    | 76791    |
| 51 | 9.806709 | 25.2   | 9.885205 | 17.6   | 9.921503 | 42.8   | 10.078497 | 64078    | 76772    |
| 52 | 806860   | 25.2   | 885100   | 17.6   | 921760   | 42.8   | 078240    | 64100    | 76754    |
| 53 | 807011   | 25.2   | 884994   | 17.6   | 922017   | 42.8   | 077983    | 64123    | 76735    |
| 54 | 807163   | 25.2   | 884889   | 17.6   | 922274   | 42.8   | 077726    | 64146    | 76717    |
| 55 | 807314   | 25.2   | 884783   | 17.6   | 922530   | 42.8   | 077470    | 64169    | 76698    |
| 56 | 807465   | 25.1   | 884677   | 17.6   | 922787   | 42.8   | 077213    | 64190    | 76679    |
| 57 | 807615   | 25.1   | 884572   | 17.6   | 923044   | 42.8   | 076956    | 64212    | 76661    |
| 58 | 807766   | 25.1   | 884466   | 17.6   | 923300   | 42.8   | 076700    | 64234    | 76642    |
| 59 | 807917   | 25.1   | 884360   | 17.6   | 923557   | 42.7   | 076443    | 64256    | 76623    |
| 60 | 808067   |        | 884254   |        | 923813   |        | 076187    | 64279    | 76604    |
|    | Cosine.  |        | Sine.    |        |          |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (40°) Natural Sines.

61

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 9.808067 | 25.1   | 9.884254 | 17.7   | 9.923813 | 42.7   | 10.076187 | 64279    | 76604    | 60 |
| 1  | 808218   | 25.1   | 884148   | 17.7   | 924070   | 42.7   | 075930    | 64301    | 76586    | 59 |
| 2  | 808368   | 25.1   | 884042   | 17.7   | 924327   | 42.7   | 075673    | 64323    | 76567    | 58 |
| 3  | 808519   | 25.0   | 883936   | 17.7   | 924583   | 42.7   | 075417    | 64346    | 76548    | 57 |
| 4  | 808669   | 25.0   | 883829   | 17.7   | 924840   | 42.7   | 075160    | 64368    | 76530    | 56 |
| 5  | 808819   | 25.0   | 883723   | 17.7   | 925096   | 42.7   | 074904    | 64390    | 76511    | 55 |
| 6  | 808969   | 25.0   | 883617   | 17.7   | 925352   | 42.7   | 074648    | 64412    | 76492    | 54 |
| 7  | 809119   | 25.0   | 883510   | 17.7   | 925609   | 42.7   | 074391    | 64435    | 76473    | 53 |
| 8  | 809269   | 25.0   | 883404   | 17.7   | 925865   | 42.7   | 074135    | 64457    | 76455    | 52 |
| 9  | 809419   | 24.9   | 883297   | 17.8   | 926122   | 42.7   | 073878    | 64479    | 76436    | 51 |
| 10 | 809569   | 24.9   | 883191   | 17.8   | 926378   | 42.7   | 073622    | 64501    | 76417    | 50 |
| 11 | 9.809718 | 24.9   | 9.883084 | 17.8   | 9.926634 | 42.7   | 10.073366 | 64524    | 76398    | 49 |
| 12 | 809868   | 24.9   | 882977   | 17.8   | 926890   | 42.7   | 073110    | 64546    | 76380    | 48 |
| 13 | 810017   | 24.9   | 882871   | 17.8   | 927147   | 42.7   | 072853    | 64568    | 76361    | 47 |
| 14 | 810167   | 24.9   | 882764   | 17.8   | 927403   | 42.7   | 072597    | 64590    | 76342    | 46 |
| 15 | 810316   | 24.8   | 882657   | 17.8   | 927659   | 42.7   | 072341    | 64612    | 76323    | 45 |
| 16 | 810465   | 24.8   | 882550   | 17.8   | 927915   | 42.7   | 072085    | 64635    | 76304    | 44 |
| 17 | 810614   | 24.8   | 882443   | 17.8   | 928171   | 42.7   | 071829    | 64657    | 76286    | 43 |
| 18 | 810763   | 24.8   | 882336   | 17.9   | 928427   | 42.7   | 071573    | 64679    | 76267    | 42 |
| 19 | 810912   | 24.8   | 882229   | 17.9   | 928683   | 42.7   | 071317    | 64701    | 76248    | 41 |
| 20 | 811061   | 24.8   | 882121   | 17.9   | 928940   | 42.7   | 071060    | 64723    | 76229    | 40 |
| 21 | 9.811210 | 24.8   | 9.882014 | 17.9   | 9.929196 | 42.7   | 10.070804 | 64746    | 76210    | 39 |
| 22 | 811358   | 24.7   | 881907   | 17.9   | 929452   | 42.7   | 070548    | 64768    | 76192    | 38 |
| 23 | 811507   | 24.7   | 881799   | 17.9   | 929708   | 42.7   | 070292    | 64790    | 76173    | 37 |
| 24 | 811655   | 24.7   | 881692   | 17.9   | 929964   | 42.6   | 070036    | 64812    | 76154    | 36 |
| 25 | 811804   | 24.7   | 881584   | 17.9   | 930220   | 42.6   | 069780    | 64834    | 76135    | 35 |
| 26 | 811952   | 24.7   | 881477   | 17.9   | 930475   | 42.6   | 069525    | 64856    | 76116    | 34 |
| 27 | 812100   | 24.7   | 881369   | 17.9   | 930731   | 42.6   | 069269    | 64878    | 76097    | 33 |
| 28 | 812248   | 24.7   | 881261   | 17.9   | 930987   | 42.6   | 069013    | 64901    | 76078    | 32 |
| 29 | 812396   | 24.6   | 881153   | 18.0   | 931243   | 42.6   | 068757    | 64923    | 76059    | 31 |
| 30 | 812544   | 24.6   | 881046   | 18.0   | 931499   | 42.6   | 068501    | 64945    | 76041    | 30 |
| 31 | 9.812692 | 24.6   | 9.880938 | 18.0   | 9.931755 | 42.6   | 10.068245 | 64967    | 76022    | 29 |
| 32 | 812840   | 24.6   | 880830   | 18.0   | 932010   | 42.6   | 067990    | 64989    | 76003    | 28 |
| 33 | 812988   | 24.6   | 880722   | 18.0   | 932266   | 42.6   | 067734    | 65011    | 75984    | 27 |
| 34 | 813135   | 24.6   | 880613   | 18.0   | 932522   | 42.6   | 067478    | 65033    | 75965    | 26 |
| 35 | 813283   | 24.6   | 880505   | 18.0   | 932778   | 42.6   | 067222    | 65055    | 75946    | 25 |
| 36 | 813430   | 24.6   | 880397   | 18.0   | 933033   | 42.6   | 066967    | 65077    | 75927    | 24 |
| 37 | 813578   | 24.5   | 880289   | 18.1   | 933289   | 42.6   | 066711    | 65100    | 75908    | 23 |
| 38 | 813725   | 24.5   | 880180   | 18.1   | 933545   | 42.6   | 066455    | 65122    | 75889    | 22 |
| 39 | 813872   | 24.5   | 880072   | 18.1   | 933800   | 42.6   | 066200    | 65144    | 75870    | 21 |
| 40 | 814019   | 24.5   | 879963   | 18.1   | 934056   | 42.6   | 065944    | 65166    | 75851    | 20 |
| 41 | 9.814166 | 24.5   | 9.879655 | 18.1   | 9.934311 | 42.6   | 10.065689 | 65188    | 75832    | 19 |
| 42 | 814313   | 24.5   | 879746   | 18.1   | 934567   | 42.6   | 065433    | 65210    | 75813    | 18 |
| 43 | 814460   | 24.4   | 879637   | 18.1   | 934823   | 42.6   | 065177    | 65232    | 75794    | 17 |
| 44 | 814607   | 24.4   | 879529   | 18.1   | 935078   | 42.6   | 064922    | 65254    | 75775    | 16 |
| 45 | 814753   | 24.4   | 879420   | 18.1   | 935333   | 42.6   | 064667    | 65276    | 75756    | 15 |
| 46 | 814900   | 24.4   | 879311   | 18.1   | 935589   | 42.6   | 064411    | 65298    | 75738    | 14 |
| 47 | 815046   | 24.4   | 879202   | 18.2   | 935844   | 42.6   | 064156    | 65320    | 75719    | 13 |
| 48 | 815193   | 24.4   | 879093   | 18.2   | 936100   | 42.6   | 063900    | 65342    | 75700    | 12 |
| 49 | 815339   | 24.4   | 878984   | 18.2   | 936355   | 42.6   | 063645    | 65364    | 75680    | 11 |
| 50 | 815485   | 24.4   | 878875   | 18.2   | 936610   | 42.6   | 063390    | 65386    | 75661    | 10 |
| 51 | 9.815631 | 24.3   | 9.878766 | 18.2   | 9.936866 | 42.5   | 10.063134 | 65408    | 75642    | 9  |
| 52 | 815778   | 24.3   | 878666   | 18.2   | 937121   | 42.5   | 062879    | 65430    | 75623    | 8  |
| 53 | 815924   | 24.3   | 878547   | 18.2   | 937376   | 42.5   | 062624    | 65452    | 75604    | 7  |
| 54 | 816069   | 24.3   | 878438   | 18.2   | 937632   | 42.5   | 062368    | 65474    | 75585    | 6  |
| 55 | 816215   | 24.3   | 878328   | 18.2   | 937887   | 42.5   | 062113    | 65496    | 75566    | 5  |
| 56 | 816361   | 24.3   | 878219   | 18.3   | 938142   | 42.5   | 061858    | 65518    | 75547    | 4  |
| 57 | 816507   | 24.2   | 878109   | 18.3   | 938398   | 42.5   | 061602    | 65540    | 75528    | 3  |
| 58 | 816652   | 24.2   | 877999   | 18.3   | 938653   | 42.5   | 061347    | 65562    | 75509    | 2  |
| 59 | 816798   | 24.2   | 877890   | 18.3   | 938908   | 42.5   | 061092    | 65584    | 75490    | 1  |
| 60 | 816943   | 24.2   | 877780   | 18.3   | 939163   | 42.5   | 060837    | 65606    | 75471    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

49 Degrees.



|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |    |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|----|
| 0  | 816943   | 24.2   | 9.877780 | 18.3   | 9.939163 | 42.5   | 10.060837 | 65606    | 75471    | 60 |
| 1  | 817088   | 24.2   | 877670   | 18.3   | 939418   | 42.5   | 060582    | 65628    | 75452    | 59 |
| 2  | 817233   | 24.2   | 877560   | 18.3   | 939673   | 42.5   | 060327    | 65650    | 75433    | 58 |
| 3  | 817379   | 24.2   | 877450   | 18.3   | 939928   | 42.5   | 060072    | 65672    | 75414    | 57 |
| 4  | 817524   | 24.1   | 877340   | 18.3   | 940183   | 42.5   | 059817    | 65694    | 75395    | 56 |
| 5  | 817668   | 24.1   | 877230   | 18.4   | 940438   | 42.5   | 059562    | 65716    | 75376    | 55 |
| 6  | 817813   | 24.1   | 877120   | 18.4   | 940694   | 42.5   | 059306    | 65738    | 75356    | 54 |
| 7  | 817958   | 24.1   | 877010   | 18.4   | 940949   | 42.5   | 059051    | 65759    | 75337    | 53 |
| 8  | 818103   | 24.1   | 876899   | 18.4   | 941204   | 42.5   | 058796    | 65781    | 75318    | 52 |
| 9  | 818247   | 24.1   | 876789   | 18.4   | 941458   | 42.5   | 058542    | 65803    | 75299    | 51 |
| 10 | 818392   | 24.1   | 876678   | 18.4   | 941714   | 42.5   | 058286    | 65825    | 75280    | 50 |
| 11 | 9.818536 | 24.1   | 9.876568 | 18.4   | 9.941968 | 42.5   | 10.058032 | 65847    | 75261    | 49 |
| 12 | 818681   | 24.0   | 876467   | 18.4   | 942223   | 42.5   | 057777    | 65869    | 75241    | 48 |
| 13 | 818825   | 24.0   | 876347   | 18.4   | 942478   | 42.5   | 057522    | 65891    | 75222    | 47 |
| 14 | 818969   | 24.0   | 876236   | 18.5   | 942733   | 42.5   | 057267    | 65913    | 75203    | 46 |
| 15 | 819113   | 24.0   | 876125   | 18.5   | 942988   | 42.5   | 057012    | 65935    | 75184    | 45 |
| 16 | 819257   | 24.0   | 876014   | 18.5   | 943243   | 42.5   | 056757    | 65956    | 75165    | 44 |
| 17 | 819401   | 24.0   | 875904   | 18.5   | 943498   | 42.5   | 056502    | 65978    | 75146    | 43 |
| 18 | 819545   | 23.9   | 875793   | 18.5   | 943752   | 42.5   | 056248    | 66000    | 75126    | 42 |
| 19 | 819689   | 23.9   | 875682   | 18.5   | 944007   | 42.5   | 055993    | 66022    | 75107    | 41 |
| 20 | 819832   | 23.9   | 875571   | 18.5   | 944262   | 42.5   | 055738    | 66044    | 75088    | 40 |
| 21 | 9.819976 | 23.9   | 9.875459 | 18.5   | 9.944517 | 42.5   | 10.055483 | 66066    | 75069    | 39 |
| 22 | 820120   | 23.9   | 875348   | 18.5   | 944771   | 42.4   | 055229    | 66088    | 75050    | 38 |
| 23 | 820263   | 23.9   | 875237   | 18.5   | 945026   | 42.4   | 054974    | 66109    | 75030    | 37 |
| 24 | 820406   | 23.9   | 875126   | 18.6   | 945281   | 42.4   | 054719    | 66131    | 75011    | 36 |
| 25 | 820550   | 23.8   | 875014   | 18.6   | 945535   | 42.4   | 054465    | 66153    | 74992    | 35 |
| 26 | 820693   | 23.8   | 874903   | 18.6   | 945790   | 42.4   | 054210    | 66175    | 74973    | 34 |
| 27 | 820836   | 23.8   | 874791   | 18.6   | 946045   | 42.4   | 053955    | 66197    | 74953    | 33 |
| 28 | 820979   | 23.8   | 874680   | 18.6   | 946299   | 42.4   | 053701    | 66218    | 74934    | 32 |
| 29 | 821122   | 23.8   | 874568   | 18.6   | 946554   | 42.4   | 053446    | 66240    | 74915    | 31 |
| 30 | 821265   | 23.8   | 874456   | 18.6   | 946808   | 42.4   | 053192    | 66262    | 74896    | 30 |
| 31 | 9.821407 | 23.8   | 9.874344 | 18.6   | 9.947063 | 42.4   | 10.052937 | 66284    | 74876    | 29 |
| 32 | 821550   | 23.8   | 874232   | 18.7   | 947318   | 42.4   | 052682    | 66306    | 74857    | 28 |
| 33 | 821693   | 23.8   | 874121   | 18.7   | 947572   | 42.4   | 052428    | 66327    | 74838    | 27 |
| 34 | 821835   | 23.7   | 874009   | 18.7   | 947826   | 42.4   | 052174    | 66349    | 74818    | 26 |
| 35 | 821977   | 23.7   | 873896   | 18.7   | 948081   | 42.4   | 051919    | 66371    | 74799    | 25 |
| 36 | 822120   | 23.7   | 873784   | 18.7   | 948336   | 42.4   | 051664    | 66393    | 74780    | 24 |
| 37 | 822262   | 23.7   | 873672   | 18.7   | 948590   | 42.4   | 051410    | 66414    | 74760    | 23 |
| 38 | 822404   | 23.7   | 873560   | 18.7   | 948844   | 42.4   | 051156    | 66436    | 74741    | 22 |
| 39 | 822546   | 23.7   | 873448   | 18.7   | 949099   | 42.4   | 050901    | 66458    | 74722    | 21 |
| 40 | 822688   | 23.7   | 873335   | 18.7   | 949353   | 42.4   | 050647    | 66480    | 74703    | 20 |
| 41 | 9.822830 | 23.6   | 9.873223 | 18.7   | 9.949607 | 42.4   | 10.050393 | 66501    | 74683    | 19 |
| 42 | 822972   | 23.6   | 873110   | 18.7   | 949862   | 42.4   | 050138    | 66523    | 74663    | 18 |
| 43 | 823114   | 23.6   | 872998   | 18.8   | 950116   | 42.4   | 049884    | 66545    | 74644    | 17 |
| 44 | 823255   | 23.6   | 872885   | 18.8   | 950370   | 42.4   | 049630    | 66566    | 74625    | 16 |
| 45 | 823397   | 23.6   | 872772   | 18.8   | 950625   | 42.4   | 049375    | 66588    | 74606    | 15 |
| 46 | 823539   | 23.6   | 872659   | 18.8   | 950879   | 42.4   | 049121    | 66610    | 74586    | 14 |
| 47 | 823680   | 23.5   | 872547   | 18.8   | 951133   | 42.4   | 048867    | 66632    | 74567    | 13 |
| 48 | 823821   | 23.5   | 872434   | 18.8   | 951388   | 42.4   | 048612    | 66653    | 74548    | 12 |
| 49 | 823963   | 23.5   | 872321   | 18.8   | 951642   | 42.4   | 048358    | 66675    | 74529    | 11 |
| 50 | 824104   | 23.5   | 872208   | 18.8   | 951896   | 42.4   | 048104    | 66697    | 74509    | 10 |
| 51 | 9.824245 | 23.5   | 9.872095 | 18.9   | 9.952150 | 42.4   | 10.047850 | 66718    | 74489    | 9  |
| 52 | 824386   | 23.5   | 871981   | 18.9   | 952405   | 42.4   | 047595    | 66740    | 74470    | 8  |
| 53 | 824527   | 23.5   | 871868   | 18.9   | 952659   | 42.4   | 047341    | 66762    | 74451    | 7  |
| 54 | 824668   | 23.4   | 871755   | 18.9   | 952913   | 42.4   | 047087    | 66783    | 74431    | 6  |
| 55 | 824808   | 23.4   | 871641   | 18.9   | 953167   | 42.3   | 046833    | 66805    | 74412    | 5  |
| 56 | 824949   | 23.4   | 871528   | 18.9   | 953421   | 42.3   | 046579    | 66827    | 74392    | 4  |
| 57 | 825090   | 23.4   | 871414   | 18.9   | 953675   | 42.3   | 046325    | 66848    | 74373    | 3  |
| 58 | 825230   | 23.4   | 871301   | 18.9   | 953929   | 42.3   | 046071    | 66870    | 74353    | 2  |
| 59 | 825371   | 23.4   | 871187   | 18.9   | 954183   | 42.3   | 045817    | 66891    | 74334    | 1  |
| 60 | 825511   | 23.4   | 871073   | 18.9   | 954437   | 42.3   | 045563    | 66913    | 74314    | 0  |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |    |

TABLE II. Log. Sines and Tangents. (42°) Natural Sines.

63

| °  | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 825511   | 23.4   | 9.871073 | 19.0   | 9.954437 | 42.3   | 10.045563 | 66913    | 74314    |
| 1  | 825651   | 23.3   | 870960   | 19.0   | 954691   | 42.3   | 045309    | 66935    | 74295    |
| 2  | 825791   | 23.3   | 870846   | 19.0   | 954945   | 42.3   | 045055    | 66956    | 74276    |
| 3  | 825931   | 23.3   | 870732   | 19.0   | 955200   | 42.3   | 044800    | 66978    | 74256    |
| 4  | 826071   | 23.3   | 870618   | 19.0   | 955454   | 42.3   | 044546    | 66999    | 74237    |
| 5  | 826211   | 23.3   | 870504   | 19.0   | 955707   | 42.3   | 044293    | 67021    | 74217    |
| 6  | 826351   | 23.3   | 870390   | 19.0   | 955961   | 42.3   | 044039    | 67043    | 74198    |
| 7  | 826491   | 23.3   | 870276   | 19.0   | 956215   | 42.3   | 043785    | 67064    | 74178    |
| 8  | 826631   | 23.3   | 870161   | 19.0   | 956469   | 42.3   | 043531    | 67086    | 74159    |
| 9  | 826770   | 23.2   | 870047   | 19.0   | 956723   | 42.3   | 043277    | 67107    | 74139    |
| 10 | 826910   | 23.2   | 869933   | 19.1   | 956977   | 42.3   | 043023    | 67129    | 74120    |
| 11 | 827049   | 23.2   | 9.869818 | 19.1   | 9.957231 | 42.3   | 10.042769 | 67151    | 74100    |
| 12 | 827189   | 23.2   | 869704   | 19.1   | 957485   | 42.3   | 042515    | 67172    | 74080    |
| 13 | 827328   | 23.2   | 869589   | 19.1   | 957739   | 42.3   | 042261    | 67194    | 74061    |
| 14 | 827467   | 23.2   | 869474   | 19.1   | 957993   | 42.3   | 042007    | 67215    | 74041    |
| 15 | 827606   | 23.2   | 869360   | 19.1   | 958246   | 42.3   | 041754    | 67237    | 74022    |
| 16 | 827745   | 23.2   | 869245   | 19.1   | 958500   | 42.3   | 041500    | 67258    | 74002    |
| 17 | 827884   | 23.1   | 869130   | 19.1   | 958754   | 42.3   | 041246    | 67280    | 73983    |
| 18 | 828023   | 23.1   | 869015   | 19.1   | 959008   | 42.3   | 040992    | 67301    | 73963    |
| 19 | 828162   | 23.1   | 868900   | 19.2   | 959262   | 42.3   | 040738    | 67323    | 73944    |
| 20 | 828301   | 23.1   | 868785   | 19.2   | 959516   | 42.3   | 040484    | 67344    | 73924    |
| 21 | 9.828439 | 23.1   | 9.868670 | 19.2   | 9.959769 | 42.3   | 10.040231 | 67366    | 73904    |
| 22 | 828578   | 23.1   | 868555   | 19.2   | 960023   | 42.3   | 039977    | 67387    | 73885    |
| 23 | 828716   | 23.1   | 868440   | 19.2   | 960277   | 42.3   | 039723    | 67409    | 73865    |
| 24 | 828855   | 23.0   | 868324   | 19.2   | 960531   | 42.3   | 039469    | 67430    | 73846    |
| 25 | 828993   | 23.0   | 868209   | 19.2   | 960784   | 42.3   | 039216    | 67452    | 73826    |
| 26 | 829131   | 23.0   | 868093   | 19.2   | 961038   | 42.3   | 038962    | 67473    | 73806    |
| 27 | 829269   | 23.0   | 867978   | 19.2   | 961291   | 42.3   | 038709    | 67495    | 73787    |
| 28 | 829407   | 23.0   | 867862   | 19.3   | 961545   | 42.3   | 038455    | 67516    | 73767    |
| 29 | 829545   | 23.0   | 867747   | 19.3   | 961799   | 42.3   | 038201    | 67538    | 73747    |
| 30 | 829683   | 23.0   | 867631   | 19.3   | 962052   | 42.3   | 037948    | 67559    | 73728    |
| 31 | 9.829821 | 22.9   | 9.867515 | 19.3   | 9.962306 | 42.3   | 10.037694 | 67580    | 73708    |
| 32 | 829959   | 22.9   | 867399   | 19.3   | 962560   | 42.3   | 037440    | 67602    | 73688    |
| 33 | 830097   | 22.9   | 867283   | 19.3   | 962813   | 42.3   | 037187    | 67623    | 73669    |
| 34 | 830234   | 22.9   | 867167   | 19.3   | 963067   | 42.3   | 036933    | 67645    | 73649    |
| 35 | 830372   | 22.9   | 867051   | 19.3   | 963320   | 42.3   | 036680    | 67666    | 73629    |
| 36 | 830509   | 22.9   | 866935   | 19.3   | 963574   | 42.3   | 036426    | 67688    | 73610    |
| 37 | 830646   | 22.9   | 866819   | 19.4   | 963827   | 42.3   | 036173    | 67709    | 73590    |
| 38 | 830784   | 22.9   | 866703   | 19.4   | 964081   | 42.3   | 035919    | 67730    | 73570    |
| 39 | 830921   | 22.8   | 866586   | 19.4   | 964335   | 42.3   | 035665    | 67752    | 73551    |
| 40 | 831058   | 22.8   | 866470   | 19.4   | 964588   | 42.3   | 035412    | 67773    | 73531    |
| 41 | 9.831195 | 22.8   | 9.866353 | 19.4   | 9.964842 | 42.2   | 10.035158 | 67795    | 73511    |
| 42 | 831332   | 22.8   | 866237   | 19.4   | 965095   | 42.2   | 034905    | 67816    | 73491    |
| 43 | 831469   | 22.8   | 866120   | 19.4   | 965349   | 42.2   | 034651    | 67837    | 73472    |
| 44 | 831606   | 22.8   | 866004   | 19.4   | 965602   | 42.2   | 034398    | 67859    | 73452    |
| 45 | 831742   | 22.8   | 865887   | 19.5   | 965855   | 42.2   | 034145    | 67880    | 73432    |
| 46 | 831879   | 22.8   | 865770   | 19.5   | 966109   | 42.2   | 033891    | 67901    | 73413    |
| 47 | 832015   | 22.7   | 865653   | 19.5   | 966362   | 42.2   | 033638    | 67923    | 73393    |
| 48 | 832152   | 22.7   | 865536   | 19.5   | 966616   | 42.2   | 033384    | 67944    | 73373    |
| 49 | 832288   | 22.7   | 865419   | 19.5   | 966869   | 42.2   | 033131    | 67965    | 73353    |
| 50 | 832425   | 22.7   | 865302   | 19.5   | 967123   | 42.2   | 032877    | 67987    | 73333    |
| 51 | 9.832561 | 22.7   | 9.865185 | 19.5   | 9.967376 | 42.2   | 10.032624 | 68008    | 73314    |
| 52 | 832697   | 22.7   | 865068   | 19.5   | 967629   | 42.2   | 032371    | 68029    | 73294    |
| 53 | 832833   | 22.7   | 864950   | 19.5   | 967883   | 42.2   | 032117    | 68051    | 73274    |
| 54 | 832969   | 22.6   | 864833   | 19.6   | 968136   | 42.2   | 031864    | 68072    | 73254    |
| 55 | 833105   | 22.6   | 864716   | 19.6   | 968389   | 42.2   | 031611    | 68093    | 73234    |
| 56 | 833241   | 22.6   | 864598   | 19.6   | 968643   | 42.2   | 031357    | 68115    | 73215    |
| 57 | 833377   | 22.6   | 864481   | 19.6   | 968896   | 42.2   | 031104    | 68136    | 73195    |
| 58 | 833512   | 22.6   | 864363   | 19.6   | 969149   | 42.2   | 030851    | 68157    | 73175    |
| 59 | 833648   | 22.6   | 864245   | 19.6   | 969403   | 42.2   | 030597    | 68179    | 73155    |
| 60 | 833783   | 22.6   | 864127   | 19.6   | 969656   | 42.2   | 030344    | 68200    | 73135    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

47 Degrees.

|    | Sine.    | D. 10" | Cosine.  | D. 10" | Tang.    | D. 10" | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|----------|--------|-----------|----------|----------|
| 0  | 9.833783 | 22.6   | 9.864127 | 19.6   | 9.969656 | 42.2   | 10.030344 | 68200    | 73135    |
| 1  | 833919   | 22.5   | 864010   | 19.6   | 969909   | 42.2   | 030091    | 68221    | 73116    |
| 2  | 834054   | 22.5   | 863892   | 19.7   | 970162   | 42.2   | 029838    | 68242    | 73096    |
| 3  | 834189   | 22.5   | 863774   | 19.7   | 970416   | 42.2   | 029584    | 68264    | 73076    |
| 4  | 834325   | 22.5   | 863656   | 19.7   | 970669   | 42.2   | 029331    | 68285    | 73056    |
| 5  | 834460   | 22.5   | 863538   | 19.7   | 970922   | 42.2   | 029078    | 68306    | 73036    |
| 6  | 834595   | 22.5   | 863419   | 19.7   | 971175   | 42.2   | 028825    | 68327    | 73016    |
| 7  | 834730   | 22.5   | 863301   | 19.7   | 971429   | 42.2   | 028571    | 68349    | 72996    |
| 8  | 834865   | 22.5   | 863183   | 19.7   | 971682   | 42.2   | 028318    | 68370    | 72976    |
| 9  | 834999   | 22.4   | 863064   | 19.7   | 971935   | 42.2   | 028065    | 68391    | 72957    |
| 10 | 835134   | 22.4   | 862946   | 19.7   | 972188   | 42.2   | 027812    | 68412    | 72937    |
| 11 | 9.835269 | 22.4   | 9.862827 | 19.8   | 9.972441 | 42.2   | 10.027559 | 68434    | 72917    |
| 12 | 835403   | 22.4   | 862709   | 19.8   | 972694   | 42.2   | 027306    | 68455    | 72897    |
| 13 | 835538   | 22.4   | 862590   | 19.8   | 972948   | 42.2   | 027052    | 68476    | 72877    |
| 14 | 835672   | 22.4   | 862471   | 19.8   | 973201   | 42.2   | 026799    | 68497    | 72857    |
| 15 | 835807   | 22.4   | 862353   | 19.8   | 973454   | 42.2   | 026546    | 68518    | 72837    |
| 16 | 835941   | 22.4   | 862234   | 19.8   | 973707   | 42.2   | 026293    | 68539    | 72817    |
| 17 | 836075   | 22.3   | 862115   | 19.8   | 973960   | 42.2   | 026040    | 68561    | 72797    |
| 18 | 836209   | 22.3   | 861996   | 19.8   | 974213   | 42.2   | 025787    | 68582    | 72777    |
| 19 | 836343   | 22.3   | 861877   | 19.8   | 974466   | 42.2   | 025534    | 68603    | 72757    |
| 20 | 836477   | 22.3   | 861758   | 19.9   | 974719   | 42.2   | 025281    | 68624    | 72737    |
| 21 | 9.836611 | 22.3   | 9.861638 | 19.9   | 9.974973 | 42.2   | 10.025027 | 68645    | 72717    |
| 22 | 836745   | 22.3   | 861619   | 19.9   | 975226   | 42.2   | 024774    | 68666    | 72697    |
| 23 | 836878   | 22.3   | 861500   | 19.9   | 975479   | 42.2   | 024521    | 68688    | 72677    |
| 24 | 837012   | 22.2   | 861380   | 19.9   | 975732   | 42.2   | 024268    | 68709    | 72657    |
| 25 | 837146   | 22.2   | 861261   | 19.9   | 975985   | 42.2   | 024015    | 68730    | 72637    |
| 26 | 837279   | 22.2   | 861141   | 19.9   | 976238   | 42.2   | 023762    | 68751    | 72617    |
| 27 | 837412   | 22.2   | 861022   | 19.9   | 976491   | 42.2   | 023509    | 68772    | 72597    |
| 28 | 837546   | 22.2   | 860902   | 19.9   | 976744   | 42.2   | 023256    | 68793    | 72577    |
| 29 | 837679   | 22.2   | 860782   | 19.9   | 976997   | 42.2   | 023003    | 68814    | 72557    |
| 30 | 837812   | 22.2   | 860662   | 20.0   | 977250   | 42.2   | 022750    | 68835    | 72537    |
| 31 | 9.837945 | 22.2   | 9.860442 | 20.0   | 9.977503 | 42.2   | 10.022497 | 68857    | 72517    |
| 32 | 838078   | 22.1   | 860522   | 20.0   | 977756   | 42.2   | 022244    | 68878    | 72497    |
| 33 | 838211   | 22.1   | 860402   | 20.0   | 978009   | 42.2   | 021991    | 68899    | 72477    |
| 34 | 838344   | 22.1   | 860282   | 20.0   | 978262   | 42.2   | 021738    | 68920    | 72457    |
| 35 | 838477   | 22.1   | 859962   | 20.0   | 978515   | 42.2   | 021485    | 68941    | 72437    |
| 36 | 838610   | 22.1   | 859842   | 20.0   | 978768   | 42.2   | 021232    | 68962    | 72417    |
| 37 | 838742   | 22.1   | 859721   | 20.0   | 979021   | 42.2   | 020979    | 68983    | 72397    |
| 38 | 838875   | 22.1   | 859601   | 20.1   | 979274   | 42.2   | 020726    | 69004    | 72377    |
| 39 | 839007   | 22.1   | 859480   | 20.1   | 979527   | 42.2   | 020473    | 69025    | 72357    |
| 40 | 839140   | 22.0   | 859360   | 20.1   | 979780   | 42.2   | 020220    | 69046    | 72337    |
| 41 | 9.839272 | 22.0   | 9.859239 | 20.1   | 9.980033 | 42.2   | 10.019967 | 69067    | 72317    |
| 42 | 839404   | 22.0   | 859119   | 20.1   | 980286   | 42.2   | 019714    | 69088    | 72297    |
| 43 | 839536   | 22.0   | 858998   | 20.1   | 980538   | 42.2   | 019462    | 69109    | 72277    |
| 44 | 839668   | 22.0   | 858877   | 20.1   | 980791   | 42.1   | 019209    | 69130    | 72257    |
| 45 | 839800   | 22.0   | 858756   | 20.2   | 981044   | 42.1   | 018956    | 69151    | 72236    |
| 46 | 839932   | 22.0   | 858635   | 20.2   | 981297   | 42.1   | 018703    | 69172    | 72216    |
| 47 | 840064   | 21.9   | 858514   | 20.2   | 981550   | 42.1   | 018450    | 69193    | 72196    |
| 48 | 840196   | 21.9   | 858393   | 20.2   | 981803   | 42.1   | 018197    | 69214    | 72176    |
| 49 | 840328   | 21.9   | 858272   | 20.2   | 982056   | 42.1   | 017944    | 69235    | 72156    |
| 50 | 840459   | 21.9   | 858151   | 20.2   | 982309   | 42.1   | 017691    | 69256    | 72136    |
| 51 | 9.840591 | 21.9   | 9.858029 | 20.2   | 9.982562 | 42.1   | 10.017438 | 69277    | 72116    |
| 52 | 840722   | 21.9   | 857908   | 20.2   | 982814   | 42.1   | 017186    | 69298    | 72096    |
| 53 | 840854   | 21.9   | 857786   | 20.2   | 983067   | 42.1   | 016933    | 69319    | 72076    |
| 54 | 840985   | 21.9   | 857665   | 20.3   | 983320   | 42.1   | 016680    | 69340    | 72056    |
| 55 | 841116   | 21.9   | 857543   | 20.3   | 983573   | 42.1   | 016427    | 69361    | 72036    |
| 56 | 841247   | 21.8   | 857422   | 20.3   | 983826   | 42.1   | 016174    | 69382    | 72016    |
| 57 | 841378   | 21.8   | 857300   | 20.3   | 984079   | 42.1   | 015921    | 69403    | 71996    |
| 58 | 841509   | 21.8   | 857178   | 20.3   | 984331   | 42.1   | 015669    | 69424    | 71976    |
| 59 | 841640   | 21.8   | 857056   | 20.3   | 984584   | 42.1   | 015416    | 69445    | 71956    |
| 60 | 841771   | 21.8   | 856934   | 20.3   | 984837   | 42.1   | 015163    | 69466    | 71936    |
|    | Cosine.  |        | Sine.    |        | Cotang.  |        | Tang.     | N. cos.  | N. sine. |

TABLE II. Log. Sines and Tangents. (44°) Natural Sines.

65

|    | Sine.    | D. 10' | Cosine.  | D. 10' | Tang.     | D. 10' | Cotang.   | N. sine. | N. cos.  |
|----|----------|--------|----------|--------|-----------|--------|-----------|----------|----------|
| 0  | 9.841771 |        | 9.856934 |        | 9.984837  |        | 10.015163 | 69466    | 71934    |
| 1  | 841902   | 21.8   | 856812   | 20.3   | 985090    | 42.1   | 014910    | 69487    | 71914    |
| 2  | 842033   | 21.8   | 856690   | 20.3   | 985343    | 42.1   | 014657    | 69508    | 71894    |
| 3  | 842163   | 21.7   | 856568   | 20.4   | 985596    | 42.1   | 014404    | 69529    | 71873    |
| 4  | 842294   | 21.7   | 856446   | 20.4   | 985848    | 42.1   | 014152    | 69549    | 71853    |
| 5  | 842424   | 21.7   | 856323   | 20.4   | 986101    | 42.1   | 013899    | 69570    | 71833    |
| 6  | 842555   | 21.7   | 856201   | 20.4   | 986354    | 42.1   | 013646    | 69591    | 71813    |
| 7  | 842685   | 21.7   | 856078   | 20.4   | 986607    | 42.1   | 013393    | 69612    | 71792    |
| 8  | 842815   | 21.7   | 855956   | 20.4   | 986860    | 42.1   | 013140    | 69633    | 71772    |
| 9  | 842946   | 21.7   | 855833   | 20.4   | 987112    | 42.1   | 012888    | 69654    | 71752    |
| 10 | 843076   | 21.7   | 855711   | 20.5   | 987365    | 42.1   | 012635    | 69675    | 71732    |
| 11 | 9.843206 |        | 9.855588 |        | 9.987618  |        | 10.012382 | 69696    | 71711    |
| 12 | 843336   | 21.6   | 855465   | 20.5   | 987871    | 42.1   | 012129    | 69717    | 71691    |
| 13 | 843466   | 21.6   | 855342   | 20.5   | 988123    | 42.1   | 011877    | 69737    | 71671    |
| 14 | 843595   | 21.6   | 855219   | 20.5   | 988376    | 42.1   | 011624    | 69758    | 71650    |
| 15 | 843725   | 21.6   | 855096   | 20.5   | 988629    | 42.1   | 011371    | 69779    | 71630    |
| 16 | 843855   | 21.6   | 854973   | 20.5   | 988882    | 42.1   | 011118    | 69800    | 71610    |
| 17 | 843984   | 21.6   | 854850   | 20.5   | 989134    | 42.1   | 010866    | 69821    | 71590    |
| 18 | 844114   | 21.5   | 854727   | 20.6   | 989387    | 42.1   | 010613    | 69842    | 71569    |
| 19 | 844243   | 21.5   | 854603   | 20.6   | 989640    | 42.1   | 010360    | 69863    | 71549    |
| 20 | 844372   | 21.5   | 854480   | 20.6   | 989893    | 42.1   | 010107    | 69883    | 71529    |
| 21 | 9.844502 |        | 9.854356 |        | 9.990145  |        | 10.009855 | 69904    | 71508    |
| 22 | 844631   | 21.5   | 854353   | 20.6   | 990398    | 42.1   | 009602    | 69925    | 71488    |
| 23 | 844760   | 21.5   | 854109   | 20.6   | 990651    | 42.1   | 009349    | 69946    | 71468    |
| 24 | 844889   | 21.5   | 853986   | 20.6   | 990903    | 42.1   | 009097    | 69966    | 71447    |
| 25 | 845018   | 21.5   | 853862   | 20.6   | 991156    | 42.1   | 008844    | 69987    | 71427    |
| 26 | 845147   | 21.5   | 853738   | 20.6   | 991409    | 42.1   | 008591    | 70008    | 71407    |
| 27 | 845276   | 21.4   | 853614   | 20.7   | 991662    | 42.1   | 008338    | 70029    | 71386    |
| 28 | 845405   | 21.4   | 853490   | 20.7   | 991914    | 42.1   | 008086    | 70049    | 71366    |
| 29 | 845533   | 21.4   | 853366   | 20.7   | 992167    | 42.1   | 007833    | 70070    | 71345    |
| 30 | 845662   | 21.4   | 853242   | 20.7   | 992420    | 42.1   | 007580    | 70091    | 71325    |
| 31 | 9.845790 |        | 9.853118 |        | 9.992672  |        | 10.007328 | 70112    | 71305    |
| 32 | 845919   | 21.4   | 852994   | 20.7   | 992925    | 42.1   | 007075    | 70132    | 71284    |
| 33 | 846047   | 21.4   | 852869   | 20.7   | 993178    | 42.1   | 006822    | 70153    | 71264    |
| 34 | 846175   | 21.4   | 852745   | 20.7   | 993430    | 42.1   | 006570    | 70174    | 71243    |
| 35 | 846304   | 21.4   | 852620   | 20.7   | 993683    | 42.1   | 006317    | 70195    | 71223    |
| 36 | 846432   | 21.3   | 852496   | 20.7   | 993936    | 42.1   | 006064    | 70215    | 71203    |
| 37 | 846560   | 21.3   | 852371   | 20.8   | 994189    | 42.1   | 005811    | 70236    | 71182    |
| 38 | 846688   | 21.3   | 852247   | 20.8   | 994441    | 42.1   | 005559    | 70257    | 71162    |
| 39 | 846816   | 21.3   | 852122   | 20.8   | 994694    | 42.1   | 005306    | 70277    | 71141    |
| 40 | 846944   | 21.3   | 851997   | 20.8   | 994947    | 42.1   | 005053    | 70298    | 71121    |
| 41 | 9.847071 |        | 9.851872 |        | 9.995199  |        | 10.004801 | 70319    | 71100    |
| 42 | 847199   | 21.3   | 851747   | 20.8   | 995452    | 42.1   | 004548    | 70339    | 71080    |
| 43 | 847327   | 21.3   | 851622   | 20.8   | 995705    | 42.1   | 004295    | 70360    | 71059    |
| 44 | 847454   | 21.2   | 851497   | 20.9   | 995957    | 42.1   | 004043    | 70381    | 71039    |
| 45 | 847582   | 21.2   | 851372   | 20.9   | 996210    | 42.1   | 003790    | 70401    | 71019    |
| 46 | 847709   | 21.2   | 851246   | 20.9   | 996463    | 42.1   | 003537    | 70422    | 70998    |
| 47 | 847836   | 21.2   | 851121   | 20.9   | 996715    | 42.1   | 003285    | 70443    | 70978    |
| 48 | 847964   | 21.2   | 850996   | 20.9   | 996968    | 42.1   | 003032    | 70463    | 70957    |
| 49 | 848091   | 21.2   | 850870   | 20.9   | 997221    | 42.1   | 002779    | 70484    | 70937    |
| 50 | 848218   | 21.2   | 850745   | 20.9   | 997473    | 42.1   | 002527    | 70505    | 70916    |
| 51 | 9.848345 |        | 9.850619 |        | 9.997726  |        | 10.002274 | 70525    | 70896    |
| 52 | 848472   | 21.2   | 850493   | 20.9   | 997979    | 42.1   | 002271    | 70546    | 70875    |
| 53 | 848599   | 21.1   | 850368   | 21.0   | 998231    | 42.1   | 002017    | 70567    | 70855    |
| 54 | 848726   | 21.1   | 850242   | 21.0   | 998484    | 42.1   | 001764    | 70587    | 70834    |
| 55 | 848852   | 21.1   | 850116   | 21.0   | 998737    | 42.1   | 001511    | 70608    | 70813    |
| 56 | 848979   | 21.1   | 849990   | 21.0   | 998989    | 42.1   | 001258    | 70628    | 70793    |
| 57 | 849106   | 21.1   | 849864   | 21.0   | 999242    | 42.1   | 001005    | 70649    | 70772    |
| 58 | 849232   | 21.1   | 849738   | 21.0   | 999495    | 42.1   | 000752    | 70670    | 70752    |
| 59 | 849359   | 21.1   | 849611   | 21.0   | 999748    | 42.1   | 000500    | 70690    | 70731    |
| 60 | 849485   | 21.1   | 849485   | 21.0   | 10.000000 |        | 000253    | 70711    | 70711    |
|    | Cosine.  |        | Sine.    |        | Cotang.   |        | Tang.     | N. cos.  | N. sine. |

45 Degrees.

TABLE III.  
LOGARITHMS OF NUMBERS,  
FROM 1 TO 110,  
INCLUDING TWELVE DECIMAL PLACES.

| N. | Log. |     |     |     |     | N. | Log. |     |     |     |     |
|----|------|-----|-----|-----|-----|----|------|-----|-----|-----|-----|
| 1  | 0.   | 000 | 000 | 000 | 000 | 36 | 1.   | 556 | 302 | 500 | 767 |
| 2  | 0.   | 301 | 029 | 995 | 644 | 37 | 1.   | 568 | 201 | 724 | 067 |
| 3  | 0.   | 477 | 121 | 254 | 720 | 38 | 1.   | 579 | 783 | 596 | 617 |
| 4  | 0.   | 602 | 059 | 991 | 328 | 39 | 1.   | 591 | 064 | 607 | 264 |
| 5  | 0.   | 698 | 970 | 004 | 336 | 40 | 1.   | 602 | 059 | 991 | 328 |
| 6  | 0.   | 778 | 151 | 250 | 384 | 41 | 1.   | 612 | 783 | 846 | 720 |
| 7  | 0.   | 845 | 098 | 040 | 014 | 42 | 1.   | 623 | 249 | 290 | 398 |
| 8  | 0.   | 903 | 089 | 986 | 992 | 43 | 1.   | 633 | 468 | 455 | 579 |
| 9  | 0.   | 954 | 242 | 509 | 440 | 44 | 1.   | 643 | 452 | 676 | 486 |
| 10 | 1.   | 000 | 000 | 000 | 000 | 45 | 1.   | 653 | 212 | 513 | 775 |
| 11 | 1.   | 041 | 392 | 685 | 158 | 46 | 1.   | 662 | 757 | 831 | 682 |
| 12 | 1.   | 079 | 181 | 246 | 048 | 47 | 1.   | 672 | 097 | 857 | 936 |
| 13 | 1.   | 113 | 943 | 352 | 309 | 48 | 1.   | 681 | 241 | 237 | 376 |
| 14 | 1.   | 146 | 128 | 035 | 678 | 49 | 1.   | 690 | 196 | 080 | 028 |
| 15 | 1.   | 176 | 091 | 259 | 059 | 50 | 1.   | 698 | 970 | 004 | 336 |
| 16 | 1.   | 204 | 119 | 982 | 656 | 51 | 1.   | 707 | 570 | 176 | 098 |
| 17 | 1.   | 230 | 448 | 921 | 378 | 52 | 1.   | 716 | 003 | 243 | 635 |
| 18 | 1.   | 255 | 272 | 505 | 103 | 53 | 1.   | 724 | 275 | 869 | 601 |
| 19 | 1.   | 278 | 753 | 600 | 953 | 54 | 1.   | 732 | 393 | 759 | 823 |
| 20 | 1.   | 301 | 029 | 995 | 664 | 55 | 1.   | 740 | 362 | 689 | 494 |
| 21 | 1.   | 322 | 219 | 294 | 734 | 56 | 1.   | 748 | 188 | 027 | 006 |
| 22 | 1.   | 342 | 422 | 680 | 822 | 57 | 1.   | 755 | 874 | 855 | 672 |
| 23 | 1.   | 361 | 727 | 836 | 076 | 58 | 1.   | 763 | 427 | 993 | 563 |
| 24 | 1.   | 380 | 211 | 241 | 712 | 59 | 1.   | 770 | 852 | 011 | 642 |
| 25 | 1.   | 397 | 940 | 008 | 672 | 60 | 1.   | 778 | 151 | 250 | 384 |
| 26 | 1.   | 414 | 973 | 347 | 971 | 61 | 1.   | 785 | 329 | 835 | 011 |
| 27 | 1.   | 431 | 363 | 764 | 159 | 62 | 1.   | 792 | 391 | 689 | 492 |
| 28 | 1.   | 447 | 158 | 031 | 342 | 63 | 1.   | 799 | 340 | 549 | 454 |
| 29 | 1.   | 462 | 397 | 997 | 899 | 64 | 1.   | 806 | 179 | 973 | 984 |
| 30 | 1.   | 477 | 121 | 254 | 720 | 65 | 1.   | 812 | 913 | 356 | 643 |
| 31 | 1.   | 491 | 361 | 693 | 834 | 66 | 1.   | 819 | 543 | 935 | 542 |
| 32 | 1.   | 505 | 149 | 978 | 320 | 67 | 1.   | 826 | 074 | 302 | 701 |
| 33 | 1.   | 518 | 513 | 939 | 878 | 68 | 1.   | 832 | 508 | 912 | 706 |
| 34 | 1.   | 531 | 478 | 917 | 042 | 69 | 1.   | 838 | 849 | 090 | 737 |
| 35 | 1.   | 544 | 068 | 044 | 350 | 70 | 1.   | 845 | 098 | 040 | 014 |

## OF NUMBERS.

67

| N. | Log. |     |     |     |     | N.  | Log. |     |     |     |     |
|----|------|-----|-----|-----|-----|-----|------|-----|-----|-----|-----|
| 71 | 1.   | 851 | 258 | 348 | 719 | 91  | 1.   | 959 | 041 | 392 | 321 |
| 72 | 1.   | 857 | 332 | 496 | 431 | 92  | 1.   | 963 | 787 | 827 | 346 |
| 73 | 1.   | 863 | 322 | 860 | 120 | 93  | 1.   | 968 | 482 | 948 | 554 |
| 74 | 1.   | 869 | 231 | 719 | 731 | 94  | 1.   | 973 | 127 | 853 | 600 |
| 75 | 1.   | 875 | 061 | 263 | 392 | 95  | 1.   | 977 | 723 | 605 | 289 |
| 76 | 1.   | 880 | 813 | 592 | 281 | 96  | 1.   | 982 | 271 | 233 | 040 |
| 77 | 1.   | 886 | 490 | 725 | 172 | 97  | 1.   | 986 | 771 | 734 | 266 |
| 78 | 1.   | 892 | 094 | 602 | 690 | 98  | 1.   | 991 | 226 | 075 | 692 |
| 79 | 1.   | 897 | 627 | 091 | 290 | 99  | 1.   | 995 | 635 | 194 | 598 |
| 80 | 1.   | 903 | 089 | 986 | 992 | 100 | 2.   | 000 | 000 | 000 | 000 |
| 81 | 1.   | 908 | 485 | 018 | 879 | 101 | 2.   | 004 | 321 | 373 | 783 |
| 82 | 1.   | 913 | 813 | 852 | 384 | 102 | 2.   | 008 | 600 | 171 | 762 |
| 83 | 1.   | 919 | 078 | 092 | 376 | 103 | 2.   | 012 | 837 | 224 | 705 |
| 84 | 1.   | 924 | 279 | 286 | 062 | 104 | 2.   | 017 | 033 | 339 | 299 |
| 85 | 1.   | 929 | 418 | 925 | 714 | 105 | 2.   | 021 | 189 | 299 | 070 |
| 86 | 1.   | 934 | 498 | 451 | 244 | 106 | 2.   | 025 | 305 | 865 | 265 |
| 87 | 1.   | 939 | 519 | 252 | 619 | 107 | 2.   | 029 | 383 | 777 | 685 |
| 88 | 1.   | 944 | 482 | 672 | 150 | 108 | 2.   | 033 | 423 | 755 | 487 |
| 89 | 1.   | 949 | 390 | 006 | 645 | 109 | 2.   | 037 | 426 | 497 | 941 |
| 90 | 1.   | 954 | 242 | 509 | 439 | 110 | 2.   | 041 | 392 | 685 | 158 |

## LOGARITHMS OF THE PRIME NUMBERS

FROM 110 TO 1129.

INCLUDING TWELVE DECIMAL PLACES.

| N.  | Log. |     |     |     |     | N.  | Log. |     |     |     |     |
|-----|------|-----|-----|-----|-----|-----|------|-----|-----|-----|-----|
| 113 | 2.   | 053 | 078 | 443 | 483 | 197 | 2.   | 294 | 466 | 266 | 162 |
| 127 | 2.   | 103 | 803 | 720 | 956 | 199 | 2.   | 298 | 853 | 076 | 410 |
| 131 | 2.   | 117 | 271 | 295 | 656 | 211 | 2.   | 324 | 282 | 455 | 298 |
| 137 | 2.   | 136 | 720 | 567 | 156 | 223 | 2.   | 348 | 304 | 863 | 222 |
| 139 | 2.   | 143 | 014 | 800 | 254 | 227 | 2.   | 356 | 025 | 857 | 189 |
| 149 | 3.   | 173 | 186 | 268 | 412 | 229 | 2.   | 359 | 835 | 482 | 343 |
| 151 | 2.   | 178 | 976 | 947 | 293 | 233 | 2.   | 367 | 355 | 922 | 471 |
| 157 | 2.   | 195 | 899 | 653 | 409 | 239 | 2.   | 378 | 397 | 902 | 352 |
| 163 | 2.   | 212 | 187 | 604 | 404 | 241 | 2.   | 382 | 017 | 042 | 576 |
| 167 | 2.   | 222 | 716 | 471 | 148 | 251 | 2.   | 399 | 673 | 721 | 509 |
| 173 | 2.   | 238 | 046 | 103 | 129 | 257 | 2.   | 409 | 933 | 123 | 332 |
| 179 | 2.   | 252 | 853 | 030 | 980 | 263 | 2.   | 419 | 955 | 748 | 490 |
| 181 | 2.   | 257 | 678 | 574 | 869 | 269 | 2.   | 429 | 752 | 261 | 993 |
| 191 | 2.   | 281 | 033 | 367 | 248 | 271 | 2.   | 432 | 969 | 290 | 877 |
| 193 | 2.   | 285 | 557 | 309 | 008 | 277 | 2.   | 442 | 479 | 768 | 999 |

| N.  | Log.               | N.  | Log.               |
|-----|--------------------|-----|--------------------|
| 281 | 2. 448 706 319 906 | 601 | 2. 778 874 471 998 |
| 283 | 2. 451 786 435 523 | 607 | 2. 783 188 691 074 |
| 293 | 2. 466 867 523 562 | 613 | 2. 787 460 556 130 |
| 307 | 2. 487 138 375 477 | 617 | 2. 790 285 164 033 |
| 311 | 2. 492 760 389 026 | 619 | 2. 791 690 648 987 |
| 313 | 2. 495 544 337 550 | 631 | 2. 800 029 359 232 |
| 317 | 2. 501 059 267 324 | 641 | 2. 806 858 879 634 |
| 331 | 2. 519 827 993 783 | 648 | 2. 808 210 973 921 |
| 337 | 2. 527 629 883 034 | 647 | 2. 810 904 280 666 |
| 347 | 2. 540 329 475 079 | 653 | 2. 814 912 981 274 |
| 349 | 2. 542 826 426 673 | 659 | 2. 818 885 490 409 |
| 353 | 2. 547 774 138 015 | 661 | 2. 820 201 459 485 |
| 359 | 2. 555 094 447 578 | 673 | 2. 828 015 064 225 |
| 367 | 2. 564 666 064 254 | 677 | 2. 830 588 667 946 |
| 373 | 2. 571 708 831 809 | 683 | 2. 834 420 703 630 |
| 379 | 2. 578 639 209 957 | 691 | 2. 839 477 902 551 |
| 383 | 2. 583 198 773 980 | 701 | 2. 845 718 017 237 |
| 389 | 2. 589 949 601 323 | 709 | 2. 850 646 235 112 |
| 397 | 2. 598 790 506 763 | 719 | 2. 856 728 890 383 |
| 401 | 2. 603 144 372 687 | 727 | 2. 861 534 410 855 |
| 409 | 2. 611 723 298 019 | 733 | 2. 865 103 970 639 |
| 419 | 2. 622 214 022 971 | 739 | 2. 868 643 643 162 |
| 421 | 2. 624 282 095 835 | 743 | 2. 870 988 813 759 |
| 431 | 2. 634 477 268 999 | 751 | 2. 875 639 937 004 |
| 433 | 2. 636 488 015 871 | 757 | 2. 879 095 879 497 |
| 439 | 2. 642 464 520 242 | 761 | 2. 881 384 656 769 |
| 443 | 2. 646 403 726 235 | 769 | 2. 885 926 339 800 |
| 449 | 2. 652 246 388 777 | 773 | 2. 888 179 493 917 |
| 457 | 2. 659 916 200 054 | 787 | 2. 895 974 732 358 |
| 461 | 2. 663 700 925 389 | 797 | 2. 901 458 321 400 |
| 463 | 2. 665 580 991 012 | 809 | 2. 977 948 459 773 |
| 467 | 2. 669 317 881 008 | 811 | 2. 909 020 854 210 |
| 479 | 2. 680 335 513 415 | 821 | 2. 914 343 157 120 |
| 487 | 2. 687 528 961 120 | 823 | 2. 915 399 835 203 |
| 491 | 2. 691 081 487 026 | 827 | 2. 917 505 509 487 |
| 499 | 2. 698 100 545 623 | 829 | 2. 918 554 530 558 |
| 503 | 2. 701 567 985 083 | 839 | 2. 923 761 960 830 |
| 509 | 2. 706 717 782 345 | 853 | 2. 930 949 031 163 |
| 521 | 2. 716 837 623 304 | 857 | 2. 932 980 821 917 |
| 523 | 2. 718 502 688 873 | 859 | 2. 933 993 163 838 |
| 541 | 2. 733 197 265 134 | 863 | 2. 936 010 794 546 |
| 547 | 2. 737 987 326 358 | 877 | 2. 942 999 593 360 |
| 557 | 2. 745 855 195 192 | 881 | 2. 944 975 908 412 |
| 563 | 2. 750 508 395 940 | 883 | 2. 945 960 703 512 |
| 569 | 2. 755 112 178 598 | 887 | 2. 947 923 619 839 |
| 571 | 2. 756 636 108 333 | 907 | 2. 957 607 287 059 |
| 577 | 2. 761 175 813 171 | 911 | 2. 959 518 376 972 |
| 587 | 2. 768 638 004 455 | 919 | 2. 963 315 513 609 |
| 593 | 2. 773 054 693 364 | 929 | 2. 968 015 713 997 |
| 599 | 2. 777 427 303 257 | 937 | 2. 971 739 590 780 |

# OF NUMBERS.

69

| N.   | Log.               | N.   | Log.               |
|------|--------------------|------|--------------------|
| 941  | 2. 973 589 620 234 | 1039 | 3. 016 615 547 558 |
| 947  | 2. 976 349 979 055 | 1049 | 3. 020 775 488 195 |
| 953  | 2. 979 092 900 639 | 1051 | 3. 021 602 716 026 |
| 967  | 2. 985 426 474 084 | 1061 | 3. 025 715 383 898 |
| 971  | 2. 987 219 229 907 | 1063 | 3. 026 533 264 523 |
| 977  | 2. 989 894 559 717 | 1069 | 3. 028 977 705 205 |
| 983  | 2. 992 553 512 733 | 1087 | 3. 036 229 513 712 |
| 991  | 2. 996 073 604 003 | 1091 | 3. 037 824 749 671 |
| 997  | 2. 998 695 158 313 | 1093 | 3. 038 620 157 372 |
| 1009 | 3. 003 891 170 203 | 1097 | 3. 040 206 627 571 |
| 1013 | 3. 005 609 445 427 | 1103 | 3. 042 575 512 437 |
| 1019 | 3. 008 174 244 007 | 1109 | 3. 044 931 546 149 |
| 1021 | 3. 009 025 742 086 | 1117 | 3. 048 053 173 103 |
| 1031 | 3. 013 258 660 430 | 1123 | 3. 050 379 756 239 |
| 1033 | 3. 014 100 321 518 | 1129 | 3. 052 693 942 370 |

It is not necessary to extend this table, as the logarithm of any one of the higher numbers can be readily computed by the following formula, which may be found in any of the standard works on algebra, namely :

$$\text{Log. } (z+1) = \text{log. } z + 0.8685889638 \left( \frac{1}{2z+1} \right)$$

The result will be true to ten decimal places for all numbers over 1000, and true to twelve decimals for all numbers over 2000.

The logarithms of composite numbers can be determined by the combination of logarithms already in the table, and the prime numbers from the formula.

Thus, the number 3083 is a prime number, find its logarithm, true to ten places of decimals.

We first find the logarithm of 3082. By factoring this number, we find that it may be composed by the multiplication of 46 into 67.

$$\text{Log. } 46 \dots\dots\dots 1. \quad 662 \, 757 \, 8316$$

$$\text{Log. } 67 \dots\dots\dots 1. \quad 826 \, 074 \, 3027$$

$$\text{Log. } 3082 \dots\dots\dots 3. \quad 488 \, 832 \, 1343$$

$$\text{Now, } \text{Log. } 3083 = 3.4888321343 + \frac{0.8685889638}{6165}$$

We give a few additional prime numbers :

|      |      |      |      |      |      |
|------|------|------|------|------|------|
| 1151 | 1223 | 1291 | 1373 | 1451 | 1511 |
| 1153 | 1229 | 1297 | 1381 | 1453 | 1523 |
| 1163 | 1231 | 1301 | 1399 | 1459 | 1531 |
| 1171 | 1237 | 1303 | 1409 | 1471 | 1543 |
| 1181 | 1249 | 1307 | 1423 | 1481 | 1549 |
| 1187 | 1259 | 1319 | 1427 | 1483 | 1553 |
| 1193 | 1277 | 1321 | 1429 | 1487 | 1559 |
| 1201 | 1279 | 1327 | 1433 | 1489 | 1567 |
| 1213 | 1283 | 1361 | 1439 | 1493 | 1571 |
| 1217 | 1289 | 1367 | 1447 | 1499 | 1579 |



## AUXILIARY LOGARITHMS.

| N.     | Log.               | N.      | Log.               |
|--------|--------------------|---------|--------------------|
| 1. 009 | 0. 003 891 170 203 | 1. 0009 | 0. 000 390 576 304 |
| 1. 008 | 0. 003 461 527 188 | 1. 0008 | 0. 000 347 233 698 |
| 1. 007 | 0. 003 030 465 635 | 1. 0007 | 0. 000 303 836 798 |
| 1. 006 | 0. 002 597 985 739 | 1. 0006 | 0. 000 260 435 561 |
| 1. 005 | 0. 002 166 071 750 | 1. 0005 | 0. 000 217 099 966 |
| 1. 004 | 0. 001 733 722 804 | 1. 0004 | 0. 000 173 690 053 |
| 1. 003 | 0. 001 300 943 017 | 1. 0003 | 0. 000 130 268 803 |
| 1. 002 | 0. 000 867 721 529 | 1. 0002 | 0. 000 086 850 213 |
| 1. 001 | 0. 000 434 077 479 | 1. 0001 | 0. 000 043 427 277 |

| N.            | Log.                   |
|---------------|------------------------|
| 1. 00009      | 0. 000 039 084 741     |
| 1. 00008      | 0. 000 034 742 166     |
| 1. 00007      | 0. 000 030 399 546     |
| 1. 00006      | 0. 000 026 056 884     |
| 1. 00005      | 0. 000 021 714 178     |
| 1. 00004      | 0. 000 017 371 430     |
| 1. 00003      | 0. 000 013 028 638     |
| 1. 00002      | 0. 000 008 685 802     |
| 1. 00001      | 0. 000 004 342 923 (a) |
| 1. 000001     | 0. 000 000 434 294 (b) |
| 1. 0000001    | 0. 000 000 043 429 (c) |
| 1. 00000001   | 0. 000 000 004 343 (d) |
| 1. 000000001  | 0. 000 000 000 434 (e) |
| 1. 0000000001 | 0. 000 000 000 043 (f) |

Number.

Log.

0. 4342944819 . . . . . —1. 637 784 298

This decimal number is the modulus of our system of logarithms. Its logarithm is very useful in correcting other logarithms, as may be seen in the Chapter on Logarithms.

# TRAVERSE TABLE.

71

| Distance. | $\frac{1}{2}$ Deg.   |       | 1 Deg.  |       | $1\frac{1}{2}$ Deg.  |       | 2 Deg.  |       |
|-----------|----------------------|-------|---------|-------|----------------------|-------|---------|-------|
|           | Lat.                 | Dep.  | Lat.    | Dep.  | Lat.                 | Dep.  | Lat.    | Dep.  |
|           |                      |       |         |       |                      |       |         |       |
| 1         | 1. 00                | 0. 01 | 1. 00   | 0. 02 | 1. 00                | 0. 03 | 1. 00   | 0. 03 |
| 2         | 2. 00                | 0. 02 | 2. 00   | 0. 03 | 2. 00                | 0. 05 | 2. 00   | 0. 07 |
| 3         | 3. 00                | 0. 03 | 3. 00   | 0. 05 | 3. 00                | 0. 08 | 3. 00   | 0. 10 |
| 4         | 4. 00                | 0. 03 | 4. 00   | 0. 07 | 4. 00                | 0. 10 | 4. 00   | 0. 14 |
| 5         | 5. 00                | 0. 04 | 5. 00   | 0. 09 | 5. 00                | 0. 13 | 5. 00   | 0. 17 |
| 6         | 6. 00                | 0. 05 | 6. 90   | 0. 10 | 6. 00                | 0. 16 | 6. 00   | 0. 21 |
| 7         | 7. 00                | 0. 06 | 7. 00   | 0. 12 | 7. 00                | 0. 18 | 7. 00   | 0. 24 |
| 8         | 8. 00                | 0. 07 | 8. 00   | 0. 14 | 8. 00                | 0. 21 | 7. 99   | 0. 28 |
| 9         | 9. 00                | 0. 08 | 9. 00   | 0. 16 | 9. 00                | 0. 24 | 8. 99   | 0. 31 |
| 10        | 10. 00               | 0. 09 | 10. 00  | 0. 17 | 10. 00               | 0. 26 | 9. 99   | 0. 35 |
| 11        | 11. 00               | 0. 10 | 11. 00  | 0. 19 | 11. 00               | 0. 28 | 10. 99  | 0. 38 |
| 12        | 12. 00               | 0. 10 | 12. 00  | 0. 21 | 12. 00               | 0. 31 | 11. 99  | 0. 42 |
| 13        | 13. 00               | 0. 11 | 13. 00  | 0. 23 | 13. 00               | 0. 34 | 12. 99  | 0. 45 |
| 14        | 14. 00               | 0. 12 | 14. 00  | 0. 24 | 14. 00               | 0. 37 | 13. 99  | 0. 49 |
| 15        | 15. 00               | 0. 13 | 15. 00  | 0. 26 | 14. 99               | 0. 39 | 14. 99  | 0. 52 |
| 16        | 16. 00               | 0. 14 | 16. 00  | 0. 28 | 15. 99               | 0. 42 | 15. 99  | 0. 56 |
| 17        | 17. 00               | 0. 15 | 17. 00  | 0. 30 | 16. 99               | 0. 45 | 16. 99  | 0. 59 |
| 18        | 18. 00               | 0. 16 | 18. 00  | 0. 31 | 17. 99               | 0. 47 | 17. 99  | 0. 63 |
| 19        | 19. 00               | 0. 17 | 19. 00  | 0. 33 | 18. 99               | 0. 50 | 18. 99  | 0. 66 |
| 20        | 20. 00               | 0. 17 | 20. 00  | 0. 35 | 19. 99               | 0. 52 | 19. 99  | 0. 70 |
| 21        | 21. 00               | 0. 18 | 21. 00  | 0. 37 | 20. 99               | 0. 55 | 20. 99  | 0. 73 |
| 22        | 22. 00               | 0. 19 | 22. 00  | 0. 38 | 21. 99               | 0. 58 | 21. 99  | 0. 77 |
| 23        | 23. 00               | 0. 20 | 23. 00  | 0. 40 | 22. 99               | 0. 60 | 22. 99  | 0. 80 |
| 24        | 24. 00               | 0. 21 | 24. 00  | 0. 42 | 23. 99               | 0. 63 | 23. 99  | 0. 84 |
| 25        | 25. 00               | 0. 22 | 25. 00  | 0. 44 | 24. 99               | 0. 65 | 24. 98  | 0. 87 |
| 26        | 26. 00               | 0. 23 | 26. 00  | 0. 45 | 25. 99               | 0. 68 | 25. 98  | 0. 91 |
| 27        | 27. 00               | 0. 24 | 27. 00  | 0. 47 | 26. 99               | 0. 71 | 26. 98  | 0. 94 |
| 28        | 28. 00               | 0. 24 | 28. 00  | 0. 49 | 27. 99               | 0. 73 | 27. 98  | 0. 98 |
| 29        | 29. 00               | 0. 25 | 29. 00  | 0. 51 | 28. 99               | 0. 76 | 28. 98  | 1. 01 |
| 30        | 30. 00               | 0. 26 | 30. 00  | 0. 52 | 29. 99               | 0. 79 | 29. 98  | 1. 05 |
| 35        | 35. 00               | 0. 31 | 34. 99  | 0. 61 | 34. 99               | 0. 92 | 34. 98  | 1. 22 |
| 40        | 40. 00               | 0. 35 | 39. 99  | 0. 70 | 39. 99               | 1. 05 | 39. 98  | 1. 40 |
| 45        | 45. 00               | 0. 39 | 44. 90  | 0. 79 | 44. 99               | 1. 18 | 44. 97  | 1. 57 |
| 50        | 50. 00               | 0. 44 | 49. 99  | 0. 87 | 49. 98               | 1. 31 | 49. 97  | 1. 74 |
| 55        | 55. 00               | 0. 48 | 54. 99  | 0. 96 | 54. 98               | 1. 44 | 54. 97  | 1. 92 |
| 60        | 60. 00               | 0. 52 | 59. 90  | 0. 05 | 59. 98               | 1. 57 | 59. 96  | 2. 09 |
| 65        | 65. 00               | 0. 57 | 64. 99  | 1. 13 | 64. 98               | 1. 70 | 64. 96  | 2. 27 |
| 70        | 70. 00               | 0. 61 | 69. 99  | 1. 22 | 69. 98               | 1. 83 | 69. 96  | 2. 44 |
| 75        | 75. 00               | 0. 65 | 74. 99  | 1. 31 | 74. 97               | 1. 96 | 74. 95  | 2. 62 |
| 80        | 80. 00               | 0. 70 | 79. 99  | 1. 40 | 79. 97               | 2. 09 | 79. 95  | 2. 79 |
| 85        | 85. 00               | 0. 74 | 84. 99  | 1. 48 | 84. 97               | 2. 23 | 84. 95  | 2. 97 |
| 90        | 90. 00               | 0. 79 | 89. 99  | 1. 57 | 89. 97               | 2. 36 | 89. 95  | 3. 14 |
| 95        | 90. 00               | 0. 83 | 94. 99  | 1. 66 | 94. 97               | 2. 49 | 94. 94  | 3. 32 |
| 100       | 100. 00              | 0. 87 | 99. 98  | 1. 75 | 99. 97               | 2. 62 | 99. 94  | 3. 49 |
|           | Dep.                 | Lat.  | Dep.    | Lat.  | Dep.                 | Lat.  | Dep.    | Lat.  |
|           | $89\frac{1}{2}$ Deg. |       | 89 Deg. |       | $88\frac{1}{2}$ Deg. |       | 88 Deg. |       |

## TRAVERSE TABLE.

| Distance. | 2½ Deg.  |       | 3 Deg.  |       | 3½ Deg.  |       | 4 Deg.  |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat. °   | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 1. 00    | 0. 04 | 1. 00   | 0. 05 | 1. 00    | 0. 06 | 1. 00   | 0. 07 |
| 2         | 2. 00    | 0. 09 | 2. 00   | 0. 10 | 2. 00    | 0. 12 | 2. 00   | 0. 14 |
| 3         | 3. 00    | 0. 13 | 3. 00   | 0. 16 | 2. 99    | 0. 18 | 2. 99   | 0. 21 |
| 4         | 4. 00    | 0. 17 | 3. 99   | 0. 21 | 3. 99    | 0. 24 | 3. 99   | 0. 28 |
| 5         | 5. 00    | 0. 22 | 4. 99   | 0. 26 | 4. 99    | 0. 31 | 4. 99   | 0. 35 |
| 6         | 5. 99    | 0. 26 | 5. 99   | 0. 31 | 5. 99    | 0. 37 | 5. 99   | 0. 42 |
| 7         | 6. 99    | 0. 31 | 6. 99   | 0. 37 | 6. 99    | 0. 43 | 6. 98   | 0. 49 |
| 8         | 7. 99    | 0. 35 | 7. 99   | 0. 42 | 7. 99    | 0. 49 | 7. 98   | 0. 56 |
| 9         | 8. 99    | 0. 39 | 8. 99   | 0. 47 | 8. 98    | 0. 55 | 8. 98   | 0. 63 |
| 10        | 9. 99    | 0. 44 | 9. 99   | 0. 52 | 9. 98    | 0. 61 | 9. 98   | 0. 70 |
| 11        | 10. 99   | 0. 48 | 10. 98  | 0. 58 | 10. 98   | 0. 67 | 10. 97  | 0. 77 |
| 12        | 11. 99   | 0. 52 | 11. 98  | 0. 63 | 11. 98   | 0. 73 | 11. 97  | 0. 84 |
| 13        | 12. 99   | 0. 57 | 12. 98  | 0. 68 | 12. 99   | 0. 79 | 12. 97  | 0. 91 |
| 14        | 13. 99   | 0. 61 | 13. 98  | 0. 73 | 13. 97   | 0. 85 | 13. 97  | 0. 98 |
| 15        | 14. 99   | 0. 65 | 14. 98  | 0. 79 | 14. 97   | 0. 92 | 14. 96  | 1. 05 |
| 16        | 15. 99   | 0. 70 | 15. 98  | 0. 84 | 15. 97   | 0. 98 | 15. 96  | 1. 12 |
| 17        | 16. 98   | 0. 74 | 16. 98  | 0. 89 | 16. 97   | 1. 04 | 16. 96  | 1. 19 |
| 18        | 17. 98   | 0. 79 | 17. 98  | 0. 94 | 17. 97   | 1. 10 | 17. 96  | 1. 26 |
| 19        | 18. 98   | 0. 83 | 18. 98  | 0. 99 | 18. 96   | 1. 16 | 18. 95  | 1. 33 |
| 20        | 19. 98   | 0. 87 | 19. 97  | 1. 05 | 19. 96   | 1. 22 | 19. 95  | 1. 40 |
| 21        | 20. 98   | 0. 92 | 20. 97  | 1. 10 | 20. 96   | 1. 28 | 20. 95  | 1. 46 |
| 22        | 21. 98   | 0. 96 | 21. 97  | 1. 15 | 21. 96   | 1. 34 | 21. 95  | 1. 53 |
| 23        | 22. 98   | 1. 00 | 22. 97  | 1. 20 | 22. 96   | 1. 40 | 22. 94  | 1. 60 |
| 24        | 23. 98   | 1. 05 | 23. 97  | 1. 26 | 23. 96   | 1. 47 | 23. 94  | 1. 67 |
| 25        | 24. 98   | 1. 09 | 24. 97  | 1. 31 | 24. 95   | 1. 53 | 24. 94  | 1. 74 |
| 26        | 25. 98   | 1. 13 | 25. 96  | 1. 36 | 25. 95   | 1. 59 | 25. 94  | 1. 81 |
| 27        | 26. 97   | 1. 18 | 26. 96  | 1. 41 | 26. 95   | 1. 65 | 26. 93  | 1. 88 |
| 28        | 27. 97   | 1. 22 | 27. 96  | 1. 47 | 27. 95   | 1. 71 | 27. 93  | 1. 95 |
| 29        | 28. 97   | 1. 26 | 28. 96  | 1. 52 | 28. 95   | 1. 77 | 28. 93  | 2. 02 |
| 30        | 29. 97   | 1. 31 | 29. 96  | 1. 57 | 29. 94   | 1. 83 | 29. 93  | 2. 09 |
| 35        | 34. 97   | 1. 53 | 34. 95  | 1. 83 | 34. 93   | 2. 14 | 34. 91  | 2. 44 |
| 40        | 39. 96   | 1. 75 | 39. 95  | 2. 09 | 39. 93   | 2. 44 | 39. 90  | 2. 79 |
| 45        | 44. 96   | 1. 96 | 44. 94  | 2. 36 | 44. 92   | 2. 75 | 44. 89  | 3. 14 |
| 50        | 49. 95   | 2. 18 | 49. 93  | 2. 62 | 49. 91   | 3. 05 | 49. 88  | 3. 49 |
| 55        | 54. 95   | 2. 40 | 54. 92  | 2. 88 | 54. 90   | 3. 36 | 54. 87  | 3. 84 |
| 60        | 59. 94   | 2. 62 | 59. 92  | 3. 14 | 59. 89   | 3. 66 | 59. 83  | 4. 19 |
| 65        | 64. 94   | 2. 84 | 64. 91  | 3. 40 | 64. 88   | 3. 97 | 64. 84  | 4. 53 |
| 70        | 69. 93   | 3. 05 | 69. 90  | 3. 66 | 69. 87   | 4. 27 | 69. 83  | 4. 88 |
| 75        | 74. 93   | 3. 27 | 74. 90  | 3. 93 | 74. 86   | 4. 58 | 74. 82  | 5. 23 |
| 80        | 79. 92   | 3. 49 | 79. 89  | 4. 19 | 79. 85   | 4. 88 | 79. 81  | 5. 58 |
| 85        | 84. 92   | 3. 71 | 84. 88  | 4. 45 | 84. 84   | 5. 19 | 84. 79  | 5. 93 |
| 90        | 89. 91   | 3. 93 | 89. 88  | 4. 71 | 89. 83   | 5. 49 | 89. 78  | 6. 28 |
| 95        | 94. 91   | 4. 14 | 94. 87  | 4. 97 | 94. 82   | 5. 80 | 94. 77  | 6. 63 |
| 100       | 99. 91   | 4. 36 | 99. 86  | 5. 23 | 99. 81   | 6. 10 | 99. 76  | 6. 98 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 87½ Deg. |       | 87 Deg. |       | 86½ Deg. |       | 86 Deg. |       |

# TRAVERSE TABLE.

73

| Distance. | 4½ Deg.  |       | 5 Deg.  |       | 5½ Deg.  |       | 6 Deg.  |        |
|-----------|----------|-------|---------|-------|----------|-------|---------|--------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.   |
| 1         | 1. 00    | 0. 08 | 1. 00   | 0. 09 | 1. 00    | 0. 10 | 0. 99   | 0. 10  |
| 2         | 1. 99    | 0. 16 | 1. 99   | 0. 17 | 1. 99    | 0. 19 | 1. 99   | 0. 21  |
| 3         | 2. 99    | 0. 24 | 2. 99   | 0. 26 | 2. 99    | 0. 29 | 2. 98   | 0. 31  |
| 4         | 3. 99    | 0. 31 | 3. 98   | 0. 35 | 3. 98    | 0. 38 | 3. 98   | 0. 41  |
| 5         | 4. 98    | 0. 39 | 4. 98   | 0. 44 | 4. 98    | 0. 48 | 4. 97   | 0. 52  |
| 6         | 5. 98    | 0. 47 | 5. 98   | 0. 52 | 5. 97    | 0. 58 | 5. 97   | 0. 63  |
| 7         | 6. 98    | 0. 55 | 6. 97   | 0. 61 | 6. 97    | 0. 67 | 6. 96   | 0. 73  |
| 8         | 7. 98    | 0. 63 | 7. 97   | 0. 70 | 7. 96    | 0. 76 | 7. 96   | 0. 84  |
| 9         | 8. 97    | 0. 71 | 8. 97   | 0. 78 | 8. 96    | 0. 86 | 8. 95   | 0. 94  |
| 10        | 9. 97    | 0. 78 | 9. 96   | 0. 87 | 9. 95    | 0. 96 | 9. 95   | 1. 05  |
| 11        | 10. 97   | 0. 86 | 10. 96  | 0. 96 | 10. 95   | 1. 05 | 10. 94  | 1. 15  |
| 12        | 11. 96   | 0. 94 | 11. 95  | 1. 05 | 11. 94   | 1. 15 | 11. 93  | 1. 25  |
| 13        | 12. 96   | 1. 02 | 12. 95  | 1. 13 | 12. 94   | 1. 25 | 12. 93  | 1. 36  |
| 14        | 13. 96   | 1. 10 | 13. 95  | 1. 22 | 13. 94   | 1. 34 | 13. 92  | 1. 46  |
| 15        | 14. 95   | 1. 18 | 14. 94  | 1. 31 | 14. 93   | 1. 44 | 14. 92  | 1. 57  |
| 16        | 15. 95   | 1. 26 | 15. 94  | 1. 39 | 15. 93   | 1. 53 | 15. 91  | 1. 67  |
| 17        | 16. 95   | 1. 33 | 16. 94  | 1. 48 | 16. 92   | 1. 63 | 16. 91  | 1. 78  |
| 18        | 17. 94   | 1. 41 | 17. 93  | 1. 57 | 17. 92   | 1. 73 | 17. 90  | 1. 88  |
| 19        | 18. 94   | 1. 49 | 18. 93  | 1. 66 | 18. 91   | 1. 82 | 18. 90  | 1. 99  |
| 20        | 19. 94   | 1. 57 | 19. 92  | 1. 74 | 19. 91   | 1. 92 | 19. 89  | 2. 09  |
| 21        | 20. 94   | 1. 65 | 20. 92  | 1. 83 | 20. 90   | 2. 01 | 20. 88  | 2. 20  |
| 22        | 21. 93   | 1. 73 | 21. 92  | 1. 92 | 21. 90   | 2. 11 | 21. 88  | 2. 30  |
| 23        | 22. 93   | 1. 80 | 22. 91  | 2. 00 | 22. 89   | 2. 20 | 22. 87  | 2. 40  |
| 24        | 23. 93   | 1. 88 | 23. 91  | 2. 09 | 23. 89   | 2. 30 | 23. 87  | 2. 51  |
| 25        | 24. 92   | 1. 96 | 24. 90  | 2. 18 | 24. 88   | 2. 40 | 24. 86  | 2. 61  |
| 26        | 25. 92   | 2. 04 | 25. 90  | 2. 27 | 25. 88   | 2. 49 | 25. 86  | 2. 72  |
| 27        | 26. 92   | 2. 12 | 26. 90  | 2. 36 | 26. 88   | 2. 59 | 26. 85  | 2. 82  |
| 28        | 27. 91   | 2. 20 | 27. 89  | 2. 44 | 27. 87   | 2. 68 | 27. 85  | 2. 93  |
| 29        | 28. 91   | 2. 28 | 28. 89  | 2. 53 | 28. 87   | 2. 78 | 28. 84  | 3. 03  |
| 30        | 29. 21   | 2. 35 | 29. 89  | 2. 61 | 29. 86   | 2. 88 | 29. 84  | 3. 14  |
| 35        | 34. 89   | 2. 75 | 34. 87  | 3. 05 | 34. 84   | 3. 35 | 34. 81  | 3. 66  |
| 40        | 39. 88   | 3. 14 | 39. 85  | 3. 49 | 39. 82   | 3. 83 | 39. 78  | 4. 18  |
| 45        | 44. 86   | 3. 53 | 44. 83  | 3. 92 | 44. 79   | 4. 31 | 44. 75  | 4. 70  |
| 50        | 49. 85   | 3. 92 | 49. 81  | 4. 36 | 49. 77   | 4. 79 | 49. 73  | 5. 23  |
| 55        | 54. 85   | 4. 08 | 54. 79  | 4. 79 | 54. 75   | 5. 27 | 54. 70  | 5. 75  |
| 60        | 59. 84   | 4. 45 | 59. 77  | 5. 23 | 59. 72   | 5. 75 | 59. 67  | 6. 27  |
| 65        | 64. 82   | 4. 82 | 64. 75  | 5. 67 | 64. 70   | 6. 23 | 64. 64  | 6. 79  |
| 70        | 69. 81   | 5. 19 | 69. 73  | 6. 10 | 69. 68   | 6. 71 | 69. 62  | 7. 32  |
| 75        | 74. 79   | 5. 65 | 74. 71  | 6. 54 | 74. 65   | 7. 19 | 74. 59  | 7. 84  |
| 80        | 79. 78   | 5. 93 | 79. 70  | 6. 97 | 79. 63   | 7. 67 | 76. 56  | 8. 36  |
| 85        | 84. 77   | 6. 30 | 84. 68  | 7. 41 | 84. 61   | 8. 15 | 84. 53  | 8. 88  |
| 90        | 89. 75   | 6. 67 | 89. 66  | 7. 84 | 89. 59   | 8. 63 | 89. 51  | 9. 41  |
| 95        | 94. 74   | 7. 04 | 94. 64  | 8. 28 | 94. 56   | 9. 11 | 94. 48  | 9. 93  |
| 100       | 99. 73   | 7. 41 | 99. 62  | 8. 72 | 99. 54   | 9. 58 | 99. 45  | 10. 43 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.   |
|           | 84½ Deg. |       | 85 Deg. |       | 84½ Deg. |       | 84 Deg. |        |

## TRAVERSE TABLE.

| Distance. | 6½ Deg.  |       | 7 Deg.  |       | 7½ Deg.  |       | 8 Deg.  |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
|           |          |       |         |       |          |       |         |       |
| 1         | 0.99     | 0.11  | 0.99    | 0.12  | 0.99     | 0.13  | 0.99    | 0.14  |
| 2         | 1.99     | 0.23  | 1.99    | 0.24  | 1.98     | 0.26  | 1.98    | 0.28  |
| 3         | 2.98     | 0.34  | 2.98    | 0.37  | 2.97     | 0.39  | 2.97    | 0.42  |
| 4         | 3.97     | 0.45  | 3.97    | 0.49  | 3.97     | 0.52  | 3.96    | 0.56  |
| 5         | 4.97     | 0.57  | 4.96    | 0.61  | 4.96     | 0.65  | 4.95    | 0.70  |
| 6         | 5.96     | 0.68  | 5.96    | 0.73  | 5.95     | 0.78  | 5.94    | 0.84  |
| 7         | 6.96     | 0.79  | 6.95    | 0.85  | 6.94     | 0.91  | 6.93    | 0.97  |
| 8         | 7.95     | 0.91  | 7.94    | 0.97  | 7.93     | 1.04  | 7.92    | 1.11  |
| 9         | 8.94     | 1.02  | 8.93    | 1.10  | 8.92     | 1.17  | 8.91    | 1.25  |
| 10        | 9.94     | 1.13  | 9.93    | 1.22  | 9.91     | 1.31  | 9.90    | 1.39  |
| 11        | 10.93    | 1.25  | 10.92   | 1.34  | 10.91    | 1.44  | 10.89   | 1.53  |
| 12        | 11.92    | 1.36  | 11.91   | 1.46  | 11.90    | 1.57  | 11.88   | 1.67  |
| 13        | 12.92    | 1.47  | 12.90   | 1.58  | 12.89    | 1.70  | 12.87   | 1.81  |
| 14        | 13.91    | 1.59  | 13.90   | 1.71  | 13.88    | 1.83  | 13.86   | 1.95  |
| 15        | 14.90    | 1.70  | 14.89   | 1.83  | 14.87    | 1.96  | 14.85   | 2.09  |
| 16        | 15.90    | 1.81  | 15.88   | 1.95  | 15.86    | 2.09  | 15.84   | 2.23  |
| 17        | 16.89    | 1.92  | 16.87   | 2.07  | 16.85    | 2.22  | 16.83   | 2.37  |
| 18        | 17.88    | 2.04  | 17.87   | 2.19  | 17.85    | 2.35  | 17.82   | 2.51  |
| 19        | 18.88    | 2.15  | 18.86   | 2.32  | 18.84    | 2.48  | 18.82   | 2.64  |
| 20        | 19.87    | 2.26  | 19.85   | 2.44  | 19.83    | 2.61  | 19.81   | 2.78  |
| 21        | 20.87    | 2.38  | 20.84   | 2.56  | 20.82    | 2.74  | 20.80   | 2.92  |
| 22        | 21.86    | 2.49  | 21.84   | 2.68  | 21.81    | 2.87  | 21.79   | 3.06  |
| 23        | 22.85    | 2.60  | 22.83   | 2.80  | 22.80    | 3.00  | 22.78   | 3.20  |
| 24        | 23.85    | 2.72  | 23.82   | 2.92  | 23.79    | 3.13  | 23.77   | 3.34  |
| 25        | 24.84    | 2.83  | 24.81   | 3.05  | 24.79    | 3.26  | 24.76   | 3.48  |
| 26        | 25.83    | 2.94  | 25.81   | 3.17  | 25.78    | 3.39  | 25.75   | 3.62  |
| 27        | 26.83    | 3.06  | 26.80   | 3.29  | 26.77    | 3.52  | 26.74   | 3.76  |
| 28        | 27.82    | 3.17  | 27.79   | 3.41  | 27.76    | 3.65  | 27.73   | 3.90  |
| 29        | 28.81    | 3.28  | 28.78   | 3.53  | 28.75    | 3.79  | 28.72   | 4.04  |
| 30        | 29.81    | 3.40  | 29.78   | 3.66  | 29.74    | 3.92  | 29.71   | 4.18  |
| 35        | 34.78    | 3.96  | 34.74   | 4.27  | 34.70    | 4.57  | 34.66   | 4.87  |
| 40        | 39.74    | 4.53  | 39.70   | 4.87  | 39.66    | 5.22  | 39.61   | 5.57  |
| 45        | 44.71    | 5.09  | 44.67   | 5.48  | 44.62    | 5.87  | 44.56   | 6.26  |
| 50        | 49.68    | 5.66  | 49.63   | 6.09  | 49.57    | 6.53  | 49.51   | 6.96  |
| 55        | 54.65    | 6.23  | 54.59   | 6.70  | 55.58    | 6.70  | 54.46   | 7.65  |
| 60        | 59.61    | 6.79  | 59.55   | 7.31  | 59.55    | 7.31  | 59.42   | 8.35  |
| 65        | 64.58    | 7.36  | 64.52   | 7.92  | 64.52    | 7.92  | 64.37   | 9.05  |
| 70        | 69.55    | 7.92  | 69.48   | 8.53  | 69.48    | 8.53  | 69.32   | 9.74  |
| 75        | 74.52    | 8.49  | 74.44   | 9.14  | 74.44    | 9.14  | 74.27   | 10.44 |
| 80        | 79.49    | 9.06  | 79.40   | 9.75  | 79.40    | 9.75  | 79.22   | 11.13 |
| 85        | 84.45    | 9.62  | 84.37   | 10.36 | 84.37    | 10.36 | 84.17   | 11.83 |
| 90        | 89.42    | 10.19 | 89.33   | 10.97 | 89.33    | 10.97 | 89.12   | 12.53 |
| 95        | 94.39    | 10.75 | 94.29   | 11.58 | 94.29    | 11.58 | 94.08   | 13.22 |
| 100       | 99.36    | 11.32 | 99.25   | 12.19 | 99.25    | 12.19 | 99.03   | 13.92 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 83½ Deg. |       | 83 Deg. |       | 82½ Deg. |       | 82 Deg. |       |

# TRAVERSE TABLE.

75

| Distance. | 8½ Deg.  |       | 9 Deg.  |       | 9 ½ Deg. |       | 10 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.99     | 0.15  | 0.99    | 0.16  | 0.99     | 0.17  | 0.98    | 0.17  |
| 2         | 1.98     | 0.30  | 1.98    | 0.31  | 1.97     | 0.33  | 1.97    | 0.35  |
| 3         | 2.97     | 0.44  | 2.96    | 0.47  | 2.96     | 0.50  | 2.95    | 0.52  |
| 4         | 3.96     | 0.59  | 3.95    | 0.63  | 3.95     | 0.66  | 3.94    | 0.69  |
| 5         | 4.95     | 0.74  | 4.94    | 0.78  | 4.93     | 0.83  | 4.92    | 0.87  |
| 6         | 5.93     | 0.89  | 5.93    | 0.94  | 5.92     | 0.99  | 5.91    | 1.04  |
| 7         | 6.92     | 1.03  | 6.91    | 1.10  | 6.90     | 1.16  | 6.89    | 1.22  |
| 8         | 7.91     | 1.18  | 7.90    | 1.25  | 7.89     | 1.32  | 7.88    | 1.39  |
| 9         | 8.90     | 1.33  | 8.89    | 1.41  | 8.88     | 1.49  | 8.86    | 1.56  |
| 10        | 9.89     | 1.48  | 9.88    | 1.56  | 9.86     | 1.65  | 9.85    | 1.74  |
| 11        | 10.88    | 1.63  | 10.86   | 1.72  | 10.85    | 1.82  | 10.83   | 1.91  |
| 12        | 11.87    | 1.77  | 11.85   | 1.88  | 11.84    | 1.98  | 11.82   | 2.08  |
| 13        | 12.86    | 1.92  | 12.84   | 2.03  | 12.82    | 2.15  | 12.80   | 2.26  |
| 14        | 13.85    | 2.07  | 13.83   | 2.19  | 13.81    | 2.31  | 13.79   | 2.43  |
| 15        | 14.84    | 2.22  | 14.82   | 2.35  | 14.79    | 2.48  | 14.77   | 2.60  |
| 16        | 15.82    | 2.36  | 15.80   | 2.50  | 15.78    | 2.64  | 15.76   | 2.78  |
| 17        | 16.81    | 2.51  | 16.79   | 2.66  | 16.77    | 2.81  | 16.74   | 2.95  |
| 18        | 17.80    | 2.66  | 17.78   | 2.82  | 17.75    | 2.97  | 17.73   | 3.13  |
| 19        | 18.79    | 2.81  | 18.77   | 2.97  | 18.74    | 3.14  | 18.71   | 3.30  |
| 20        | 19.78    | 2.96  | 19.75   | 3.13  | 19.73    | 3.30  | 19.70   | 3.47  |
| 21        | 20.77    | 3.10  | 20.74   | 3.29  | 20.71    | 3.47  | 20.68   | 3.65  |
| 22        | 21.76    | 3.25  | 21.73   | 3.44  | 21.70    | 3.63  | 21.67   | 3.82  |
| 23        | 22.75    | 3.40  | 22.72   | 3.60  | 22.68    | 3.80  | 22.65   | 3.99  |
| 24        | 23.74    | 3.55  | 23.70   | 3.75  | 23.67    | 3.96  | 23.64   | 4.17  |
| 25        | 24.73    | 3.70  | 24.69   | 3.91  | 24.66    | 4.13  | 24.62   | 4.34  |
| 26        | 25.71    | 3.84  | 25.68   | 4.07  | 25.64    | 4.29  | 25.61   | 4.51  |
| 27        | 26.70    | 3.99  | 26.67   | 4.22  | 26.63    | 4.46  | 26.59   | 4.69  |
| 28        | 27.69    | 4.14  | 27.66   | 4.38  | 27.62    | 4.62  | 27.57   | 4.86  |
| 29        | 28.68    | 4.29  | 28.64   | 4.54  | 28.60    | 4.79  | 28.56   | 5.04  |
| 30        | 29.67    | 4.43  | 29.63   | 4.69  | 29.59    | 4.95  | 29.54   | 5.21  |
| 35        | 34.62    | 5.17  | 34.57   | 5.48  | 34.52    | 5.78  | 34.47   | 6.08  |
| 40        | 39.56    | 5.91  | 39.51   | 6.26  | 39.45    | 6.60  | 39.39   | 6.95  |
| 45        | 44.51    | 6.65  | 44.45   | 7.04  | 44.38    | 7.43  | 44.32   | 7.81  |
| 50        | 49.45    | 7.39  | 49.38   | 7.82  | 49.32    | 8.25  | 49.24   | 8.68  |
| 55        | 54.40    | 8.13  | 54.32   | 8.60  | 54.25    | 9.08  | 54.16   | 9.95  |
| 60        | 59.34    | 8.87  | 59.26   | 9.39  | 59.18    | 9.90  | 59.09   | 10.42 |
| 65        | 64.29    | 9.61  | 64.20   | 10.17 | 64.11    | 10.73 | 64.01   | 11.29 |
| 70        | 69.23    | 10.35 | 69.14   | 10.95 | 69.04    | 11.55 | 68.94   | 12.16 |
| 75        | 74.18    | 11.09 | 74.08   | 11.73 | 73.97    | 12.38 | 73.86   | 13.02 |
| 80        | 79.12    | 11.82 | 79.02   | 12.51 | 78.90    | 13.20 | 78.78   | 13.89 |
| 85        | 84.07    | 12.56 | 83.95   | 13.30 | 83.83    | 14.03 | 83.71   | 14.76 |
| 90        | 89.01    | 13.30 | 88.89   | 14.08 | 88.77    | 14.85 | 88.63   | 15.63 |
| 95        | 93.96    | 14.04 | 93.83   | 14.86 | 93.70    | 15.68 | 93.56   | 16.50 |
| 100       | 98.90    | 14.78 | 98.77   | 15.64 | 98.63    | 16.50 | 98.48   | 17.36 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 81½ Deg. |       | 81 Deg. |       | 80½ Deg. |       | 80 Deg. |       |

## TRAVERSE TABLE.

| Distance. | 10½ Deg. |        | 11 Deg. |        | 11½ Deg. |        | 12 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
| 1         | 0. 98    | 0. 18  | 0. 98   | 0. 19  | 0. 98    | 0. 20  | 0. 98   | 0. 21  |
| 2         | 1. 97    | 0. 36  | 1. 96   | 0. 38  | 1. 96    | 0. 40  | 1. 96   | 0. 42  |
| 3         | 2. 95    | 0. 55  | 2. 94   | 0. 57  | 2. 94    | 0. 60  | 2. 93   | 0. 62  |
| 4         | 3. 93    | 0. 73  | 3. 93   | 0. 76  | 3. 92    | 0. 80  | 3. 91   | 0. 83  |
| 5         | 4. 92    | 0. 91  | 4. 91   | 0. 95  | 4. 90    | 1. 00  | 4. 89   | 1. 04  |
| 6         | 5. 90    | 1. 09  | 5. 89   | 1. 14  | 5. 88    | 1. 20  | 5. 87   | 1. 25  |
| 7         | 6. 88    | 1. 28  | 6. 87   | 1. 34  | 6. 86    | 1. 40  | 6. 85   | 1. 46  |
| 8         | 7. 87    | 1. 46  | 7. 85   | 1. 53  | 7. 84    | 1. 59  | 7. 83   | 1. 66  |
| 9         | 8. 85    | 1. 64  | 8. 83   | 1. 72  | 8. 82    | 1. 79  | 8. 80   | 1. 87  |
| 10        | 9. 83    | 1. 82  | 9. 82   | 1. 91  | 9. 80    | 1. 99  | 9. 78   | 2. 08  |
| 11        | 10. 82   | 2. 00  | 10. 80  | 2. 10  | 10. 78   | 2. 19  | 10. 76  | 2. 29  |
| 12        | 11. 80   | 2. 19  | 11. 78  | 2. 29  | 11. 76   | 2. 39  | 11. 74  | 2. 49  |
| 13        | 12. 78   | 2. 37  | 12. 76  | 2. 48  | 12. 74   | 2. 59  | 12. 72  | 2. 70  |
| 14        | 13. 77   | 2. 55  | 13. 74  | 2. 67  | 13. 72   | 2. 79  | 13. 69  | 2. 91  |
| 15        | 14. 75   | 2. 73  | 14. 72  | 2. 87  | 14. 70   | 2. 99  | 14. 67  | 3. 12  |
| 16        | 15. 73   | 2. 92  | 15. 71  | 2. 86  | 15. 68   | 3. 19  | 15. 65  | 3. 33  |
| 17        | 16. 72   | 3. 10  | 16. 69  | 3. 05  | 16. 66   | 3. 39  | 16. 63  | 3. 53  |
| 18        | 17. 70   | 3. 28  | 17. 67  | 3. 24  | 17. 64   | 3. 59  | 17. 61  | 3. 74  |
| 19        | 18. 68   | 3. 46  | 18. 65  | 3. 43  | 18. 62   | 3. 79  | 18. 58  | 3. 95  |
| 20        | 19. 67   | 3. 64  | 19. 63  | 3. 63  | 19. 60   | 3. 99  | 19. 56  | 4. 16  |
| 21        | 20. 65   | 3. 83  | 20. 61  | 3. 82  | 20. 58   | 4. 13  | 20. 54  | 4. 37  |
| 22        | 21. 63   | 4. 01  | 21. 60  | 4. 01  | 21. 56   | 4. 39  | 21. 52  | 4. 57  |
| 23        | 22. 61   | 4. 19  | 22. 58  | 4. 20  | 22. 54   | 4. 59  | 22. 50  | 4. 78  |
| 24        | 23. 60   | 4. 37  | 23. 56  | 4. 39  | 23. 52   | 4. 78  | 23. 48  | 4. 99  |
| 25        | 24. 58   | 4. 56  | 24. 54  | 4. 58  | 24. 50   | 4. 98  | 24. 45  | 5. 20  |
| 26        | 25. 56   | 4. 74  | 25. 52  | 4. 77  | 25. 48   | 5. 18  | 25. 43  | 5. 41  |
| 27        | 26. 55   | 4. 92  | 26. 50  | 4. 96  | 26. 46   | 5. 38  | 26. 41  | 5. 61  |
| 28        | 27. 53   | 5. 10  | 27. 49  | 5. 15  | 27. 44   | 5. 58  | 27. 39  | 5. 82  |
| 29        | 28. 51   | 5. 28  | 28. 47  | 5. 34  | 28. 42   | 5. 78  | 28. 37  | 6. 03  |
| 30        | 29. 50   | 5. 47  | 29. 45  | 5. 72  | 29. 40   | 5. 98  | 29. 34  | 6. 24  |
| 35        | 34. 41   | 6. 38  | 34. 36  | 6. 68  | 34. 30   | 6. 98  | 34. 24  | 7. 28  |
| 40        | 39. 33   | 7. 29  | 39. 27  | 7. 63  | 39. 20   | 7. 97  | 39. 13  | 8. 32  |
| 45        | 44. 25   | 8. 20  | 44. 17  | 8. 59  | 44. 10   | 8. 97  | 44. 02  | 9. 36  |
| 50        | 49. 16   | 9. 11  | 49. 08  | 9. 54  | 49. 00   | 9. 97  | 48. 91  | 10. 40 |
| 55        | 54. 08   | 10. 02 | 53. 99  | 10. 49 | 53. 90   | 10. 97 | 53. 80  | 11. 44 |
| 60        | 59. 00   | 10. 93 | 58. 90  | 11. 45 | 58. 80   | 11. 96 | 58. 69  | 12. 47 |
| 65        | 63. 91   | 11. 85 | 63. 81  | 12. 40 | 63. 70   | 12. 96 | 63. 58  | 13. 51 |
| 70        | 68. 83   | 12. 76 | 68. 71  | 13. 36 | 68. 59   | 13. 96 | 68. 47  | 14. 55 |
| 75        | 73. 74   | 13. 67 | 73. 62  | 14. 31 | 73. 49   | 14. 95 | 73. 36  | 15. 59 |
| 80        | 78. 66   | 14. 58 | 78. 53  | 15. 26 | 78. 39   | 15. 95 | 78. 25  | 16. 63 |
| 85        | 83. 58   | 15. 49 | 83. 44  | 16. 22 | 83. 29   | 16. 95 | 83. 14  | 17. 67 |
| 90        | 88. 49   | 16. 40 | 88. 35  | 17. 17 | 88. 19   | 17. 94 | 88. 03  | 18. 71 |
| 95        | 93. 41   | 17. 31 | 93. 25  | 18. 13 | 93. 09   | 18. 94 | 92. 92  | 19. 75 |
| 100       | 98. 33   | 18. 22 | 98. 16  | 19. 08 | 97. 99   | 19. 94 | 97. 81  | 20. 79 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 79½ Deg. |        | 79 Deg. |        | 78½ Deg. |        | 78 Deg. |        |

# TRAVERSE TABLE.

77

| Distance. | 12½ Deg. |       | 13 Deg. |       | 13½ Deg. |       | 14 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.98     | 0.22  | 0.97    | 0.23  | 0.97     | 0.23  | 0.97    | 0.24  |
| 2         | 1.95     | 0.43  | 1.95    | 0.46  | 1.95     | 0.47  | 1.94    | 0.48  |
| 3         | 2.93     | 0.65  | 2.92    | 0.67  | 2.92     | 0.70  | 2.91    | 0.73  |
| 4         | 3.91     | 0.87  | 3.90    | 0.90  | 3.89     | 0.93  | 3.88    | 0.97  |
| 5         | 4.88     | 1.08  | 4.87    | 1.12  | 4.86     | 1.17  | 4.85    | 1.21  |
| 6         | 5.86     | 1.30  | 5.85    | 1.35  | 5.83     | 1.40  | 5.82    | 1.45  |
| 7         | 6.83     | 1.52  | 6.82    | 1.57  | 6.81     | 1.63  | 6.79    | 1.69  |
| 8         | 7.81     | 1.73  | 7.80    | 1.80  | 7.78     | 1.87  | 7.76    | 1.94  |
| 9         | 8.79     | 1.95  | 8.77    | 2.02  | 8.75     | 2.10  | 8.73    | 2.18  |
| 10        | 9.76     | 2.16  | 9.74    | 2.25  | 9.72     | 2.33  | 9.70    | 2.42  |
| 11        | 10.74    | 2.38  | 10.72   | 2.47  | 10.70    | 2.57  | 10.67   | 2.66  |
| 12        | 11.72    | 2.60  | 11.69   | 2.70  | 11.67    | 2.80  | 11.64   | 2.90  |
| 13        | 12.69    | 2.81  | 12.67   | 2.92  | 12.64    | 3.03  | 12.61   | 3.15  |
| 14        | 13.67    | 3.03  | 13.64   | 3.15  | 13.61    | 3.27  | 13.58   | 3.39  |
| 15        | 14.64    | 3.25  | 14.62   | 3.37  | 14.59    | 3.50  | 14.55   | 3.63  |
| 16        | 15.62    | 3.46  | 15.59   | 3.60  | 15.56    | 3.74  | 15.52   | 3.87  |
| 17        | 16.60    | 3.68  | 16.57   | 3.82  | 16.53    | 3.97  | 16.50   | 4.11  |
| 18        | 17.57    | 3.90  | 17.54   | 4.05  | 17.50    | 4.20  | 17.47   | 4.35  |
| 19        | 18.55    | 4.11  | 18.51   | 4.27  | 18.48    | 4.44  | 18.44   | 4.60  |
| 20        | 19.53    | 4.33  | 19.49   | 4.50  | 19.45    | 4.67  | 19.41   | 4.84  |
| 21        | 20.50    | 4.55  | 20.46   | 4.72  | 20.42    | 4.90  | 20.38   | 5.08  |
| 22        | 21.48    | 4.76  | 21.44   | 4.95  | 21.39    | 5.14  | 21.35   | 5.32  |
| 23        | 22.45    | 4.98  | 22.41   | 5.17  | 22.36    | 5.37  | 22.32   | 5.56  |
| 24        | 23.43    | 5.19  | 23.38   | 5.40  | 23.34    | 5.60  | 23.29   | 5.81  |
| 25        | 24.41    | 5.41  | 24.36   | 5.62  | 24.31    | 5.84  | 24.26   | 6.05  |
| 26        | 25.38    | 5.63  | 25.33   | 5.85  | 25.28    | 6.07  | 25.23   | 6.29  |
| 27        | 26.36    | 5.84  | 26.31   | 6.07  | 26.25    | 6.30  | 26.20   | 6.53  |
| 28        | 27.34    | 6.06  | 27.28   | 6.30  | 27.23    | 6.54  | 27.17   | 6.77  |
| 29        | 28.31    | 6.28  | 28.26   | 6.52  | 28.20    | 6.77  | 28.14   | 7.02  |
| 30        | 29.29    | 6.49  | 29.23   | 6.75  | 29.17    | 7.00  | 29.11   | 7.26  |
| 35        | 34.17    | 7.58  | 34.10   | 7.87  | 34.03    | 8.17  | 33.96   | 8.47  |
| 40        | 39.05    | 8.66  | 38.97   | 9.00  | 38.89    | 9.34  | 38.81   | 9.68  |
| 45        | 43.93    | 9.74  | 43.85   | 10.12 | 43.76    | 10.51 | 43.66   | 10.89 |
| 50        | 48.81    | 10.82 | 48.72   | 11.25 | 48.62    | 11.67 | 48.51   | 12.10 |
| 55        | 53.70    | 11.90 | 53.59   | 12.37 | 53.48    | 12.84 | 53.37   | 13.31 |
| 60        | 58.58    | 12.99 | 58.46   | 13.50 | 58.34    | 14.01 | 58.22   | 14.52 |
| 65        | 63.46    | 14.07 | 63.33   | 14.62 | 63.20    | 15.17 | 63.07   | 15.72 |
| 70        | 68.34    | 15.15 | 68.21   | 15.75 | 68.07    | 16.34 | 67.92   | 16.93 |
| 75        | 73.22    | 16.23 | 73.08   | 16.87 | 72.93    | 17.50 | 72.77   | 18.14 |
| 80        | 78.10    | 17.32 | 77.95   | 18.00 | 77.79    | 18.68 | 77.62   | 19.35 |
| 85        | 82.99    | 18.40 | 82.82   | 19.12 | 82.65    | 19.84 | 82.48   | 20.56 |
| 90        | 87.87    | 19.48 | 87.69   | 20.25 | 87.51    | 21.01 | 87.33   | 21.77 |
| 95        | 92.75    | 20.56 | 92.57   | 21.37 | 92.38    | 22.18 | 92.18   | 22.98 |
| 100       | 97.63    | 21.64 | 97.44   | 22.50 | 97.24    | 23.34 | 97.03   | 24.19 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 77½ Deg. |       | 77 Deg. |       | 76½ Deg. |       | 76 Deg. |       |



## TRAVERSE TABLE.

| Distance. | 14½ Deg. |        | 15 Deg. |        | 15½ Deg. |        | 16 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
|           |          |        |         |        |          |        |         |        |
| 1         | 0. 97    | 0. 25  | 0. 97   | 0. 26  | 0. 96    | 0. 27  | 0. 96   | 0. 28  |
| 2         | 1. 94    | 0. 50  | 1. 93   | 0. 52  | 1. 93    | 0. 53  | 1. 92   | 0. 55  |
| 3         | 2. 90    | 0. 75  | 2. 90   | 0. 78  | 2. 89    | 0. 80  | 2. 88   | 0. 83  |
| 4         | 3. 87    | 1. 00  | 3. 86   | 1. 04  | 3. 85    | 1. 07  | 3. 85   | 1. 10  |
| 5         | 4. 84    | 1. 25  | 4. 83   | 1. 29  | 4. 82    | 1. 34  | 4. 81   | 1. 38  |
| 6         | 5. 81    | 1. 50  | 5. 80   | 1. 55  | 5. 78    | 1. 60  | 5. 77   | 1. 65  |
| 7         | 6. 78    | 1. 75  | 6. 76   | 1. 81  | 6. 75    | 1. 87  | 6. 73   | 1. 93  |
| 8         | 7. 75    | 2. 00  | 7. 73   | 2. 07  | 7. 71    | 2. 14  | 7. 69   | 2. 21  |
| 9         | 8. 71    | 2. 25  | 8. 69   | 2. 33  | 8. 67    | 2. 41  | 8. 65   | 2. 48  |
| 10        | 9. 68    | 2. 50  | 9. 66   | 2. 59  | 9. 64    | 2. 67  | 9. 61   | 2. 76  |
| 11        | 10. 65   | 2. 75  | 10. 63  | 2. 85  | 10. 60   | 2. 94  | 10. 57  | 3. 03  |
| 12        | 11. 62   | 3. 00  | 11. 59  | 3. 11  | 11. 56   | 3. 21  | 11. 54  | 3. 31  |
| 13        | 12. 59   | 3. 25  | 12. 56  | 3. 36  | 12. 53   | 3. 47  | 12. 50  | 3. 58  |
| 14        | 13. 55   | 3. 51  | 13. 52  | 3. 62  | 13. 49   | 3. 74  | 13. 46  | 3. 86  |
| 15        | 14. 52   | 3. 76  | 14. 49  | 3. 88  | 14. 45   | 4. 01  | 14. 42  | 4. 13  |
| 16        | 15. 49   | 4. 01  | 15. 45  | 4. 14  | 15. 42   | 4. 28  | 15. 38  | 4. 41  |
| 17        | 16. 46   | 4. 26  | 16. 42  | 4. 40  | 16. 38   | 4. 54  | 16. 34  | 4. 69  |
| 18        | 17. 43   | 4. 51  | 17. 39  | 4. 66  | 17. 35   | 4. 81  | 17. 30  | 4. 96  |
| 19        | 18. 39   | 4. 76  | 18. 35  | 4. 92  | 18. 31   | 5. 08  | 18. 26  | 5. 24  |
| 20        | 19. 36   | 5. 01  | 19. 32  | 5. 18  | 19. 27   | 5. 34  | 19. 23  | 5. 51  |
| 21        | 20. 33   | 5. 26  | 20. 28  | 5. 44  | 20. 24   | 5. 61  | 20. 19  | 5. 79  |
| 22        | 21. 30   | 5. 51  | 21. 25  | 5. 69  | 21. 20   | 5. 88  | 21. 15  | 6. 06  |
| 23        | 22. 27   | 5. 76  | 22. 22  | 5. 95  | 22. 16   | 6. 15  | 22. 11  | 6. 34  |
| 24        | 23. 24   | 6. 01  | 23. 18  | 6. 21  | 23. 13   | 6. 41  | 23. 07  | 6. 62  |
| 25        | 24. 20   | 6. 26  | 24. 15  | 6. 47  | 24. 09   | 6. 68  | 24. 03  | 6. 89  |
| 26        | 25. 17   | 6. 51  | 25. 11  | 6. 73  | 25. 05   | 6. 95  | 24. 99  | 7. 17  |
| 27        | 26. 14   | 6. 76  | 26. 08  | 6. 99  | 26. 02   | 7. 22  | 25. 95  | 7. 44  |
| 28        | 27. 11   | 7. 01  | 27. 05  | 7. 25  | 26. 98   | 7. 48  | 26. 92  | 7. 72  |
| 29        | 28. 08   | 7. 26  | 28. 01  | 7. 51  | 27. 95   | 7. 75  | 27. 88  | 7. 99  |
| 30        | 29. 04   | 7. 51  | 28. 98  | 7. 76  | 28. 91   | 8. 02  | 28. 84  | 8. 27  |
| 35        | 33. 89   | 8. 76  | 33. 81  | 9. 06  | 33. 73   | 9. 35  | 33. 64  | 9. 65  |
| 40        | 38. 73   | 10. 02 | 38. 64  | 10. 35 | 38. 55   | 10. 69 | 38. 45  | 11. 03 |
| 45        | 43. 57   | 11. 27 | 43. 47  | 11. 65 | 43. 36   | 12. 03 | 43. 26  | 12. 40 |
| 50        | 48. 41   | 12. 52 | 48. 30  | 12. 94 | 48. 18   | 13. 36 | 48. 06  | 13. 78 |
| 55        | 53. 25   | 13. 77 | 53. 13  | 14. 24 | 53. 00   | 14. 70 | 52. 87  | 15. 16 |
| 60        | 58. 09   | 15. 02 | 57. 96  | 15. 53 | 57. 82   | 16. 03 | 57. 68  | 16. 54 |
| 65        | 62. 93   | 16. 27 | 62. 79  | 16. 82 | 62. 64   | 17. 37 | 62. 48  | 17. 92 |
| 70        | 67. 77   | 17. 53 | 67. 61  | 18. 12 | 67. 45   | 18. 71 | 67. 29  | 19. 29 |
| 75        | 72. 61   | 18. 78 | 72. 44  | 19. 41 | 72. 27   | 20. 04 | 72. 09  | 20. 67 |
| 80        | 77. 45   | 20. 03 | 77. 27  | 20. 71 | 77. 09   | 21. 38 | 76. 90  | 22. 05 |
| 85        | 82. 29   | 21. 28 | 82. 10  | 22. 00 | 81. 91   | 22. 72 | 81. 71  | 23. 43 |
| 90        | 87. 13   | 22. 53 | 86. 93  | 23. 29 | 86. 73   | 24. 05 | 86. 51  | 24. 81 |
| 95        | 91. 97   | 23. 79 | 91. 76  | 24. 59 | 91. 54   | 25. 39 | 91. 32  | 26. 19 |
| 100       | 96. 81   | 25. 04 | 96. 59  | 25. 88 | 96. 36   | 26. 72 | 96. 13  | 27. 56 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 75½ Deg. |        | 75 Deg. |        | 74½ Deg. |        | 74 Deg. |        |

# TRAVERSE TABLE.

79

| Distance. | 16½ Deg. |       | 17 Deg. |       | 17½ Deg. |       | 18 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.96     | 0.28  | 0.96    | 0.29  | 0.95     | 0.30  | 0.95    | 0.31  |
| 2         | 1.92     | 0.57  | 1.91    | 0.58  | 1.91     | 0.60  | 1.90    | 0.62  |
| 3         | 2.88     | 0.85  | 2.87    | 0.88  | 2.86     | 0.90  | 2.85    | 0.93  |
| 4         | 3.84     | 1.14  | 3.83    | 1.17  | 3.81     | 1.20  | 3.80    | 1.24  |
| 5         | 4.79     | 1.42  | 4.78    | 1.46  | 4.77     | 1.50  | 4.76    | 1.55  |
| 6         | 5.75     | 1.70  | 5.74    | 1.75  | 5.72     | 1.80  | 5.71    | 1.85  |
| 7         | 6.71     | 1.99  | 6.69    | 2.05  | 6.68     | 2.10  | 6.66    | 2.16  |
| 8         | 7.67     | 2.27  | 7.65    | 2.34  | 7.63     | 2.41  | 7.61    | 2.47  |
| 9         | 8.63     | 2.56  | 8.61    | 2.63  | 8.58     | 2.71  | 8.56    | 2.78  |
| 10        | 9.59     | 2.84  | 9.56    | 2.92  | 9.54     | 3.01  | 9.51    | 3.09  |
| 11        | 10.55    | 3.12  | 10.52   | 3.22  | 10.49    | 3.31  | 10.46   | 3.40  |
| 12        | 11.51    | 3.41  | 11.48   | 3.51  | 11.44    | 3.61  | 11.41   | 3.71  |
| 13        | 12.46    | 3.69  | 12.43   | 3.80  | 12.40    | 3.91  | 12.36   | 4.02  |
| 14        | 13.42    | 3.98  | 13.39   | 4.09  | 13.35    | 4.21  | 13.31   | 4.33  |
| 15        | 14.38    | 4.26  | 14.34   | 4.39  | 14.31    | 4.51  | 14.27   | 4.64  |
| 16        | 15.34    | 4.54  | 15.30   | 4.68  | 15.26    | 4.81  | 15.22   | 4.94  |
| 17        | 16.30    | 4.83  | 16.26   | 4.97  | 16.21    | 5.11  | 16.17   | 5.25  |
| 18        | 17.26    | 5.11  | 17.21   | 5.26  | 17.17    | 5.41  | 17.12   | 5.56  |
| 19        | 18.22    | 5.40  | 18.17   | 5.56  | 18.12    | 5.71  | 18.07   | 5.87  |
| 20        | 19.18    | 5.68  | 19.13   | 5.85  | 19.07    | 6.01  | 19.02   | 6.18  |
| 21        | 20.14    | 5.96  | 20.08   | 6.14  | 20.03    | 6.31  | 19.97   | 6.49  |
| 22        | 21.09    | 6.25  | 21.04   | 6.43  | 20.98    | 6.62  | 20.92   | 6.80  |
| 23        | 22.05    | 6.53  | 21.99   | 6.72  | 21.94    | 6.92  | 21.87   | 7.11  |
| 24        | 23.01    | 6.82  | 22.95   | 7.02  | 22.89    | 7.22  | 22.83   | 7.42  |
| 25        | 23.97    | 7.10  | 23.91   | 7.30  | 23.84    | 7.52  | 23.78   | 7.73  |
| 26        | 24.93    | 7.38  | 24.86   | 7.60  | 24.80    | 7.82  | 24.73   | 8.03  |
| 27        | 25.89    | 7.67  | 25.82   | 7.89  | 25.75    | 8.12  | 25.68   | 8.34  |
| 28        | 26.85    | 7.95  | 26.78   | 8.19  | 26.70    | 8.42  | 26.63   | 8.65  |
| 29        | 27.81    | 8.24  | 27.73   | 8.48  | 27.66    | 8.72  | 27.58   | 8.96  |
| 30        | 28.76    | 8.52  | 28.69   | 8.77  | 28.61    | 9.02  | 28.53   | 9.27  |
| 35        | 33.56    | 9.94  | 33.47   | 10.23 | 33.38    | 10.52 | 33.29   | 10.82 |
| 40        | 38.35    | 11.36 | 38.25   | 11.69 | 38.15    | 12.03 | 38.04   | 12.36 |
| 45        | 43.15    | 12.78 | 43.03   | 13.16 | 42.92    | 13.53 | 42.80   | 13.91 |
| 50        | 47.94    | 14.20 | 47.82   | 14.62 | 47.69    | 15.04 | 47.55   | 15.45 |
| 55        | 52.74    | 15.62 | 52.60   | 16.08 | 52.45    | 16.54 | 52.31   | 17.00 |
| 60        | 57.53    | 17.04 | 57.38   | 17.54 | 57.22    | 18.04 | 57.06   | 18.54 |
| 65        | 62.32    | 18.46 | 62.16   | 19.00 | 61.99    | 19.55 | 61.82   | 20.09 |
| 70        | 67.12    | 19.88 | 66.94   | 20.47 | 66.76    | 21.05 | 66.57   | 21.63 |
| 75        | 71.91    | 21.30 | 71.72   | 21.93 | 71.53    | 22.55 | 71.33   | 23.18 |
| 80        | 76.71    | 22.72 | 76.50   | 23.39 | 76.30    | 24.06 | 76.08   | 24.72 |
| 85        | 81.50    | 24.14 | 81.29   | 24.85 | 81.07    | 25.56 | 80.84   | 26.27 |
| 90        | 86.29    | 25.56 | 86.07   | 26.31 | 85.83    | 27.06 | 85.60   | 27.81 |
| 95        | 91.09    | 26.98 | 90.85   | 27.78 | 90.60    | 28.57 | 90.35   | 29.36 |
| 100       | 95.88    | 28.40 | 95.63   | 29.24 | 95.37    | 30.07 | 95.11   | 30.90 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 73½ Deg. |       | 73 Deg. |       | 72½ Deg. |       | 72 Deg. |       |

## TRAVERSE TABLE.

| Distance. | 18½ Deg. |       | 19 Deg. |       | 19½ Deg. |       | 20 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
|           |          |       |         |       |          |       |         |       |
| 1         | 0.95     | 0.32  | 0.95    | 0.33  | 0.94     | 0.33  | 0.94    | 0.34  |
| 2         | 1.90     | 0.63  | 1.89    | 0.65  | 1.89     | 0.67  | 1.88    | 0.68  |
| 3         | 2.84     | 0.95  | 2.84    | 0.98  | 2.83     | 1.00  | 2.82    | 1.03  |
| 4         | 3.79     | 1.27  | 3.78    | 1.30  | 3.77     | 1.34  | 3.76    | 1.37  |
| 5         | 4.74     | 1.59  | 4.73    | 1.63  | 4.71     | 1.67  | 4.70    | 1.71  |
| 6         | 5.69     | 1.90  | 5.67    | 1.95  | 5.66     | 2.00  | 5.64    | 2.05  |
| 7         | 6.64     | 2.22  | 6.62    | 2.28  | 6.60     | 2.34  | 6.58    | 2.39  |
| 8         | 7.59     | 2.54  | 7.56    | 2.60  | 7.54     | 2.67  | 7.52    | 2.74  |
| 9         | 8.53     | 2.86  | 8.51    | 2.93  | 8.48     | 3.01  | 8.46    | 3.08  |
| 10        | 9.48     | 3.17  | 9.46    | 3.26  | 9.43     | 3.34  | 9.40    | 3.42  |
| 11        | 10.43    | 3.49  | 10.40   | 3.58  | 10.37    | 3.67  | 10.34   | 3.76  |
| 12        | 11.38    | 3.81  | 11.35   | 3.91  | 11.31    | 4.01  | 11.28   | 4.10  |
| 13        | 12.33    | 4.12  | 12.29   | 4.23  | 12.25    | 4.34  | 12.22   | 4.45  |
| 14        | 13.28    | 4.44  | 13.24   | 4.56  | 13.20    | 4.67  | 13.16   | 4.79  |
| 15        | 14.22    | 4.76  | 14.18   | 4.88  | 14.14    | 5.01  | 14.10   | 5.13  |
| 16        | 15.17    | 5.08  | 15.13   | 5.21  | 15.08    | 5.34  | 15.04   | 5.47  |
| 17        | 16.12    | 5.39  | 16.07   | 5.53  | 16.02    | 5.67  | 15.97   | 5.81  |
| 18        | 17.07    | 5.71  | 17.02   | 5.86  | 16.97    | 6.01  | 16.91   | 6.16  |
| 19        | 18.02    | 6.03  | 17.96   | 6.19  | 17.91    | 6.34  | 17.85   | 6.50  |
| 20        | 18.97    | 6.35  | 18.91   | 6.51  | 18.85    | 6.68  | 18.79   | 6.84  |
| 21        | 19.91    | 6.66  | 19.86   | 6.84  | 19.80    | 7.01  | 19.73   | 7.18  |
| 22        | 20.86    | 6.98  | 20.80   | 7.16  | 20.74    | 7.34  | 20.67   | 7.52  |
| 23        | 21.81    | 7.30  | 21.75   | 7.49  | 21.68    | 7.68  | 21.61   | 7.87  |
| 24        | 22.76    | 7.62  | 22.69   | 7.81  | 22.62    | 8.01  | 22.55   | 8.21  |
| 25        | 23.71    | 7.93  | 23.64   | 8.14  | 23.57    | 8.34  | 23.49   | 8.55  |
| 26        | 24.66    | 8.25  | 24.58   | 8.46  | 24.51    | 8.68  | 24.43   | 8.89  |
| 27        | 25.60    | 8.57  | 25.53   | 8.79  | 25.45    | 9.01  | 25.37   | 9.23  |
| 28        | 26.55    | 8.88  | 26.47   | 9.12  | 26.39    | 9.34  | 26.31   | 9.58  |
| 29        | 27.50    | 9.20  | 27.42   | 9.44  | 27.34    | 9.68  | 27.25   | 9.92  |
| 30        | 28.45    | 9.52  | 28.37   | 9.77  | 28.28    | 10.01 | 28.19   | 10.26 |
| 35        | 33.19    | 11.11 | 33.09   | 11.39 | 32.99    | 11.68 | 32.89   | 11.97 |
| 40        | 37.93    | 12.69 | 37.82   | 13.02 | 37.71    | 13.35 | 37.59   | 13.68 |
| 45        | 42.67    | 14.28 | 42.55   | 14.65 | 42.42    | 15.02 | 42.29   | 15.39 |
| 50        | 47.42    | 15.87 | 47.28   | 16.28 | 47.13    | 16.69 | 46.98   | 17.10 |
| 55        | 52.16    | 17.45 | 52.00   | 17.91 | 51.85    | 18.36 | 51.68   | 18.81 |
| 60        | 56.90    | 19.04 | 56.73   | 19.53 | 56.56    | 20.03 | 56.38   | 20.52 |
| 65        | 61.64    | 20.62 | 61.46   | 21.16 | 61.27    | 21.70 | 61.08   | 22.23 |
| 70        | 66.38    | 22.21 | 66.19   | 22.79 | 67.98    | 23.37 | 65.78   | 23.94 |
| 75        | 71.12    | 23.80 | 70.91   | 24.42 | 70.70    | 25.04 | 70.48   | 25.65 |
| 80        | 75.87    | 25.38 | 75.64   | 26.05 | 75.41    | 26.70 | 75.18   | 27.36 |
| 85        | 80.61    | 26.97 | 80.37   | 27.67 | 80.12    | 28.37 | 79.87   | 29.07 |
| 90        | 85.35    | 28.56 | 85.10   | 29.30 | 84.84    | 30.04 | 84.57   | 30.78 |
| 95        | 90.09    | 30.14 | 89.82   | 30.93 | 89.55    | 31.71 | 89.27   | 32.49 |
| 100       | 94.83    | 31.73 | 94.55   | 32.56 | 94.26    | 33.38 | 93.97   | 34.20 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 71½ Deg. |       | 71 Deg. |       | 70½ Deg. |       | 70 Deg. |       |

# TRAVERSE TABLE.

81

| Distance. | 20½ Deg. |        | 21 Deg. |        | 21½ Deg. |        | 22 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
| 1         | 0. 94    | 0. 35  | 0. 93   | 0. 36  | 0. 93    | 0. 37  | 0. 93   | 0. 37  |
| 2         | 1. 87    | 0. 70  | 1. 87   | 0. 72  | 1. 86    | 0. 73  | 1. 85   | 0. 75  |
| 3         | 2. 81    | 1. 05  | 2. 80   | 1. 08  | 2. 79    | 1. 10  | 2. 78   | 1. 12  |
| 4         | 3. 75    | 1. 40  | 3. 73   | 1. 43  | 3. 72    | 1. 47  | 3. 71   | 1. 50  |
| 5         | 4. 68    | 1. 75  | 4. 67   | 1. 79  | 4. 65    | 1. 83  | 4. 64   | 1. 87  |
| 6         | 5. 62    | 2. 10  | 5. 60   | 2. 15  | 5. 58    | 2. 20  | 5. 56   | 2. 25  |
| 7         | 6. 56    | 2. 45  | 6. 54   | 2. 51  | 6. 51    | 2. 57  | 6. 49   | 2. 62  |
| 8         | 7. 49    | 2. 80  | 7. 47   | 2. 87  | 7. 44    | 2. 93  | 7. 42   | 3. 00  |
| 9         | 8. 43    | 3. 15  | 8. 40   | 3. 23  | 8. 37    | 3. 30  | 8. 34   | 3. 37  |
| 10        | 9. 37    | 3. 50  | 9. 34   | 3. 58  | 9. 30    | 3. 67  | 9. 27   | 3. 75  |
| 11        | 10. 30   | 3. 85  | 10. 27  | 3. 94  | 10. 23   | 4. 03  | 10. 20  | 4. 12  |
| 12        | 11. 24   | 4. 20  | 11. 20  | 4. 30  | 11. 17   | 4. 40  | 11. 13  | 4. 50  |
| 13        | 12. 18   | 4. 55  | 12. 14  | 4. 66  | 12. 10   | 4. 76  | 12. 05  | 4. 87  |
| 14        | 13. 11   | 4. 90  | 13. 07  | 5. 02  | 13. 03   | 5. 13  | 12. 98  | 5. 24  |
| 15        | 14. 05   | 5. 25  | 14. 00  | 5. 38  | 13. 96   | 5. 50  | 13. 91  | 5. 62  |
| 16        | 14. 99   | 5. 60  | 14. 94  | 5. 73  | 14. 89   | 5. 86  | 14. 83  | 5. 99  |
| 17        | 15. 92   | 5. 95  | 15. 87  | 6. 09  | 15. 82   | 6. 23  | 15. 76  | 6. 37  |
| 18        | 16. 86   | 6. 30  | 16. 80  | 6. 45  | 16. 75   | 6. 60  | 16. 69  | 6. 74  |
| 19        | 17. 80   | 6. 65  | 17. 74  | 6. 81  | 17. 68   | 6. 96  | 17. 62  | 7. 12  |
| 20        | 18. 73   | 7. 00  | 18. 67  | 7. 17  | 18. 61   | 7. 33  | 18. 54  | 7. 49  |
| 21        | 19. 67   | 7. 35  | 19. 61  | 7. 53  | 19. 54   | 7. 70  | 19. 47  | 7. 87  |
| 22        | 20. 61   | 7. 70  | 20. 54  | 7. 88  | 20. 47   | 8. 06  | 20. 40  | 8. 24  |
| 23        | 21. 54   | 8. 05  | 21. 47  | 8. 24  | 21. 40   | 8. 43  | 21. 33  | 8. 62  |
| 24        | 22. 48   | 8. 40  | 22. 41  | 8. 60  | 22. 33   | 8. 80  | 22. 25  | 8. 99  |
| 25        | 23. 42   | 8. 76  | 23. 34  | 8. 96  | 23. 26   | 9. 16  | 23. 18  | 9. 37  |
| 26        | 24. 35   | 9. 11  | 24. 27  | 9. 32  | 24. 19   | 9. 53  | 24. 11  | 9. 74  |
| 27        | 25. 29   | 9. 46  | 25. 21  | 9. 68  | 25. 12   | 9. 90  | 25. 03  | 10. 11 |
| 28        | 26. 23   | 9. 81  | 26. 14  | 10. 08 | 26. 05   | 10. 26 | 25. 96  | 10. 49 |
| 29        | 27. 16   | 10. 16 | 27. 07  | 10. 39 | 26. 98   | 10. 63 | 26. 89  | 10. 86 |
| 30        | 28. 10   | 10. 51 | 28. 01  | 10. 75 | 27. 91   | 11. 00 | 27. 82  | 11. 24 |
| 35        | 32. 78   | 12. 26 | 32. 68  | 12. 54 | 32. 56   | 12. 83 | 32. 45  | 13. 11 |
| 40        | 37. 47   | 14. 01 | 37. 34  | 14. 33 | 37. 22   | 14. 66 | 37. 09  | 14. 98 |
| 45        | 42. 15   | 15. 76 | 42. 01  | 16. 13 | 41. 87   | 16. 49 | 41. 72  | 16. 86 |
| 50        | 46. 83   | 17. 51 | 46. 68  | 17. 92 | 46. 52   | 18. 33 | 46. 36  | 18. 73 |
| 55        | 51. 52   | 19. 26 | 51. 35  | 19. 71 | 51. 17   | 20. 16 | 51. 00  | 20. 60 |
| 60        | 56. 20   | 21. 01 | 56. 01  | 21. 50 | 55. 83   | 21. 99 | 55. 63  | 22. 48 |
| 65        | 60. 88   | 22. 76 | 60. 68  | 23. 29 | 60. 48   | 23. 82 | 60. 27  | 24. 35 |
| 70        | 65. 57   | 24. 51 | 65. 35  | 25. 09 | 65. 13   | 25. 66 | 64. 90  | 26. 22 |
| 75        | 70. 25   | 26. 27 | 70. 02  | 26. 88 | 69. 78   | 27. 49 | 69. 54  | 28. 10 |
| 80        | 74. 93   | 28. 02 | 74. 69  | 28. 67 | 74. 43   | 29. 32 | 74. 17  | 29. 97 |
| 85        | 79. 62   | 29. 77 | 79. 35  | 30. 46 | 79. 09   | 31. 15 | 78. 81  | 31. 84 |
| 90        | 84. 30   | 31. 52 | 84. 02  | 32. 25 | 83. 74   | 32. 99 | 83. 45  | 33. 71 |
| 95        | 98. 98   | 33. 27 | 88. 69  | 34. 04 | 88. 39   | 34. 82 | 88. 08  | 35. 59 |
| 100       | 93. 67   | 35. 02 | 93. 36  | 35. 84 | 93. 04   | 36. 65 | 92. 72  | 37. 46 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 69½ Deg. |        | 69 Deg. |        | 68½ Deg. |        | 68 Deg. |        |

## TRAVERSE TABLE.

| Distance. | 22½ Deg. |       | 23 Deg. |       | 23½ Deg. |       | 24 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.90     | 0.38  | 0.92    | 0.39  | 0.92     | 0.40  | 0.91    | 0.41  |
| 2         | 1.85     | 0.77  | 1.84    | 0.78  | 1.83     | 0.80  | 1.83    | 0.81  |
| 3         | 2.77     | 1.15  | 2.76    | 1.17  | 2.75     | 1.20  | 2.74    | 1.22  |
| 4         | 3.70     | 1.53  | 3.68    | 1.56  | 3.67     | 1.59  | 3.65    | 1.63  |
| 5         | 4.62     | 1.91  | 4.60    | 1.95  | 4.59     | 1.99  | 4.57    | 2.03  |
| 6         | 5.54     | 2.30  | 5.52    | 2.34  | 5.50     | 2.39  | 5.48    | 2.44  |
| 7         | 6.47     | 2.68  | 6.44    | 2.74  | 6.42     | 2.79  | 6.39    | 2.85  |
| 8         | 7.39     | 3.06  | 7.36    | 3.13  | 7.34     | 3.19  | 7.31    | 3.25  |
| 9         | 8.31     | 3.44  | 8.28    | 3.52  | 8.25     | 3.59  | 8.22    | 3.66  |
| 10        | 9.24     | 3.83  | 9.20    | 3.91  | 9.17     | 3.99  | 9.14    | 4.07  |
| 11        | 10.16    | 4.21  | 10.13   | 4.30  | 10.09    | 4.39  | 10.05   | 4.47  |
| 12        | 11.09    | 4.59  | 11.05   | 4.69  | 11.00    | 4.78  | 10.96   | 4.88  |
| 13        | 12.01    | 4.97  | 11.97   | 5.08  | 11.92    | 5.18  | 11.88   | 5.29  |
| 14        | 12.93    | 5.36  | 12.89   | 5.47  | 12.84    | 5.58  | 12.79   | 5.69  |
| 15        | 13.86    | 5.74  | 13.81   | 5.86  | 13.76    | 5.98  | 13.70   | 6.10  |
| 16        | 14.78    | 6.12  | 14.73   | 6.25  | 14.67    | 6.38  | 14.62   | 6.51  |
| 17        | 15.71    | 6.51  | 15.65   | 6.64  | 15.59    | 6.78  | 15.53   | 6.92  |
| 18        | 16.63    | 6.89  | 16.57   | 7.03  | 16.51    | 7.18  | 16.44   | 7.32  |
| 19        | 17.55    | 7.27  | 17.49   | 7.42  | 17.42    | 7.58  | 17.36   | 7.73  |
| 20        | 18.48    | 7.65  | 18.41   | 7.81  | 18.34    | 7.97  | 18.27   | 8.13  |
| 21        | 19.40    | 8.04  | 19.33   | 8.21  | 19.26    | 8.37  | 19.18   | 8.54  |
| 22        | 20.33    | 8.42  | 20.25   | 8.60  | 20.18    | 8.77  | 20.10   | 8.95  |
| 23        | 21.25    | 8.80  | 21.17   | 8.99  | 21.09    | 9.17  | 21.01   | 9.35  |
| 24        | 22.17    | 9.18  | 22.09   | 9.38  | 22.01    | 9.57  | 21.93   | 9.76  |
| 25        | 23.10    | 9.57  | 23.01   | 9.77  | 22.93    | 9.97  | 22.84   | 10.17 |
| 26        | 24.02    | 9.95  | 23.93   | 10.16 | 23.84    | 10.37 | 23.75   | 10.58 |
| 27        | 24.94    | 10.33 | 24.85   | 10.55 | 24.76    | 10.77 | 24.67   | 10.98 |
| 28        | 25.87    | 10.72 | 25.77   | 10.94 | 25.68    | 11.16 | 25.58   | 11.39 |
| 29        | 26.79    | 11.10 | 26.69   | 11.33 | 26.59    | 11.56 | 26.49   | 11.80 |
| 30        | 27.72    | 11.48 | 27.62   | 11.52 | 27.51    | 11.96 | 27.41   | 12.20 |
| 35        | 32.34    | 13.39 | 32.22   | 13.68 | 32.10    | 13.96 | 31.97   | 14.24 |
| 40        | 36.96    | 15.31 | 36.82   | 15.63 | 36.68    | 15.95 | 36.54   | 16.27 |
| 45        | 41.57    | 17.22 | 41.42   | 17.58 | 41.27    | 17.94 | 41.11   | 18.30 |
| 50        | 46.19    | 19.13 | 46.03   | 19.54 | 45.85    | 19.94 | 45.68   | 20.34 |
| 55        | 50.81    | 21.05 | 50.63   | 21.49 | 50.44    | 21.93 | 50.24   | 22.37 |
| 60        | 55.43    | 22.96 | 55.23   | 23.44 | 55.02    | 23.92 | 54.81   | 24.40 |
| 65        | 60.05    | 24.87 | 59.83   | 25.40 | 59.61    | 25.92 | 59.38   | 26.44 |
| 70        | 64.67    | 26.79 | 64.44   | 27.35 | 64.19    | 27.91 | 63.95   | 28.47 |
| 75        | 69.29    | 28.70 | 69.04   | 29.30 | 68.78    | 29.91 | 68.52   | 30.51 |
| 80        | 73.91    | 30.61 | 73.64   | 31.26 | 73.36    | 31.90 | 73.08   | 32.54 |
| 85        | 78.53    | 32.53 | 78.24   | 33.21 | 77.95    | 33.89 | 77.65   | 34.57 |
| 90        | 83.15    | 34.44 | 82.85   | 35.17 | 82.54    | 35.89 | 82.22   | 36.61 |
| 95        | 87.77    | 36.35 | 87.45   | 37.12 | 87.12    | 37.88 | 86.79   | 38.64 |
| 100       | 92.39    | 38.27 | 92.05   | 39.07 | 91.71    | 39.87 | 91.35   | 40.67 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 67½ Deg. |       | 67 Deg. |       | 66½ Deg. |       | 66 Deg. |       |

# TRAVERSE TABLE.

83

| Distance. | 24½ Deg. |        | 25 Deg. |        | 25½ Deg. |        | 26 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
| 1         | 0. 91    | 0. 41  | 0. 91   | 0. 42  | 0. 90    | 0. 43  | 0. 90   | 0. 44  |
| 2         | 1. 82    | 0. 83  | 1. 81   | 0. 85  | 1. 81    | 0. 86  | 1. 80   | 0. 88  |
| 3         | 2. 73    | 1. 24  | 2. 72   | 1. 27  | 2. 71    | 1. 29  | 2. 70   | 1. 32  |
| 4         | 3. 64    | 1. 66  | 3. 63   | 1. 69  | 3. 61    | 1. 72  | 3. 60   | 1. 75  |
| 5         | 4. 55    | 2. 07  | 4. 53   | 2. 11  | 4. 51    | 2. 15  | 4. 49   | 2. 19  |
| 6         | 5. 46    | 2. 49  | 5. 44   | 2. 54  | 5. 42    | 2. 58  | 5. 39   | 2. 63  |
| 7         | 6. 37    | 2. 90  | 6. 34   | 2. 96  | 6. 32    | 3. 01  | 6. 29   | 3. 07  |
| 8         | 7. 28    | 3. 32  | 7. 25   | 3. 38  | 7. 22    | 3. 44  | 7. 19   | 3. 51  |
| 9         | 8. 19    | 3. 73  | 8. 16   | 3. 80  | 8. 12    | 3. 87  | 8. 09   | 3. 95  |
| 10        | 9. 10    | 4. 15  | 9. 06   | 4. 23  | 9. 03    | 4. 31  | 8. 99   | 4. 38  |
| 11        | 10. 01   | 4. 56  | 9. 97   | 4. 65  | 9. 93    | 4. 74  | 9. 89   | 4. 82  |
| 12        | 10. 92   | 4. 98  | 10. 88  | 5. 07  | 10. 83   | 5. 17  | 10. 79  | 5. 26  |
| 13        | 11. 83   | 5. 39  | 11. 78  | 5. 49  | 11. 73   | 5. 60  | 11. 68  | 5. 70  |
| 14        | 12. 74   | 5. 81  | 12. 69  | 5. 92  | 12. 64   | 6. 03  | 12. 58  | 6. 14  |
| 15        | 13. 65   | 6. 22  | 13. 59  | 6. 34  | 13. 54   | 6. 46  | 13. 48  | 6. 58  |
| 16        | 14. 56   | 6. 64  | 14. 50  | 6. 76  | 14. 44   | 6. 89  | 14. 38  | 7. 01  |
| 17        | 15. 47   | 7. 05  | 15. 41  | 7. 18  | 15. 34   | 7. 32  | 15. 28  | 7. 45  |
| 18        | 16. 38   | 7. 46  | 16. 31  | 7. 61  | 16. 25   | 7. 75  | 16. 18  | 7. 89  |
| 19        | 17. 19   | 7. 88  | 17. 22  | 8. 03  | 17. 15   | 8. 18  | 17. 08  | 8. 33  |
| 20        | 18. 20   | 8. 29  | 18. 13  | 8. 45  | 18. 05   | 8. 61  | 17. 98  | 8. 77  |
| 21        | 19. 11   | 8. 71  | 19. 03  | 8. 87  | 18. 95   | 9. 04  | 18. 87  | 9. 21  |
| 22        | 20. 02   | 9. 12  | 19. 94  | 9. 30  | 19. 86   | 9. 47  | 19. 77  | 9. 64  |
| 23        | 20. 93   | 9. 54  | 20. 85  | 9. 72  | 20. 76   | 9. 90  | 20. 67  | 10. 08 |
| 24        | 21. 84   | 9. 95  | 21. 75  | 10. 14 | 21. 66   | 10. 33 | 21. 57  | 10. 52 |
| 25        | 22. 75   | 10. 37 | 22. 66  | 10. 57 | 22. 56   | 10. 76 | 22. 47  | 10. 96 |
| 26        | 23. 66   | 10. 78 | 23. 56  | 10. 99 | 23. 47   | 11. 19 | 23. 37  | 11. 40 |
| 27        | 24. 57   | 11. 20 | 24. 47  | 11. 41 | 24. 37   | 11. 62 | 24. 27  | 11. 84 |
| 28        | 25. 48   | 11. 61 | 25. 38  | 11. 83 | 25. 27   | 12. 05 | 25. 17  | 12. 27 |
| 29        | 26. 39   | 12. 03 | 26. 28  | 12. 26 | 26. 17   | 12. 48 | 26. 06  | 12. 71 |
| 30        | 27. 30   | 12. 44 | 27. 19  | 12. 68 | 27. 08   | 12. 92 | 26. 96  | 13. 15 |
| 35        | 31. 85   | 14. 51 | 31. 72  | 14. 79 | 31. 59   | 15. 07 | 31. 46  | 15. 34 |
| 40        | 36. 40   | 16. 59 | 36. 25  | 16. 90 | 36. 10   | 17. 22 | 35. 95  | 17. 53 |
| 45        | 40. 95   | 18. 66 | 40. 78  | 19. 02 | 40. 62   | 19. 37 | 40. 45  | 19. 73 |
| 50        | 45. 50   | 20. 73 | 45. 32  | 21. 13 | 45. 13   | 21. 53 | 44. 94  | 21. 92 |
| 55        | 50. 05   | 22. 81 | 49. 85  | 23. 24 | 49. 64   | 23. 68 | 49. 43  | 24. 11 |
| 60        | 54. 60   | 24. 88 | 54. 38  | 25. 36 | 54. 16   | 25. 83 | 53. 93  | 26. 30 |
| 65        | 59. 15   | 26. 96 | 58. 91  | 27. 47 | 58. 67   | 27. 98 | 58. 42  | 28. 49 |
| 70        | 63. 70   | 29. 03 | 63. 44  | 29. 58 | 63. 18   | 30. 14 | 62. 92  | 30. 69 |
| 75        | 68. 25   | 31. 10 | 67. 97  | 31. 70 | 67. 69   | 32. 29 | 67. 41  | 32. 88 |
| 80        | 72. 80   | 33. 18 | 72. 50  | 33. 81 | 72. 21   | 34. 44 | 71. 90  | 35. 07 |
| 85        | 77. 35   | 35. 25 | 77. 04  | 35. 92 | 76. 72   | 36. 59 | 76. 40  | 37. 26 |
| 90        | 81. 90   | 37. 32 | 81. 57  | 38. 04 | 81. 23   | 38. 75 | 80. 89  | 39. 45 |
| 95        | 86. 45   | 39. 40 | 86. 10  | 40. 15 | 85. 75   | 40. 90 | 85. 39  | 41. 65 |
| 100       | 91. 00   | 41. 47 | 90. 63  | 42. 26 | 90. 26   | 43. 05 | 89. 88  | 43. 84 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 65½ Deg. |        | 66 Deg. |        | 66½ Deg. |        | 67 Deg. |        |

## TRAVERSE TABLE.

| Distance. | 26½ Deg. |        | 27 Deg. |        | 27½ Deg. |        | 28 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
|           |          |        |         |        |          |        |         |        |
| 1         | 0. 89.   | 0. 45  | 0. 89   | 0. 45  | 0. 89    | 0. 46  | 0. 88   | 0. 47  |
| 2         | 1. 79    | 0. 89  | 1. 78   | 0. 91  | 1. 77    | 0. 92  | 1. 77   | 0. 94  |
| 3         | 2. 68    | 1. 34  | 2. 67   | 1. 36  | 2. 66    | 1. 39  | 2. 65   | 1. 41  |
| 4         | 3. 58    | 1. 78  | 3. 56   | 1. 82  | 3. 55    | 1. 85  | 3. 53   | 1. 88  |
| 5         | 4. 57    | 2. 23  | 4. 45   | 2. 27  | 4. 44    | 2. 31  | 4. 41   | 2. 35  |
| 6         | 5. 37    | 2. 68  | 5. 35   | 2. 72  | 5. 32    | 2. 77  | 5. 30   | 2. 82  |
| 7         | 6. 26    | 3. 12  | 6. 24   | 3. 18  | 6. 21    | 3. 23  | 6. 18   | 3. 29  |
| 8         | 7. 16    | 3. 47  | 7. 13   | 3. 63  | 7. 10    | 3. 69  | 7. 06   | 3. 76  |
| 9         | 8. 05    | 4. 02  | 8. 02   | 4. 09  | 7. 98    | 4. 16  | 7. 95   | 4. 23  |
| 10        | 8. 95    | 4. 46  | 8. 91   | 4. 54  | 8. 87    | 4. 62  | 8. 83   | 4. 69  |
| 11        | 9. 84    | 4. 91  | 9. 80   | 4. 99  | 9. 76    | 5. 08  | 9. 71   | 5. 16  |
| 12        | 10. 74   | 5. 35  | 10. 69  | 5. 45  | 10. 64   | 5. 54  | 10. 60  | 5. 63  |
| 13        | 11. 63   | 5. 80  | 11. 58  | 5. 90  | 11. 53   | 6. 00  | 11. 48  | 6. 10  |
| 14        | 12. 53   | 6. 25  | 12. 47  | 6. 36  | 12. 42   | 6. 49  | 12. 36  | 6. 57  |
| 15        | 13. 42   | 6. 69  | 13. 37  | 6. 81  | 13. 31   | 6. 93  | 13. 24  | 7. 04  |
| 16        | 14. 32   | 7. 14  | 14. 26  | 7. 26  | 14. 19   | 7. 39  | 14. 13  | 7. 51  |
| 17        | 15. 21   | 7. 59  | 15. 15  | 7. 72  | 15. 08   | 7. 85  | 15. 01  | 7. 98  |
| 18        | 16. 11   | 8. 03  | 16. 04  | 8. 17  | 15. 97   | 8. 31  | 15. 89  | 8. 45  |
| 19        | 17. 00   | 8. 48  | 16. 93  | 8. 63  | 16. 85   | 8. 77  | 16. 78  | 8. 92  |
| 20        | 17. 90   | 8. 92  | 17. 82  | 9. 08  | 17. 74   | 9. 23  | 17. 66  | 9. 39  |
| 21        | 18. 79   | 9. 37  | 18. 71  | 9. 53  | 18. 63   | 9. 70  | 18. 54  | 9. 86  |
| 22        | 19. 69   | 9. 82  | 19. 60  | 9. 99  | 19. 51   | 10. 16 | 19. 42  | 10. 33 |
| 23        | 20. 58   | 10. 26 | 20. 49  | 10. 44 | 20. 40   | 10. 62 | 20. 31  | 10. 80 |
| 24        | 21. 48   | 10. 71 | 21. 38  | 10. 90 | 21. 29   | 11. 08 | 21. 19  | 11. 27 |
| 25        | 22. 37   | 11. 15 | 22. 28  | 11. 35 | 22. 18   | 11. 54 | 22. 07  | 11. 74 |
| 26        | 23. 27   | 11. 60 | 23. 17  | 11. 80 | 23. 06   | 12. 01 | 22. 96  | 12. 21 |
| 27        | 24. 16   | 12. 05 | 24. 06  | 12. 26 | 23. 95   | 12. 47 | 23. 84  | 12. 68 |
| 28        | 25. 06   | 12. 49 | 24. 95  | 12. 71 | 24. 84   | 12. 93 | 24. 72  | 13. 15 |
| 29        | 25. 95   | 12. 94 | 25. 84  | 13. 17 | 25. 72   | 13. 39 | 25. 61  | 13. 61 |
| 30        | 26. 85   | 13. 39 | 26. 73  | 13. 62 | 26. 61   | 13. 85 | 26. 49  | 14. 08 |
| 35        | 31. 32   | 15. 62 | 31. 19  | 15. 89 | 31. 05   | 16. 16 | 30. 90  | 16. 43 |
| 40        | 35. 80   | 17. 85 | 35. 64  | 18. 16 | 35. 48   | 18. 47 | 35. 32  | 18. 78 |
| 45        | 40. 27   | 20. 08 | 40. 10  | 20. 43 | 39. 92   | 20. 78 | 39. 73  | 21. 13 |
| 50        | 44. 75   | 22. 31 | 44. 55  | 22. 70 | 44. 35   | 23. 09 | 44. 15  | 23. 47 |
| 55        | 49. 22   | 24. 54 | 49. 01  | 24. 97 | 48. 79   | 25. 40 | 48. 56  | 25. 82 |
| 60        | 53. 70   | 26. 77 | 53. 46  | 27. 24 | 53. 22   | 27. 70 | 52. 98  | 28. 17 |
| 65        | 58. 17   | 29. 00 | 57. 92  | 29. 51 | 57. 66   | 30. 01 | 57. 39  | 30. 52 |
| 70        | 62. 65   | 31. 23 | 62. 37  | 31. 78 | 62. 09   | 32. 32 | 61. 81  | 32. 86 |
| 75        | 67. 12   | 33. 46 | 66. 83  | 34. 05 | 66. 53   | 34. 63 | 66. 22  | 35. 21 |
| 80        | 71. 59   | 35. 70 | 71. 28  | 36. 32 | 70. 96   | 36. 94 | 70. 64  | 37. 56 |
| 85        | 76. 07   | 37. 93 | 75. 74  | 38. 59 | 75. 40   | 39. 25 | 75. 05  | 39. 91 |
| 90        | 80. 54   | 40. 16 | 80. 19  | 40. 86 | 79. 83   | 41. 56 | 79. 47  | 42. 25 |
| 95        | 85. 02   | 42. 39 | 84. 65  | 43. 13 | 84. 27   | 43. 87 | 83. 88  | 44. 60 |
| 100       | 89. 49   | 44. 62 | 89. 10  | 45. 40 | 88. 90   | 46. 17 | 88. 29  | 46. 95 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 68½ Deg. |        | 69 Deg. |        | 69½ Deg. |        | 70 Deg. |        |

# TRAVERSE TABLE.

85

| Distance. | 28½ Deg. |       | 29 Deg. |       | 29½ Deg. |       | 30 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
|           |          |       |         |       |          |       |         |       |
| 1         | 0.88     | 0.48  | 0.87    | 0.48  | 0.87     | 0.49  | 0.87    | 0.50  |
| 2         | 1.76     | 0.95  | 1.75    | 0.97  | 1.74     | 0.98  | 1.73    | 1.00  |
| 3         | 2.64     | 1.43  | 2.62    | 1.45  | 2.61     | 1.48  | 2.60    | 1.50  |
| 4         | 3.52     | 1.91  | 3.50    | 1.94  | 3.48     | 1.97  | 3.46    | 2.00  |
| 5         | 4.39     | 2.39  | 4.37    | 2.42  | 4.35     | 2.46  | 4.33    | 2.50  |
| 6         | 5.27     | 2.86  | 5.25    | 2.91  | 5.22     | 2.95  | 5.20    | 3.00  |
| 7         | 6.15     | 3.34  | 6.12    | 3.39  | 6.09     | 3.45  | 6.06    | 3.50  |
| 8         | 7.03     | 3.82  | 7.00    | 3.88  | 6.96     | 3.94  | 6.93    | 4.00  |
| 9         | 7.91     | 4.29  | 7.87    | 4.36  | 7.83     | 4.43  | 7.79    | 4.50  |
| 10        | 8.79     | 4.77  | 8.75    | 4.85  | 8.70     | 4.92  | 8.66    | 5.00  |
| 11        | 9.67     | 5.25  | 9.62    | 5.33  | 9.57     | 5.42  | 9.53    | 5.50  |
| 12        | 10.55    | 5.73  | 10.50   | 5.82  | 10.44    | 5.91  | 10.39   | 6.00  |
| 13        | 11.42    | 6.20  | 11.37   | 6.30  | 11.31    | 6.40  | 11.26   | 6.50  |
| 14        | 12.30    | 6.68  | 12.24   | 6.79  | 12.18    | 6.89  | 12.12   | 7.00  |
| 15        | 13.18    | 7.16  | 13.12   | 7.27  | 13.06    | 7.39  | 12.99   | 7.50  |
| 16        | 14.06    | 7.63  | 13.99   | 7.76  | 13.93    | 7.88  | 13.86   | 8.00  |
| 17        | 14.94    | 8.11  | 14.87   | 8.24  | 14.80    | 8.37  | 14.72   | 8.50  |
| 18        | 15.82    | 8.59  | 15.74   | 8.73  | 15.67    | 8.86  | 15.59   | 9.00  |
| 19        | 16.70    | 9.07  | 16.62   | 9.21  | 16.54    | 9.36  | 16.45   | 9.50  |
| 20        | 17.58    | 9.54  | 17.49   | 9.70  | 17.41    | 9.85  | 17.32   | 10.00 |
| 21        | 18.46    | 10.02 | 18.37   | 10.18 | 18.28    | 10.34 | 18.19   | 10.50 |
| 22        | 19.33    | 10.50 | 19.24   | 10.67 | 19.15    | 10.83 | 19.05   | 11.00 |
| 23        | 20.21    | 10.97 | 20.12   | 11.15 | 20.02    | 11.33 | 19.92   | 11.50 |
| 24        | 21.09    | 11.45 | 20.99   | 11.64 | 20.89    | 11.82 | 20.78   | 12.00 |
| 25        | 21.97    | 11.93 | 21.87   | 12.12 | 21.76    | 12.31 | 21.65   | 12.50 |
| 26        | 22.85    | 12.41 | 22.74   | 12.60 | 22.63    | 12.80 | 22.52   | 13.00 |
| 27        | 23.73    | 12.88 | 23.61   | 13.09 | 23.50    | 13.30 | 23.38   | 13.50 |
| 28        | 24.61    | 13.36 | 24.49   | 13.57 | 24.37    | 13.79 | 24.25   | 14.00 |
| 29        | 25.49    | 13.84 | 25.36   | 14.06 | 25.24    | 14.28 | 25.11   | 14.50 |
| 30        | 26.36    | 14.31 | 26.24   | 14.54 | 26.11    | 14.77 | 25.98   | 15.00 |
| 35        | 30.76    | 16.70 | 30.61   | 16.97 | 30.46    | 17.23 | 30.31   | 17.50 |
| 40        | 35.15    | 19.09 | 34.98   | 19.39 | 34.81    | 19.70 | 34.64   | 20.00 |
| 45        | 39.55    | 21.47 | 39.36   | 21.82 | 39.17    | 22.16 | 38.97   | 22.50 |
| 50        | 43.94    | 23.86 | 43.73   | 24.24 | 43.52    | 24.62 | 43.30   | 25.00 |
| 55        | 48.33    | 26.24 | 48.10   | 26.66 | 47.87    | 27.08 | 47.63   | 27.50 |
| 60        | 52.73    | 28.63 | 52.48   | 29.09 | 52.22    | 29.55 | 51.96   | 30.00 |
| 65        | 57.12    | 31.02 | 56.85   | 31.51 | 56.57    | 32.01 | 56.29   | 32.50 |
| 70        | 61.52    | 33.40 | 61.22   | 33.94 | 60.92    | 34.47 | 60.62   | 35.00 |
| 75        | 65.91    | 35.79 | 65.60   | 36.36 | 65.28    | 36.93 | 64.95   | 37.50 |
| 80        | 70.31    | 38.17 | 69.97   | 38.78 | 69.63    | 39.39 | 69.28   | 40.00 |
| 85        | 74.70    | 40.56 | 74.34   | 41.21 | 73.98    | 41.86 | 73.61   | 42.50 |
| 90        | 79.09    | 42.94 | 78.72   | 43.63 | 78.33    | 44.32 | 77.94   | 45.00 |
| 95        | 83.49    | 45.33 | 83.09   | 46.06 | 82.68    | 46.78 | 82.27   | 47.50 |
| 100       | 87.88    | 47.72 | 87.46   | 48.48 | 87.04    | 49.24 | 86.60   | 50.00 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 61½ Deg. |       | 61 Deg. |       | 60½ Deg. |       | 60 Deg. |       |



## TRAVERSE TABLE.

| Distance. | 30½ Deg. |       | 31 Deg. |       | 31½ Deg. |       | 32 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
|           |          |       |         |       |          |       |         |       |
| 1         | 0.86     | 0.51  | 0.86    | 0.51  | 0.85     | 0.52  | 0.85    | 0.53  |
| 2         | 1.72     | 1.02  | 1.71    | 1.03  | 1.71     | 1.04  | 1.70    | 1.06  |
| 3         | 2.58     | 1.52  | 2.57    | 1.55  | 2.56     | 1.57  | 2.54    | 1.59  |
| 4         | 3.45     | 2.03  | 3.43    | 2.06  | 3.41     | 2.09  | 3.39    | 2.12  |
| 5         | 4.31     | 2.54  | 4.29    | 2.58  | 4.26     | 2.61  | 4.24    | 2.65  |
| 6         | 5.17     | 3.05  | 5.14    | 3.09  | 5.12     | 3.13  | 5.09    | 3.18  |
| 7         | 6.03     | 3.55  | 6.00    | 3.61  | 5.97     | 3.66  | 5.94    | 3.71  |
| 8         | 6.89     | 4.06  | 6.86    | 4.12  | 6.82     | 4.18  | 6.78    | 4.24  |
| 9         | 7.75     | 4.57  | 7.71    | 4.64  | 7.67     | 4.70  | 7.63    | 4.77  |
| 10        | 8.62     | 5.08  | 8.57    | 5.15  | 8.53     | 5.22  | 8.48    | 5.30  |
| 11        | 9.48     | 5.58  | 9.43    | 5.67  | 9.38     | 5.75  | 9.33    | 5.83  |
| 12        | 10.34    | 6.09  | 10.29   | 6.18  | 10.23    | 6.27  | 10.18   | 6.36  |
| 13        | 11.20    | 6.60  | 11.14   | 6.70  | 11.08    | 6.79  | 11.02   | 6.89  |
| 14        | 12.06    | 7.11  | 12.00   | 7.21  | 11.94    | 7.31  | 11.87   | 7.42  |
| 15        | 12.92    | 7.61  | 12.86   | 7.73  | 12.79    | 7.84  | 12.72   | 7.95  |
| 16        | 13.79    | 8.12  | 13.71   | 8.24  | 13.64    | 8.36  | 13.57   | 8.48  |
| 17        | 14.65    | 8.63  | 14.57   | 8.77  | 14.49    | 8.88  | 14.42   | 9.01  |
| 18        | 15.51    | 9.14  | 15.43   | 9.27  | 15.35    | 9.40  | 15.26   | 9.54  |
| 19        | 16.37    | 9.64  | 16.29   | 9.79  | 16.20    | 9.93  | 16.11   | 10.07 |
| 20        | 17.23    | 10.15 | 17.14   | 10.30 | 17.05    | 10.45 | 16.96   | 10.60 |
| 21        | 18.09    | 10.66 | 18.00   | 10.82 | 17.91    | 10.97 | 17.81   | 11.13 |
| 22        | 18.96    | 11.17 | 18.86   | 11.33 | 18.76    | 11.49 | 18.66   | 11.66 |
| 23        | 19.82    | 11.67 | 19.71   | 11.85 | 19.61    | 12.02 | 19.51   | 12.19 |
| 24        | 20.68    | 12.18 | 20.57   | 12.36 | 20.46    | 12.54 | 20.35   | 12.72 |
| 25        | 21.54    | 12.69 | 21.43   | 12.88 | 21.32    | 13.06 | 21.20   | 13.25 |
| 26        | 22.40    | 13.20 | 22.29   | 13.39 | 22.17    | 13.58 | 22.05   | 13.78 |
| 27        | 23.26    | 13.70 | 23.14   | 13.91 | 23.02    | 14.11 | 22.90   | 14.31 |
| 28        | 24.13    | 14.21 | 24.00   | 14.42 | 23.87    | 14.63 | 23.75   | 14.84 |
| 29        | 24.99    | 14.72 | 24.86   | 14.94 | 24.73    | 15.15 | 24.59   | 15.37 |
| 30        | 25.85    | 15.23 | 25.71   | 15.45 | 25.58    | 15.67 | 25.44   | 15.90 |
| 35        | 30.16    | 17.76 | 30.00   | 18.03 | 29.84    | 18.29 | 29.68   | 18.55 |
| 40        | 34.47    | 20.30 | 34.29   | 20.60 | 34.11    | 20.90 | 33.92   | 21.20 |
| 45        | 38.77    | 22.84 | 38.57   | 23.18 | 38.37    | 23.51 | 38.16   | 23.85 |
| 50        | 43.08    | 25.38 | 42.86   | 25.75 | 42.63    | 26.12 | 42.40   | 26.50 |
| 55        | 47.39    | 27.91 | 47.14   | 28.33 | 46.90    | 28.74 | 46.64   | 29.15 |
| 60        | 51.70    | 30.45 | 51.53   | 30.90 | 51.16    | 31.35 | 50.88   | 31.80 |
| 65        | 56.01    | 32.99 | 55.72   | 33.48 | 55.42    | 33.96 | 55.12   | 34.44 |
| 70        | 60.31    | 35.53 | 60.00   | 36.05 | 59.68    | 36.57 | 59.36   | 37.09 |
| 75        | 64.62    | 38.07 | 64.29   | 38.63 | 63.95    | 39.19 | 63.60   | 39.74 |
| 80        | 68.93    | 40.60 | 68.57   | 41.20 | 68.21    | 41.80 | 67.84   | 42.39 |
| 85        | 73.24    | 43.14 | 72.86   | 43.78 | 72.47    | 44.41 | 72.08   | 45.04 |
| 90        | 77.55    | 45.68 | 77.15   | 46.35 | 76.74    | 47.02 | 76.32   | 47.69 |
| 95        | 81.85    | 48.22 | 81.43   | 48.93 | 81.00    | 49.64 | 80.56   | 50.34 |
| 100       | 86.16    | 50.75 | 85.72   | 51.50 | 85.26    | 52.25 | 84.80   | 52.59 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 50½ Deg. |       | 50 Deg. |       | 50½ Deg. |       | 50 Deg. |       |

# TRAVERSE TABLE.

87

| Distance. | 32½ Deg. |       | 33 Deg. |       | 33½ Deg. |       | 34 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.84     | 0.54  | 0.84    | 0.54  | 0.83     | 0.55  | 0.83    | 0.56  |
| 2         | 1.69     | 1.07  | 1.68    | 1.09  | 1.67     | 1.10  | 1.66    | 1.12  |
| 3         | 2.53     | 1.61  | 2.52    | 1.63  | 2.50     | 1.66  | 2.49    | 1.68  |
| 4         | 3.37     | 2.15  | 3.35    | 2.18  | 3.34     | 2.21  | 3.32    | 2.24  |
| 5         | 4.22     | 2.69  | 4.19    | 2.72  | 4.17     | 2.76  | 4.15    | 2.80  |
| 6         | 5.06     | 3.22  | 5.03    | 3.27  | 5.00     | 3.31  | 4.97    | 3.36  |
| 7         | 5.90     | 3.76  | 5.87    | 3.81  | 5.84     | 3.86  | 5.80    | 3.91  |
| 8         | 6.75     | 4.30  | 6.71    | 4.36  | 6.67     | 4.42  | 6.63    | 4.47  |
| 9         | 7.59     | 4.84  | 7.55    | 4.90  | 7.50     | 4.97  | 7.46    | 5.03  |
| 10        | 8.43     | 5.37  | 8.39    | 5.45  | 8.34     | 5.52  | 8.29    | 5.59  |
| 11        | 9.28     | 5.91  | 9.23    | 5.99  | 9.17     | 6.07  | 9.12    | 6.15  |
| 12        | 10.12    | 6.45  | 10.06   | 6.54  | 10.01    | 6.62  | 9.95    | 6.71  |
| 13        | 10.96    | 6.98  | 10.90   | 7.08  | 10.84    | 7.18  | 10.78   | 7.27  |
| 14        | 11.81    | 7.52  | 11.74   | 7.62  | 11.67    | 7.73  | 11.61   | 7.83  |
| 15        | 12.65    | 8.06  | 12.58   | 8.17  | 12.51    | 8.28  | 12.44   | 8.39  |
| 16        | 13.49    | 8.60  | 13.42   | 8.71  | 13.34    | 8.83  | 13.26   | 8.95  |
| 17        | 14.34    | 9.13  | 14.26   | 9.26  | 14.18    | 9.38  | 14.09   | 9.51  |
| 18        | 15.18    | 9.67  | 15.10   | 9.80  | 15.01    | 9.93  | 14.92   | 10.07 |
| 19        | 16.02    | 10.21 | 15.93   | 10.35 | 15.84    | 10.49 | 15.75   | 10.62 |
| 20        | 16.87    | 10.75 | 16.77   | 10.89 | 16.68    | 11.04 | 16.58   | 11.18 |
| 21        | 17.71    | 11.28 | 17.61   | 11.44 | 17.51    | 11.59 | 17.41   | 11.74 |
| 22        | 18.55    | 11.82 | 18.45   | 11.98 | 18.35    | 12.14 | 18.24   | 12.30 |
| 23        | 19.40    | 12.36 | 19.29   | 12.53 | 19.18    | 12.69 | 19.07   | 12.86 |
| 24        | 20.28    | 12.90 | 20.13   | 13.07 | 20.01    | 13.25 | 19.90   | 13.42 |
| 25        | 21.08    | 13.43 | 20.97   | 13.62 | 20.85    | 13.80 | 20.73   | 13.98 |
| 26        | 21.93    | 13.97 | 21.81   | 14.16 | 21.68    | 14.35 | 21.55   | 14.54 |
| 27        | 22.77    | 14.51 | 22.64   | 14.71 | 22.51    | 14.90 | 22.38   | 15.10 |
| 28        | 23.61    | 15.04 | 23.48   | 15.25 | 23.35    | 15.45 | 23.21   | 15.66 |
| 29        | 24.46    | 15.58 | 24.32   | 15.97 | 24.18    | 16.01 | 24.04   | 16.22 |
| 30        | 25.30    | 16.12 | 25.16   | 16.34 | 25.02    | 16.56 | 24.87   | 16.78 |
| 35        | 29.52    | 18.81 | 29.35   | 19.06 | 29.19    | 19.32 | 29.02   | 19.57 |
| 40        | 33.74    | 21.49 | 33.55   | 21.79 | 33.36    | 22.08 | 33.16   | 22.37 |
| 45        | 37.95    | 24.18 | 37.74   | 24.51 | 37.52    | 24.84 | 37.31   | 25.16 |
| 50        | 42.17    | 26.86 | 41.93   | 27.23 | 41.69    | 27.60 | 41.45   | 27.96 |
| 55        | 46.39    | 29.55 | 46.13   | 29.96 | 45.86    | 30.36 | 45.60   | 30.76 |
| 60        | 50.60    | 32.24 | 50.32   | 32.68 | 50.08    | 33.12 | 49.74   | 33.55 |
| 65        | 54.82    | 34.92 | 54.51   | 35.40 | 54.20    | 35.88 | 53.89   | 36.35 |
| 70        | 59.04    | 37.61 | 58.72   | 38.12 | 58.37    | 38.64 | 58.03   | 39.14 |
| 75        | 63.25    | 40.30 | 62.90   | 40.85 | 62.54    | 41.40 | 62.18   | 41.94 |
| 80        | 67.47    | 42.98 | 67.09   | 43.57 | 66.71    | 44.15 | 66.32   | 44.74 |
| 85        | 71.69    | 45.67 | 71.29   | 46.29 | 70.88    | 46.91 | 70.47   | 47.53 |
| 90        | 75.91    | 48.36 | 75.48   | 49.02 | 75.05    | 49.67 | 74.61   | 50.33 |
| 95        | 80.12    | 51.04 | 79.67   | 51.74 | 79.22    | 54.43 | 78.76   | 53.12 |
| 100       | 84.34    | 53.73 | 83.87   | 54.46 | 83.39    | 55.19 | 82.90   | 55.92 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 57½ Deg. |       | 57 Deg. |       | 56½ Deg. |       | 56 Deg. |       |

## TRAVERSE TABLE.

| Distance. | 34½ Deg. |       | 35 Deg. |       | 35½ Deg. |       | 36 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
|           |          |       |         |       |          |       |         |       |
| 1         | 0.82     | 0.57  | 0.82    | 0.57  | 0.81     | 0.58  | 0.81    | 0.59  |
| 2         | 1.65     | 1.13  | 1.64    | 1.15  | 1.63     | 1.16  | 1.62    | 1.18  |
| 3         | 2.47     | 1.70  | 2.46    | 1.72  | 2.44     | 1.74  | 2.43    | 1.76  |
| 4         | 3.30     | 2.27  | 3.28    | 2.29  | 3.26     | 2.32  | 3.24    | 2.35  |
| 5         | 4.12     | 2.83  | 4.10    | 2.87  | 4.07     | 2.90  | 4.05    | 2.94  |
| 6         | 4.94     | 3.40  | 4.91    | 3.44  | 4.88     | 3.48  | 4.85    | 3.53  |
| 7         | 5.77     | 3.96  | 5.73    | 4.01  | 5.70     | 4.06  | 5.66    | 4.11  |
| 8         | 6.59     | 4.53  | 6.55    | 4.59  | 6.51     | 4.65  | 6.47    | 4.70  |
| 9         | 7.42     | 5.10  | 7.37    | 5.16  | 7.33     | 5.23  | 7.28    | 5.29  |
| 10        | 8.24     | 5.66  | 8.19    | 5.74  | 8.14     | 5.81  | 8.09    | 5.88  |
| 11        | 9.07     | 6.23  | 9.01    | 6.31  | 8.96     | 6.39  | 8.90    | 6.47  |
| 12        | 9.89     | 6.80  | 9.83    | 6.88  | 9.77     | 6.97  | 9.71    | 7.05  |
| 13        | 10.71    | 7.36  | 10.65   | 7.46  | 10.58    | 7.55  | 10.52   | 7.64  |
| 14        | 11.54    | 7.93  | 11.47   | 8.03  | 11.40    | 8.13  | 11.33   | 8.23  |
| 15        | 12.36    | 8.50  | 12.29   | 8.60  | 12.21    | 8.71  | 12.14   | 8.82  |
| 16        | 13.19    | 9.06  | 13.11   | 9.18  | 13.03    | 9.29  | 12.94   | 9.40  |
| 17        | 14.01    | 9.63  | 13.93   | 9.75  | 13.84    | 9.87  | 13.75   | 9.99  |
| 18        | 14.83    | 10.20 | 14.74   | 10.32 | 14.65    | 10.45 | 14.56   | 10.58 |
| 19        | 15.66    | 10.76 | 15.56   | 10.90 | 15.47    | 11.03 | 15.37   | 11.17 |
| 20        | 16.48    | 11.33 | 16.38   | 11.47 | 16.28    | 11.61 | 16.18   | 11.76 |
| 21        | 17.31    | 11.89 | 17.20   | 12.05 | 17.10    | 12.19 | 16.99   | 12.34 |
| 22        | 18.13    | 12.46 | 18.02   | 12.62 | 17.91    | 12.78 | 17.80   | 12.93 |
| 23        | 18.95    | 13.03 | 18.84   | 13.19 | 18.72    | 13.36 | 18.61   | 13.52 |
| 24        | 19.78    | 13.59 | 19.66   | 13.77 | 19.54    | 13.94 | 19.42   | 14.11 |
| 25        | 20.60    | 14.16 | 20.48   | 14.34 | 20.35    | 14.52 | 20.23   | 14.69 |
| 26        | 21.43    | 14.73 | 21.30   | 14.91 | 21.17    | 15.10 | 21.03   | 15.28 |
| 27        | 22.25    | 15.29 | 22.12   | 15.49 | 21.98    | 15.68 | 21.84   | 15.87 |
| 28        | 23.08    | 15.86 | 22.94   | 16.06 | 22.80    | 16.26 | 22.65   | 16.46 |
| 29        | 23.90    | 16.43 | 23.76   | 16.63 | 23.61    | 16.84 | 23.46   | 17.05 |
| 30        | 24.72    | 16.99 | 24.57   | 17.21 | 24.42    | 17.42 | 24.27   | 17.63 |
| 35        | 28.84    | 19.82 | 28.67   | 20.08 | 28.49    | 20.32 | 28.32   | 20.57 |
| 40        | 32.97    | 22.66 | 32.77   | 22.94 | 32.56    | 23.23 | 32.36   | 23.51 |
| 45        | 37.09    | 25.49 | 36.86   | 25.81 | 36.64    | 26.13 | 36.41   | 26.45 |
| 50        | 41.21    | 28.32 | 40.96   | 28.68 | 40.71    | 29.04 | 40.45   | 29.39 |
| 55        | 45.33    | 31.15 | 45.05   | 31.55 | 44.78    | 31.94 | 44.50   | 32.23 |
| 60        | 49.45    | 33.98 | 49.15   | 34.41 | 48.85    | 34.84 | 48.54   | 35.27 |
| 65        | 53.57    | 36.82 | 53.24   | 37.28 | 52.92    | 37.75 | 52.59   | 38.21 |
| 70        | 57.69    | 39.65 | 57.34   | 40.15 | 56.99    | 40.65 | 56.63   | 41.14 |
| 75        | 61.81    | 42.48 | 61.44   | 43.02 | 61.06    | 43.55 | 60.68   | 44.08 |
| 80        | 65.93    | 45.31 | 65.53   | 45.89 | 65.13    | 46.46 | 64.72   | 47.02 |
| 85        | 70.05    | 48.14 | 69.63   | 48.75 | 69.20    | 49.36 | 68.77   | 49.96 |
| 90        | 74.17    | 50.98 | 73.72   | 51.62 | 73.27    | 52.26 | 72.81   | 52.90 |
| 95        | 78.29    | 53.81 | 77.82   | 54.49 | 77.34    | 55.17 | 76.86   | 55.84 |
| 100       | 82.41    | 56.64 | 81.92   | 57.36 | 81.41    | 58.07 | 80.90   | 58.78 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 55½ Deg. |       | 55 Deg. |       | 54½ Deg. |       | 54 Deg. |       |

# TRAVERSE TABLE.

89

| Distance. | 36½ Deg. |       | 37 Deg. |       | 37½ Deg. |       | 38 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.80     | 0.59  | 0.80    | 0.60  | 0.79     | 0.61  | 0.79    | 0.62  |
| 2         | 1.61     | 1.19  | 1.60    | 1.20  | 1.59     | 1.22  | 1.58    | 1.23  |
| 3         | 2.41     | 1.78  | 2.40    | 1.81  | 2.38     | 1.83  | 2.36    | 1.85  |
| 4         | 3.22     | 2.38  | 3.19    | 2.41  | 3.17     | 2.43  | 3.15    | 2.46  |
| 5         | 4.02     | 2.97  | 3.99    | 3.01  | 3.97     | 3.04  | 3.94    | 3.08  |
| 6         | 4.82     | 3.57  | 4.79    | 3.61  | 4.76     | 3.65  | 4.73    | 3.69  |
| 7         | 5.63     | 4.16  | 5.59    | 4.21  | 5.55     | 4.20  | 5.52    | 4.31  |
| 8         | 6.43     | 4.76  | 6.39    | 4.81  | 6.35     | 4.87  | 6.30    | 4.93  |
| 9         | 7.23     | 5.35  | 7.19    | 5.42  | 7.14     | 5.48  | 7.09    | 5.54  |
| 10        | 8.04     | 5.95  | 7.99    | 6.02  | 7.93     | 6.09  | 7.88    | 6.16  |
| 11        | 8.84     | 6.54  | 8.78    | 6.62  | 8.73     | 6.70  | 8.67    | 6.77  |
| 12        | 9.65     | 7.14  | 9.58    | 7.22  | 9.52     | 7.31  | 9.46    | 7.39  |
| 13        | 10.45    | 7.73  | 10.38   | 7.82  | 10.31    | 7.91  | 10.24   | 8.00  |
| 14        | 11.25    | 8.33  | 11.18   | 8.43  | 11.11    | 8.52  | 11.03   | 8.62  |
| 15        | 12.06    | 8.92  | 11.98   | 9.03  | 11.90    | 9.13  | 11.82   | 9.23  |
| 16        | 12.86    | 9.52  | 12.78   | 9.63  | 12.69    | 9.74  | 12.61   | 9.85  |
| 17        | 13.67    | 10.11 | 13.58   | 10.23 | 13.49    | 10.35 | 13.40   | 10.47 |
| 18        | 14.47    | 10.71 | 14.38   | 10.83 | 14.28    | 10.96 | 14.18   | 11.08 |
| 19        | 15.27    | 11.30 | 15.17   | 11.43 | 15.07    | 11.57 | 14.97   | 11.70 |
| 20        | 16.08    | 11.90 | 15.97   | 12.04 | 15.87    | 12.18 | 15.76   | 12.31 |
| 21        | 16.88    | 12.49 | 16.77   | 12.64 | 16.66    | 12.78 | 16.55   | 12.93 |
| 22        | 17.68    | 13.09 | 17.57   | 13.24 | 17.45    | 13.39 | 17.34   | 13.54 |
| 23        | 18.49    | 13.68 | 18.37   | 13.84 | 18.25    | 14.00 | 18.12   | 14.16 |
| 24        | 19.29    | 14.28 | 19.17   | 14.44 | 19.04    | 14.61 | 18.91   | 14.78 |
| 25        | 20.10    | 14.87 | 19.97   | 15.05 | 19.83    | 15.22 | 19.70   | 15.39 |
| 26        | 20.90    | 15.47 | 20.76   | 15.65 | 20.63    | 15.83 | 20.49   | 16.01 |
| 27        | 21.70    | 16.06 | 21.56   | 16.25 | 21.42    | 16.44 | 21.28   | 16.62 |
| 28        | 22.51    | 16.65 | 22.36   | 16.85 | 22.21    | 17.05 | 22.06   | 17.24 |
| 29        | 23.31    | 17.25 | 23.16   | 17.45 | 23.01    | 17.65 | 22.85   | 17.85 |
| 30        | 24.12    | 17.84 | 23.96   | 18.05 | 23.80    | 18.26 | 23.64   | 18.47 |
| 35        | 28.13    | 20.82 | 27.95   | 21.06 | 27.77    | 21.31 | 27.58   | 21.55 |
| 40        | 32.15    | 23.79 | 31.95   | 24.07 | 31.73    | 24.35 | 31.52   | 24.63 |
| 45        | 36.17    | 26.77 | 35.94   | 27.08 | 35.70    | 27.39 | 35.46   | 27.70 |
| 50        | 40.19    | 29.74 | 39.93   | 30.09 | 39.67    | 30.44 | 39.40   | 30.78 |
| 55        | 44.21    | 32.72 | 43.92   | 33.10 | 43.63    | 33.48 | 43.34   | 33.86 |
| 60        | 48.23    | 35.69 | 47.92   | 36.11 | 47.60    | 36.53 | 47.28   | 36.94 |
| 65        | 52.25    | 38.66 | 51.91   | 39.12 | 51.57    | 39.57 | 51.22   | 40.02 |
| 70        | 56.27    | 41.64 | 55.90   | 42.13 | 55.53    | 42.61 | 55.16   | 43.10 |
| 75        | 60.29    | 44.61 | 59.90   | 45.14 | 59.50    | 45.66 | 59.10   | 46.17 |
| 80        | 64.31    | 47.59 | 63.89   | 48.15 | 63.47    | 48.70 | 63.04   | 49.25 |
| 85        | 68.33    | 50.56 | 67.88   | 51.15 | 67.43    | 51.74 | 66.98   | 52.33 |
| 90        | 72.35    | 53.53 | 71.88   | 54.16 | 71.40    | 54.79 | 70.92   | 55.41 |
| 95        | 76.37    | 56.51 | 75.87   | 57.17 | 75.37    | 57.83 | 74.86   | 58.49 |
| 100       | 80.39    | 59.48 | 79.86   | 60.18 | 79.34    | 60.88 | 78.80   | 61.57 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 53½ Deg. |       | 53 Deg. |       | 52½ Deg. |       | 52 Deg. |       |

## TRAVERSE TABLE.

| Distance. | 38½ Deg. |        | 39 Deg. |        | 39½ Deg. |        | 40 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
|           |          |        |         |        |          |        |         |        |
| 1         | 0. 78    | 0. 62  | 0. 78   | 0. 63  | 0. 77    | 0. 64  | 0. 77   | 0. 64  |
| 2         | 1. 57    | 1. 24  | 1. 55   | 1. 26  | 1. 54    | 1. 27  | 1. 53   | 1. 29  |
| 3         | 2. 35    | 1. 87  | 2. 33   | 1. 89  | 2. 31    | 1. 91  | 2. 30   | 1. 93  |
| 4         | 3. 13    | 2. 49  | 3. 11   | 2. 52  | 3. 09    | 2. 54  | 3. 06   | 2. 57  |
| 5         | 3. 91    | 3. 11  | 3. 89   | 3. 15  | 3. 86    | 3. 18  | 3. 83   | 3. 21  |
| 6         | 4. 70    | 3. 74  | 4. 66   | 3. 98  | 4. 63    | 3. 82  | 4. 60   | 3. 86  |
| 7         | 5. 48    | 4. 36  | 5. 44   | 4. 41  | 5. 40    | 4. 45  | 5. 36   | 4. 50  |
| 8         | 6. 26    | 4. 98  | 6. 22   | 5. 03  | 6. 17    | 5. 09  | 6. 13   | 5. 14  |
| 9         | 7. 04    | 5. 60  | 6. 99   | 5. 66  | 6. 94    | 5. 72  | 6. 89   | 5. 79  |
| 10        | 7. 83    | 6. 23  | 7. 77   | 6. 29  | 7. 72    | 6. 36  | 7. 66   | 6. 43  |
| 11        | 8. 61    | 6. 85  | 8. 55   | 6. 92  | 8. 49    | 7. 00  | 8. 43   | 7. 07  |
| 12        | 9. 39    | 7. 47  | 9. 33   | 7. 55  | 9. 26    | 7. 63  | 9. 19   | 7. 71  |
| 13        | 10. 17   | 8. 09  | 10. 10  | 8. 18  | 10. 03   | 8. 27  | 9. 96   | 8. 36  |
| 14        | 10. 96   | 8. 72  | 10. 88  | 8. 81  | 10. 80   | 8. 91  | 10. 72  | 9. 00  |
| 15        | 11. 74   | 9. 34  | 11. 66  | 9. 44  | 11. 57   | 9. 54  | 11. 49  | 9. 64  |
| 16        | 12. 52   | 9. 96  | 12. 43  | 10. 07 | 12. 35   | 10. 18 | 12. 26  | 10. 28 |
| 17        | 13. 30   | 10. 58 | 13. 21  | 10. 70 | 13. 12   | 10. 81 | 13. 02  | 10. 93 |
| 18        | 14. 09   | 11. 21 | 13. 99  | 11. 33 | 13. 89   | 11. 45 | 13. 79  | 11. 57 |
| 19        | 14. 87   | 11. 83 | 14. 77  | 11. 96 | 14. 66   | 12. 09 | 14. 55  | 12. 21 |
| 20        | 15. 65   | 12. 45 | 15. 54  | 12. 59 | 15. 43   | 12. 72 | 15. 32  | 12. 86 |
| 21        | 16. 43   | 13. 07 | 16. 32  | 13. 22 | 16. 20   | 13. 36 | 16. 09  | 13. 50 |
| 22        | 17. 22   | 13. 70 | 17. 10  | 13. 84 | 16. 98   | 13. 99 | 16. 85  | 14. 14 |
| 23        | 18. 00   | 14. 32 | 17. 87  | 14. 47 | 17. 75   | 14. 63 | 17. 62  | 14. 78 |
| 24        | 18. 78   | 14. 94 | 18. 65  | 15. 10 | 18. 52   | 15. 27 | 18. 39  | 15. 43 |
| 25        | 19. 57   | 15. 56 | 19. 43  | 15. 73 | 19. 29   | 15. 90 | 19. 15  | 16. 07 |
| 26        | 20. 35   | 16. 19 | 20. 21  | 16. 36 | 20. 06   | 16. 54 | 19. 92  | 16. 71 |
| 27        | 21. 13   | 16. 81 | 20. 98  | 16. 99 | 20. 83   | 17. 17 | 20. 68  | 17. 36 |
| 28        | 21. 91   | 17. 43 | 21. 76  | 17. 62 | 21. 61   | 17. 81 | 21. 45  | 18. 00 |
| 29        | 22. 70   | 18. 05 | 22. 54  | 18. 25 | 22. 38   | 18. 45 | 22. 22  | 18. 64 |
| 30        | 23. 48   | 18. 68 | 23. 31  | 18. 88 | 23. 15   | 19. 08 | 22. 98  | 19. 28 |
| 35        | 27. 39   | 21. 79 | 27. 20  | 22. 03 | 27. 01   | 22. 26 | 26. 81  | 22. 50 |
| 40        | 31. 30   | 24. 90 | 31. 09  | 25. 17 | 30. 86   | 25. 44 | 30. 64  | 25. 71 |
| 45        | 35. 22   | 28. 01 | 34. 97  | 28. 32 | 34. 72   | 28. 62 | 34. 47  | 28. 93 |
| 50        | 39. 13   | 31. 13 | 38. 86  | 31. 47 | 38. 58   | 31. 80 | 38. 30  | 32. 14 |
| 55        | 43. 04   | 34. 24 | 42. 74  | 34. 61 | 42. 44   | 34. 98 | 42. 13  | 35. 35 |
| 60        | 46. 96   | 37. 35 | 46. 63  | 37. 76 | 46. 30   | 38. 16 | 45. 96  | 38. 57 |
| 65        | 50. 87   | 40. 46 | 50. 51  | 40. 91 | 50. 16   | 41. 35 | 49. 79  | 41. 78 |
| 70        | 54. 78   | 43. 58 | 54. 40  | 44. 05 | 54. 01   | 44. 53 | 53. 62  | 45. 00 |
| 75        | 58. 70   | 46. 69 | 58. 29  | 47. 20 | 57. 87   | 47. 71 | 57. 45  | 48. 21 |
| 80        | 62. 61   | 49. 80 | 62. 17  | 50. 35 | 61. 73   | 50. 89 | 61. 28  | 51. 42 |
| 85        | 66. 52   | 52. 91 | 66. 06  | 53. 49 | 65. 59   | 54. 07 | 65. 11  | 54. 64 |
| 90        | 70. 43   | 56. 03 | 69. 94  | 56. 64 | 69. 45   | 57. 25 | 68. 94  | 57. 85 |
| 95        | 74. 35   | 59. 14 | 73. 83  | 59. 79 | 73. 30   | 60. 43 | 72. 77  | 61. 06 |
| 100       | 78. 26   | 62. 25 | 77. 71  | 62. 93 | 77. 16   | 63. 61 | 76. 60  | 64. 28 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 51½ Deg. |        | 51 Deg. |        | 50½ Deg. |        | 50 Deg. |        |

# TRAVERSE TABLE.

91

| Distance. | 40½ Deg. |       | 41 Deg. |       | 41½ Deg. |       | 42 Deg. |       |
|-----------|----------|-------|---------|-------|----------|-------|---------|-------|
|           | Lat.     | Dep.  | Lat.    | Dep.  | Lat.     | Dep.  | Lat.    | Dep.  |
| 1         | 0.76     | 0.65  | 0.75    | 0.66  | 0.75     | 0.66  | 0.74    | 0.67  |
| 2         | 1.52     | 1.30  | 1.51    | 1.31  | 1.50     | 1.33  | 1.49    | 1.34  |
| 3         | 2.28     | 1.95  | 2.26    | 1.97  | 2.25     | 1.99  | 2.23    | 2.01  |
| 4         | 3.04     | 2.60  | 3.02    | 2.62  | 3.00     | 2.65  | 2.97    | 2.68  |
| 5         | 3.80     | 3.25  | 3.77    | 3.28  | 3.74     | 3.31  | 3.72    | 3.35  |
| 6         | 4.56     | 3.90  | 4.53    | 3.94  | 4.49     | 3.98  | 4.46    | 4.01  |
| 7         | 5.32     | 4.55  | 5.28    | 4.59  | 5.24     | 4.64  | 5.20    | 4.68  |
| 8         | 6.08     | 5.20  | 6.04    | 5.25  | 5.99     | 5.30  | 5.95    | 5.35  |
| 9         | 6.84     | 5.84  | 6.79    | 5.90  | 6.74     | 5.96  | 6.69    | 6.02  |
| 10        | 7.60     | 6.49  | 7.55    | 6.56  | 7.49     | 6.63  | 7.43    | 6.69  |
| 11        | 8.36     | 7.14  | 8.30    | 7.22  | 8.24     | 7.29  | 8.17    | 7.36  |
| 12        | 9.12     | 7.79  | 9.06    | 7.87  | 8.99     | 7.95  | 8.92    | 8.03  |
| 13        | 9.89     | 8.44  | 9.81    | 8.53  | 9.74     | 8.61  | 9.66    | 8.70  |
| 14        | 10.65    | 9.09  | 10.57   | 9.18  | 10.49    | 9.28  | 10.40   | 9.37  |
| 15        | 11.41    | 9.74  | 11.32   | 9.84  | 11.23    | 9.94  | 11.15   | 10.04 |
| 16        | 12.17    | 10.39 | 12.08   | 10.50 | 11.98    | 10.60 | 11.89   | 10.71 |
| 17        | 12.93    | 11.04 | 12.83   | 11.15 | 12.73    | 11.26 | 12.63   | 11.38 |
| 18        | 13.69    | 11.69 | 13.58   | 11.81 | 13.48    | 11.93 | 13.38   | 12.04 |
| 19        | 14.45    | 12.34 | 14.34   | 12.47 | 14.23    | 12.59 | 14.12   | 12.71 |
| 20        | 15.21    | 12.99 | 15.09   | 13.12 | 14.98    | 13.25 | 14.86   | 13.38 |
| 21        | 15.97    | 13.64 | 15.85   | 13.78 | 15.73    | 13.91 | 15.61   | 14.05 |
| 22        | 16.73    | 14.29 | 16.60   | 14.43 | 16.48    | 14.58 | 16.35   | 14.72 |
| 23        | 17.49    | 14.94 | 17.36   | 15.09 | 17.23    | 15.24 | 17.09   | 15.39 |
| 24        | 18.25    | 15.59 | 18.11   | 15.75 | 17.97    | 15.90 | 17.84   | 16.06 |
| 25        | 19.01    | 16.24 | 18.87   | 16.40 | 18.72    | 16.57 | 18.58   | 16.73 |
| 26        | 19.77    | 16.89 | 19.62   | 17.06 | 19.47    | 17.23 | 19.32   | 17.40 |
| 27        | 20.53    | 17.54 | 20.38   | 17.71 | 20.22    | 17.89 | 20.06   | 18.07 |
| 28        | 21.29    | 18.18 | 21.13   | 18.37 | 20.97    | 18.55 | 20.81   | 18.74 |
| 29        | 22.05    | 18.83 | 21.89   | 19.03 | 21.72    | 19.22 | 21.55   | 19.40 |
| 30        | 22.81    | 19.48 | 22.64   | 19.68 | 22.47    | 19.88 | 22.29   | 20.07 |
| 35        | 26.61    | 22.73 | 26.41   | 22.96 | 26.21    | 23.19 | 26.01   | 23.42 |
| 40        | 30.42    | 25.98 | 30.19   | 26.24 | 29.96    | 26.50 | 29.73   | 26.77 |
| 45        | 34.22    | 29.23 | 33.96   | 29.52 | 33.70    | 29.82 | 33.44   | 30.11 |
| 50        | 38.02    | 32.47 | 37.74   | 32.80 | 37.45    | 33.13 | 37.16   | 33.46 |
| 55        | 41.82    | 35.72 | 41.51   | 36.08 | 41.19    | 36.44 | 40.87   | 36.80 |
| 60        | 45.62    | 38.97 | 45.28   | 39.36 | 44.94    | 39.76 | 44.59   | 40.15 |
| 65        | 49.43    | 42.21 | 49.06   | 42.64 | 48.68    | 43.07 | 48.30   | 43.49 |
| 70        | 53.23    | 45.46 | 52.83   | 45.92 | 52.43    | 46.38 | 52.02   | 46.84 |
| 75        | 57.03    | 48.71 | 56.60   | 49.20 | 56.17    | 49.70 | 55.74   | 50.18 |
| 80        | 60.83    | 51.96 | 60.38   | 52.48 | 59.92    | 53.01 | 59.45   | 53.53 |
| 85        | 64.63    | 55.20 | 64.15   | 55.76 | 63.66    | 56.32 | 63.17   | 56.88 |
| 90        | 68.44    | 58.45 | 67.92   | 59.05 | 67.41    | 59.64 | 66.88   | 60.22 |
| 95        | 72.24    | 61.70 | 71.70   | 62.33 | 71.15    | 62.95 | 70.60   | 63.57 |
| 100       | 76.04    | 64.98 | 75.47   | 65.61 | 74.90    | 66.26 | 74.31   | 66.91 |
|           | Dep.     | Lat.  | Dep.    | Lat.  | Dep.     | Lat.  | Dep.    | Lat.  |
|           | 40½ Deg. |       | 41 Deg. |       | 41½ Deg. |       | 42 Deg. |       |

## TRAVERSE TABLE.

| Distance. | 42½ Deg. |        | 43 Deg. |        | 43½ Deg. |        | 44 Deg. |        |
|-----------|----------|--------|---------|--------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   | Lat.     | Dep.   | Lat.    | Dep.   |
|           |          |        |         |        |          |        |         |        |
| 1         | 0. 74    | 0. 68  | 0. 73   | 0. 68  | 0. 73    | 0. 69  | 0. 72   | 0. 69  |
| 2         | 1. 47    | 1. 35  | 1. 46   | 1. 36  | 1. 45    | 1. 38  | 1. 44   | 1. 39  |
| 3         | 2. 21    | 2. 03  | 2. 19   | 2. 05  | 2. 18    | 2. 07  | 2. 16   | 2. 08  |
| 4         | 2. 95    | 2. 70  | 2. 93   | 2. 73  | 2. 90    | 2. 75  | 2. 88   | 2. 78  |
| 5         | 3. 69    | 3. 38  | 3. 66   | 3. 41  | 3. 63    | 3. 44  | 3. 60   | 3. 47  |
| 6         | 4. 42    | 4. 05  | 4. 39   | 4. 09  | 4. 35    | 4. 13  | 4. 32   | 4. 17  |
| 7         | 5. 16    | 4. 73  | 5. 12   | 4. 77  | 5. 08    | 4. 82  | 5. 04   | 4. 86  |
| 8         | 5. 90    | 5. 40  | 5. 85   | 5. 46  | 5. 80    | 5. 51  | 5. 75   | 5. 56  |
| 9         | 6. 64    | 6. 08  | 6. 58   | 6. 14  | 6. 53    | 6. 20  | 6. 47   | 6. 25  |
| 10        | 7. 37    | 6. 76  | 7. 31   | 6. 82  | 7. 25    | 6. 88  | 7. 19   | 6. 95  |
| 11        | 8. 11    | 7. 43  | 8. 04   | 7. 50  | 7. 98    | 7. 57  | 7. 91   | 7. 64  |
| 12        | 8. 85    | 8. 11  | 8. 78   | 8. 18  | 8. 70    | 8. 26  | 8. 63   | 8. 34  |
| 13        | 9. 58    | 8. 78  | 9. 51   | 8. 87  | 9. 43    | 8. 95  | 9. 35   | 9. 03  |
| 14        | 10. 32   | 9. 46  | 10. 24  | 9. 55  | 10. 16   | 9. 64  | 10. 07  | 9. 73  |
| 15        | 11. 06   | 10. 13 | 10. 97  | 10. 23 | 10. 88   | 10. 33 | 10. 79  | 10. 42 |
| 16        | 11. 80   | 10. 81 | 11. 70  | 10. 91 | 11. 61   | 11. 01 | 11. 51  | 11. 11 |
| 17        | 12. 53   | 11. 48 | 12. 43  | 11. 59 | 12. 33   | 11. 70 | 12. 23  | 11. 81 |
| 18        | 13. 27   | 12. 16 | 13. 16  | 12. 28 | 13. 06   | 12. 39 | 12. 95  | 12. 50 |
| 19        | 14. 01   | 12. 84 | 13. 90  | 12. 96 | 13. 78   | 13. 08 | 13. 67  | 13. 20 |
| 20        | 14. 75   | 13. 51 | 14. 63  | 13. 64 | 14. 51   | 13. 77 | 14. 39  | 13. 89 |
| 21        | 15. 48   | 14. 19 | 15. 36  | 14. 32 | 15. 23   | 14. 46 | 15. 11  | 14. 59 |
| 22        | 16. 22   | 14. 86 | 16. 09  | 15. 00 | 15. 96   | 15. 14 | 15. 83  | 15. 28 |
| 23        | 16. 96   | 15. 54 | 16. 82  | 15. 69 | 16. 88   | 15. 83 | 16. 54  | 15. 98 |
| 24        | 17. 69   | 16. 21 | 17. 55  | 16. 37 | 17. 41   | 16. 52 | 17. 26  | 16. 67 |
| 25        | 18. 43   | 16. 89 | 18. 28  | 17. 05 | 18. 13   | 17. 21 | 17. 98  | 17. 37 |
| 26        | 19. 17   | 17. 57 | 19. 02  | 17. 73 | 18. 86   | 17. 90 | 18. 70  | 18. 06 |
| 27        | 19. 91   | 18. 24 | 19. 75  | 18. 41 | 19. 59   | 18. 59 | 19. 42  | 18. 76 |
| 28        | 20. 64   | 18. 92 | 20. 48  | 19. 10 | 20. 31   | 19. 27 | 20. 14  | 19. 45 |
| 29        | 21. 38   | 19. 59 | 21. 21  | 19. 78 | 21. 04   | 19. 96 | 20. 86  | 20. 15 |
| 30        | 22. 12   | 20. 27 | 21. 94  | 20. 46 | 21. 76   | 20. 65 | 21. 58  | 20. 84 |
| 35        | 25. 80   | 23. 65 | 25. 60  | 23. 87 | 25. 39   | 24. 09 | 25. 18  | 24. 31 |
| 40        | 29. 49   | 27. 02 | 29. 25  | 27. 28 | 29. 01   | 27. 53 | 28. 77  | 27. 79 |
| 45        | 33. 18   | 30. 40 | 32. 91  | 30. 69 | 32. 64   | 30. 98 | 32. 37  | 31. 26 |
| 50        | 36. 86   | 33. 78 | 36. 57  | 34. 10 | 36. 27   | 34. 42 | 35. 57  | 34. 73 |
| 55        | 40. 55   | 37. 16 | 40. 22  | 37. 51 | 39. 90   | 37. 86 | 39. 96  | 38. 21 |
| 60        | 44. 24   | 40. 54 | 43. 88  | 40. 92 | 43. 52   | 41. 30 | 43. 16  | 41. 68 |
| 65        | 47. 92   | 43. 91 | 47. 54  | 44. 33 | 47. 15   | 44. 74 | 46. 76  | 45. 15 |
| 70        | 51. 61   | 47. 29 | 51. 19  | 47. 74 | 50. 78   | 48. 18 | 50. 35  | 48. 63 |
| 75        | 55. 30   | 50. 67 | 54. 85  | 51. 15 | 54. 40   | 51. 63 | 53. 95  | 52. 10 |
| 80        | 58. 98   | 54. 05 | 58. 51  | 54. 56 | 58. 03   | 55. 07 | 57. 55  | 55. 57 |
| 85        | 62. 67   | 57. 43 | 62. 17  | 57. 97 | 61. 66   | 58. 51 | 61. 14  | 59. 05 |
| 90        | 66. 35   | 60. 80 | 65. 82  | 61. 38 | 65. 28   | 61. 95 | 64. 74  | 62. 52 |
| 95        | 70. 04   | 64. 18 | 69. 48  | 64. 79 | 68. 91   | 65. 39 | 68. 34  | 65. 99 |
| 100       | 73. 73   | 67. 56 | 73. 14  | 68. 20 | 72. 54   | 68. 84 | 71. 93  | 69. 47 |
|           | Dep.     | Lat.   | Dep.    | Lat.   | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 47½ Deg. |        | 47 Deg. |        | 46½ Deg. |        | 46 Deg. |        |

# TRAVERSE TABLE.

93

| Distance. | 44½ Deg. |        | 45 Deg. |        |
|-----------|----------|--------|---------|--------|
|           | Lat.     | Dep.   | Lat.    | Dep.   |
|           |          |        |         |        |
| 1         | 0. 71    | 0. 70  | 0. 71   | 0. 71  |
| 2         | 1. 43    | 1. 40  | 1. 41   | 1. 41  |
| 3         | 2. 14    | 2. 10  | 2. 12   | 2. 12  |
| 4         | 2. 85    | 2. 80  | 2. 83   | 2. 83  |
| 5         | 3. 57    | 3. 50  | 3. 54   | 3. 54  |
| 6         | 4. 28    | 4. 21  | 4. 24   | 4. 24  |
| 7         | 4. 99    | 4. 91  | 4. 95   | 4. 95  |
| 8         | 5. 71    | 5. 61  | 5. 66   | 5. 66  |
| 9         | 6. 42    | 6. 31  | 6. 36   | 6. 36  |
| 10        | 7. 13    | 7. 01  | 7. 07   | 7. 07  |
| 11        | 7. 85    | 7. 71  | 7. 78   | 7. 78  |
| 12        | 8. 56    | 8. 41  | 8. 49   | 8. 49  |
| 13        | 9. 27    | 9. 11  | 9. 19   | 9. 19  |
| 14        | 9. 99    | 9. 81  | 9. 90   | 9. 90  |
| 15        | 10. 70   | 10. 51 | 10. 61  | 10. 61 |
| 16        | 11. 41   | 11. 21 | 11. 31  | 11. 31 |
| 17        | 12. 13   | 11. 92 | 12. 02  | 12. 02 |
| 18        | 12. 84   | 12. 62 | 12. 73  | 12. 73 |
| 19        | 13. 55   | 13. 32 | 13. 43  | 13. 43 |
| 20        | 14. 26   | 14. 02 | 14. 14  | 14. 14 |
| 21        | 14. 98   | 14. 72 | 14. 85  | 14. 85 |
| 22        | 15. 69   | 15. 42 | 15. 56  | 15. 56 |
| 23        | 16. 40   | 16. 12 | 16. 26  | 16. 26 |
| 24        | 17. 12   | 16. 82 | 16. 97  | 16. 97 |
| 25        | 17. 83   | 17. 52 | 17. 68  | 17. 68 |
| 26        | 18. 54   | 18. 22 | 18. 38  | 18. 38 |
| 27        | 19. 26   | 18. 92 | 19. 09  | 19. 09 |
| 28        | 19. 97   | 19. 63 | 19. 80  | 19. 80 |
| 29        | 20. 68   | 20. 33 | 20. 51  | 20. 51 |
| 30        | 21. 40   | 21. 03 | 21. 21  | 21. 21 |
| 35        | 24. 96   | 24. 53 | 24. 75  | 24. 75 |
| 40        | 28. 53   | 28. 04 | 28. 28  | 28. 28 |
| 45        | 32. 10   | 31. 54 | 31. 82  | 31. 82 |
| 50        | 35. 66   | 35. 05 | 35. 36  | 35. 36 |
| 55        | 39. 23   | 38. 55 | 38. 89  | 38. 89 |
| 60        | 42. 79   | 42. 05 | 42. 43  | 42. 43 |
| 65        | 46. 36   | 45. 56 | 45. 96  | 45. 96 |
| 70        | 49. 93   | 49. 06 | 49. 50  | 49. 50 |
| 75        | 53. 49   | 52. 57 | 53. 03  | 53. 03 |
| 80        | 57. 06   | 56. 07 | 56. 57  | 56. 57 |
| 85        | 60. 63   | 59. 58 | 60. 10  | 60. 10 |
| 90        | 64. 19   | 63. 08 | 63. 64  | 63. 64 |
| 95        | 67. 76   | 66. 59 | 67. 18  | 67. 18 |
| 100       | 71. 33   | 70. 09 | 70. 71  | 70. 71 |
|           | Dep.     | Lat.   | Dep.    | Lat.   |
|           | 44½ Deg. |        | 45 Deg. |        |



|    | 0° | 1°  | 2°  | 3°  | 4°  | 5°  | 6°  | 7°  | 8°  | 9°  | 10° | 11° | 12° | 13° | 14° | 15° |
|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0  | 0  | 60  | 120 | 180 | 240 | 300 | 361 | 421 | 482 | 542 | 603 | 664 | 725 | 787 | 848 | 910 |
| 1  | 1  | 61  | 121 | 181 | 241 | 301 | 362 | 422 | 483 | 543 | 604 | 665 | 726 | 788 | 850 | 911 |
| 2  | 2  | 62  | 122 | 182 | 242 | 302 | 363 | 423 | 484 | 544 | 605 | 666 | 727 | 789 | 851 | 913 |
| 3  | 3  | 63  | 123 | 183 | 243 | 303 | 364 | 424 | 485 | 545 | 606 | 667 | 728 | 790 | 852 | 914 |
| 4  | 4  | 64  | 124 | 184 | 244 | 304 | 365 | 425 | 486 | 546 | 607 | 668 | 729 | 791 | 853 | 915 |
| 5  | 5  | 65  | 125 | 185 | 245 | 305 | 366 | 426 | 487 | 547 | 608 | 669 | 730 | 792 | 854 | 916 |
| 6  | 6  | 66  | 126 | 186 | 246 | 306 | 367 | 427 | 488 | 548 | 609 | 670 | 731 | 793 | 855 | 917 |
| 7  | 7  | 67  | 127 | 187 | 247 | 307 | 368 | 428 | 489 | 549 | 610 | 671 | 732 | 794 | 856 | 918 |
| 8  | 8  | 68  | 128 | 188 | 248 | 308 | 369 | 429 | 490 | 550 | 611 | 672 | 734 | 795 | 857 | 919 |
| 9  | 9  | 69  | 129 | 189 | 249 | 309 | 370 | 430 | 491 | 551 | 612 | 673 | 735 | 796 | 858 | 920 |
| 10 | 10 | 70  | 130 | 190 | 250 | 310 | 371 | 431 | 492 | 552 | 613 | 674 | 736 | 797 | 859 | 921 |
| 11 | 11 | 71  | 131 | 191 | 251 | 311 | 372 | 432 | 493 | 553 | 614 | 675 | 737 | 798 | 860 | 922 |
| 12 | 12 | 72  | 132 | 192 | 252 | 312 | 373 | 433 | 494 | 554 | 615 | 676 | 738 | 799 | 861 | 923 |
| 13 | 13 | 73  | 133 | 193 | 253 | 313 | 374 | 434 | 495 | 555 | 616 | 677 | 739 | 800 | 862 | 924 |
| 14 | 14 | 74  | 134 | 194 | 254 | 314 | 375 | 435 | 496 | 556 | 617 | 678 | 740 | 801 | 863 | 925 |
| 15 | 15 | 75  | 135 | 195 | 255 | 315 | 376 | 436 | 497 | 557 | 618 | 679 | 741 | 802 | 864 | 926 |
| 16 | 16 | 76  | 136 | 196 | 256 | 316 | 377 | 437 | 498 | 558 | 619 | 680 | 742 | 803 | 865 | 927 |
| 17 | 17 | 77  | 137 | 197 | 257 | 317 | 378 | 438 | 499 | 559 | 620 | 681 | 743 | 804 | 866 | 928 |
| 18 | 18 | 78  | 138 | 198 | 258 | 318 | 379 | 439 | 500 | 560 | 621 | 682 | 744 | 805 | 867 | 929 |
| 19 | 19 | 79  | 139 | 199 | 259 | 319 | 380 | 440 | 501 | 561 | 622 | 683 | 745 | 806 | 868 | 930 |
| 20 | 20 | 80  | 140 | 200 | 260 | 320 | 381 | 441 | 502 | 562 | 623 | 684 | 746 | 807 | 869 | 931 |
| 21 | 21 | 81  | 141 | 201 | 261 | 321 | 382 | 442 | 503 | 563 | 624 | 685 | 747 | 808 | 870 | 932 |
| 22 | 22 | 82  | 142 | 202 | 262 | 322 | 383 | 443 | 504 | 565 | 625 | 687 | 748 | 809 | 871 | 933 |
| 23 | 23 | 83  | 143 | 203 | 263 | 323 | 384 | 444 | 505 | 566 | 626 | 688 | 749 | 810 | 872 | 934 |
| 24 | 24 | 84  | 144 | 204 | 264 | 324 | 385 | 445 | 506 | 567 | 627 | 689 | 750 | 811 | 873 | 935 |
| 25 | 25 | 85  | 145 | 205 | 265 | 325 | 386 | 446 | 507 | 568 | 628 | 690 | 751 | 812 | 874 | 936 |
| 26 | 26 | 86  | 146 | 206 | 266 | 326 | 387 | 447 | 508 | 569 | 629 | 691 | 752 | 813 | 875 | 937 |
| 27 | 27 | 87  | 147 | 207 | 267 | 327 | 388 | 448 | 509 | 570 | 631 | 692 | 753 | 815 | 876 | 938 |
| 28 | 28 | 88  | 148 | 208 | 268 | 328 | 389 | 449 | 510 | 571 | 632 | 693 | 754 | 816 | 877 | 939 |
| 29 | 29 | 89  | 149 | 209 | 269 | 330 | 390 | 450 | 511 | 572 | 633 | 694 | 755 | 817 | 878 | 941 |
| 30 | 30 | 90  | 150 | 210 | 270 | 331 | 391 | 451 | 512 | 573 | 634 | 695 | 756 | 818 | 879 | 942 |
| 31 | 31 | 91  | 151 | 211 | 271 | 332 | 392 | 452 | 513 | 574 | 635 | 696 | 757 | 819 | 880 | 943 |
| 32 | 32 | 92  | 152 | 212 | 272 | 333 | 393 | 453 | 514 | 575 | 636 | 697 | 758 | 820 | 882 | 944 |
| 33 | 33 | 93  | 153 | 213 | 273 | 334 | 394 | 454 | 515 | 576 | 637 | 698 | 759 | 821 | 883 | 945 |
| 34 | 34 | 94  | 154 | 214 | 274 | 335 | 395 | 455 | 516 | 577 | 638 | 699 | 760 | 822 | 884 | 946 |
| 35 | 35 | 95  | 155 | 215 | 275 | 336 | 396 | 456 | 517 | 578 | 639 | 700 | 761 | 823 | 885 | 947 |
| 36 | 36 | 96  | 156 | 216 | 276 | 337 | 397 | 457 | 518 | 579 | 640 | 701 | 762 | 824 | 886 | 948 |
| 37 | 37 | 97  | 157 | 217 | 277 | 338 | 398 | 458 | 519 | 580 | 641 | 702 | 763 | 825 | 887 | 949 |
| 38 | 38 | 98  | 158 | 218 | 278 | 339 | 399 | 459 | 520 | 581 | 642 | 703 | 764 | 826 | 888 | 950 |
| 39 | 39 | 99  | 159 | 219 | 279 | 340 | 400 | 460 | 521 | 582 | 643 | 704 | 765 | 827 | 889 | 951 |
| 40 | 40 | 100 | 160 | 220 | 280 | 341 | 401 | 461 | 522 | 583 | 644 | 705 | 766 | 828 | 890 | 952 |
| 41 | 41 | 101 | 161 | 221 | 281 | 342 | 402 | 462 | 523 | 584 | 645 | 706 | 767 | 829 | 891 | 953 |
| 42 | 42 | 102 | 162 | 222 | 282 | 343 | 403 | 463 | 524 | 585 | 646 | 707 | 768 | 830 | 892 | 954 |
| 43 | 43 | 103 | 163 | 223 | 283 | 344 | 404 | 464 | 525 | 586 | 647 | 708 | 769 | 831 | 893 | 955 |
| 44 | 44 | 104 | 164 | 224 | 284 | 345 | 405 | 465 | 526 | 587 | 648 | 709 | 770 | 832 | 894 | 956 |
| 45 | 45 | 105 | 165 | 225 | 285 | 346 | 406 | 466 | 527 | 588 | 649 | 710 | 771 | 833 | 895 | 957 |
| 46 | 46 | 106 | 166 | 226 | 286 | 347 | 407 | 467 | 528 | 589 | 650 | 711 | 772 | 834 | 896 | 958 |
| 47 | 47 | 107 | 167 | 227 | 287 | 348 | 408 | 468 | 529 | 590 | 651 | 712 | 773 | 835 | 897 | 959 |
| 48 | 48 | 108 | 168 | 228 | 288 | 349 | 409 | 469 | 530 | 591 | 652 | 713 | 774 | 836 | 898 | 960 |
| 49 | 49 | 109 | 169 | 229 | 289 | 350 | 410 | 470 | 531 | 592 | 653 | 714 | 775 | 837 | 899 | 961 |
| 50 | 50 | 110 | 170 | 230 | 290 | 351 | 411 | 471 | 532 | 593 | 654 | 715 | 777 | 838 | 900 | 962 |
| 51 | 51 | 111 | 171 | 231 | 291 | 352 | 412 | 472 | 533 | 594 | 655 | 716 | 778 | 839 | 901 | 963 |
| 52 | 52 | 112 | 172 | 232 | 292 | 353 | 413 | 473 | 534 | 595 | 656 | 717 | 779 | 840 | 902 | 964 |
| 53 | 53 | 113 | 173 | 233 | 293 | 354 | 414 | 474 | 535 | 596 | 657 | 718 | 780 | 841 | 903 | 965 |
| 54 | 54 | 114 | 174 | 234 | 294 | 355 | 415 | 475 | 536 | 597 | 658 | 719 | 781 | 842 | 904 | 966 |
| 55 | 55 | 115 | 175 | 235 | 295 | 356 | 416 | 477 | 537 | 598 | 659 | 720 | 782 | 843 | 905 | 968 |
| 56 | 56 | 116 | 176 | 236 | 296 | 357 | 417 | 478 | 538 | 599 | 660 | 721 | 783 | 844 | 906 | 969 |
| 57 | 57 | 117 | 177 | 237 | 297 | 358 | 418 | 479 | 539 | 600 | 661 | 722 | 784 | 845 | 907 | 970 |
| 58 | 58 | 118 | 178 | 238 | 298 | 359 | 419 | 480 | 540 | 601 | 662 | 723 | 785 | 846 | 908 | 971 |
| 59 | 59 | 119 | 179 | 239 | 299 | 360 | 420 | 481 | 541 | 602 | 663 | 724 | 786 | 847 | 909 | 972 |

TABLE IV.

## Meridional Parts.

95

|    | 16°  | 17°  | 18°  | 19°  | 20°  | 21°  | 22°  | 23°  | 24°  | 25°  | 26°  | 27°  | 28°  |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0  | 973  | 1035 | 1098 | 1161 | 1225 | 1289 | 1354 | 1419 | 1484 | 1550 | 1616 | 1684 | 1751 |
| 1  | 974  | 1036 | 1099 | 1163 | 1226 | 1290 | 1355 | 1420 | 1485 | 1551 | 1618 | 1685 | 1752 |
| 2  | 975  | 1037 | 1100 | 1164 | 1227 | 1291 | 1356 | 1421 | 1486 | 1552 | 1619 | 1686 | 1753 |
| 3  | 976  | 1038 | 1101 | 1165 | 1228 | 1292 | 1357 | 1422 | 1487 | 1553 | 1620 | 1687 | 1755 |
| 4  | 977  | 1039 | 1102 | 1166 | 1229 | 1293 | 1358 | 1423 | 1488 | 1554 | 1621 | 1688 | 1756 |
| 5  | 978  | 1041 | 1103 | 1167 | 1230 | 1295 | 1359 | 1424 | 1490 | 1556 | 1622 | 1689 | 1757 |
| 6  | 979  | 1042 | 1105 | 1168 | 1232 | 1296 | 1360 | 1425 | 1491 | 1557 | 1623 | 1690 | 1758 |
| 7  | 980  | 1043 | 1106 | 1169 | 1233 | 1297 | 1361 | 1426 | 1492 | 1558 | 1624 | 1692 | 1759 |
| 8  | 981  | 1044 | 1107 | 1170 | 1234 | 1298 | 1362 | 1427 | 1493 | 1559 | 1625 | 1693 | 1760 |
| 9  | 982  | 1045 | 1108 | 1171 | 1235 | 1299 | 1363 | 1428 | 1494 | 1560 | 1626 | 1694 | 1761 |
| 10 | 983  | 1046 | 1109 | 1172 | 1236 | 1300 | 1364 | 1430 | 1495 | 1561 | 1628 | 1695 | 1762 |
| 11 | 984  | 1047 | 1110 | 1173 | 1237 | 1301 | 1366 | 1431 | 1496 | 1562 | 1629 | 1696 | 1764 |
| 12 | 985  | 1048 | 1111 | 1174 | 1238 | 1302 | 1367 | 1432 | 1497 | 1563 | 1630 | 1697 | 1765 |
| 13 | 986  | 1049 | 1112 | 1175 | 1239 | 1303 | 1368 | 1433 | 1498 | 1564 | 1631 | 1698 | 1766 |
| 14 | 987  | 1050 | 1113 | 1176 | 1240 | 1304 | 1369 | 1434 | 1499 | 1565 | 1632 | 1699 | 1767 |
| 15 | 988  | 1051 | 1114 | 1177 | 1241 | 1305 | 1370 | 1435 | 1500 | 1567 | 1633 | 1700 | 1768 |
| 16 | 989  | 1052 | 1115 | 1178 | 1242 | 1306 | 1371 | 1436 | 1502 | 1568 | 1634 | 1701 | 1769 |
| 17 | 990  | 1053 | 1116 | 1179 | 1243 | 1307 | 1372 | 1437 | 1503 | 1569 | 1635 | 1703 | 1770 |
| 18 | 991  | 1054 | 1117 | 1181 | 1244 | 1308 | 1373 | 1438 | 1504 | 1570 | 1637 | 1704 | 1772 |
| 19 | 993  | 1055 | 1118 | 1182 | 1245 | 1310 | 1374 | 1439 | 1505 | 1571 | 1638 | 1705 | 1773 |
| 20 | 994  | 1056 | 1119 | 1183 | 1246 | 1311 | 1375 | 1440 | 1506 | 1572 | 1639 | 1706 | 1774 |
| 21 | 995  | 1057 | 1120 | 1184 | 1248 | 1312 | 1376 | 1441 | 1507 | 1573 | 1640 | 1707 | 1775 |
| 22 | 996  | 1058 | 1121 | 1185 | 1249 | 1313 | 1377 | 1443 | 1508 | 1574 | 1641 | 1708 | 1776 |
| 23 | 997  | 1059 | 1122 | 1186 | 1250 | 1314 | 1379 | 1444 | 1509 | 1575 | 1642 | 1709 | 1777 |
| 24 | 998  | 1060 | 1123 | 1187 | 1251 | 1315 | 1380 | 1445 | 1510 | 1577 | 1643 | 1711 | 1778 |
| 25 | 999  | 1161 | 1125 | 1188 | 1252 | 1316 | 1381 | 1446 | 1511 | 1578 | 1644 | 1712 | 1780 |
| 26 | 1000 | 1063 | 1126 | 1189 | 1253 | 1317 | 1382 | 1447 | 1513 | 1579 | 1645 | 1713 | 1781 |
| 27 | 1001 | 1064 | 1127 | 1190 | 1254 | 1318 | 1383 | 1448 | 1514 | 1580 | 1647 | 1714 | 1782 |
| 28 | 1002 | 1065 | 1128 | 1191 | 1255 | 1319 | 1384 | 1449 | 1515 | 1581 | 1648 | 1715 | 1783 |
| 29 | 1003 | 1066 | 1129 | 1192 | 1256 | 1320 | 1385 | 1450 | 1516 | 1582 | 1649 | 1716 | 1784 |
| 30 | 1004 | 1067 | 1130 | 1193 | 1257 | 1321 | 1386 | 1451 | 1517 | 1583 | 1650 | 1717 | 1785 |
| 31 | 1005 | 1068 | 1131 | 1194 | 1258 | 1322 | 1387 | 1452 | 1518 | 1584 | 1651 | 1718 | 1786 |
| 32 | 1006 | 1069 | 1132 | 1195 | 1259 | 1324 | 1388 | 1453 | 1519 | 1585 | 1652 | 1720 | 1787 |
| 33 | 1007 | 1070 | 1133 | 1196 | 1260 | 1325 | 1389 | 1455 | 1520 | 1586 | 1653 | 1721 | 1789 |
| 34 | 1008 | 1071 | 1134 | 1198 | 1261 | 1326 | 1390 | 1456 | 1521 | 1588 | 1654 | 1722 | 1790 |
| 35 | 1009 | 1072 | 1135 | 1199 | 1262 | 1327 | 1392 | 1457 | 1522 | 1589 | 1656 | 1723 | 1791 |
| 36 | 1010 | 1073 | 1136 | 1200 | 1264 | 1328 | 1393 | 1458 | 1524 | 1590 | 1657 | 1724 | 1792 |
| 37 | 1011 | 1074 | 1137 | 1201 | 1265 | 1329 | 1394 | 1459 | 1525 | 1591 | 1658 | 1725 | 1793 |
| 38 | 1012 | 1075 | 1138 | 1202 | 1266 | 1330 | 1395 | 1460 | 1526 | 1592 | 1659 | 1726 | 1794 |
| 39 | 1013 | 1076 | 1139 | 1203 | 1267 | 1331 | 1396 | 1461 | 1527 | 1593 | 1660 | 1727 | 1795 |
| 40 | 1014 | 1077 | 1140 | 1204 | 1268 | 1332 | 1397 | 1462 | 1528 | 1594 | 1661 | 1729 | 1797 |
| 41 | 1015 | 1078 | 1141 | 1205 | 1269 | 1333 | 1398 | 1463 | 1529 | 1595 | 1662 | 1730 | 1798 |
| 42 | 1016 | 1079 | 1142 | 1206 | 1270 | 1334 | 1399 | 1464 | 1530 | 1596 | 1663 | 1731 | 1799 |
| 43 | 1018 | 1080 | 1144 | 1207 | 1271 | 1335 | 1400 | 1465 | 1531 | 1598 | 1664 | 1732 | 1800 |
| 44 | 1019 | 1081 | 1145 | 1208 | 1272 | 1336 | 1401 | 1467 | 1532 | 1599 | 1666 | 1733 | 1801 |
| 45 | 1020 | 1082 | 1146 | 1209 | 1273 | 1338 | 1402 | 1468 | 1533 | 1600 | 1667 | 1734 | 1802 |
| 46 | 1021 | 1084 | 1147 | 1210 | 1274 | 1339 | 1403 | 1469 | 1535 | 1601 | 1668 | 1735 | 1803 |
| 47 | 1022 | 1085 | 1148 | 1211 | 1275 | 1340 | 1405 | 1470 | 1536 | 1602 | 1669 | 1736 | 1805 |
| 48 | 1023 | 1086 | 1149 | 1212 | 1276 | 1341 | 1406 | 1471 | 1537 | 1603 | 1670 | 1737 | 1806 |
| 49 | 1024 | 1087 | 1150 | 1213 | 1277 | 1342 | 1407 | 1472 | 1538 | 1604 | 1671 | 1739 | 1807 |
| 50 | 1025 | 1088 | 1151 | 1215 | 1278 | 1343 | 1408 | 1473 | 1539 | 1605 | 1672 | 1740 | 1808 |
| 51 | 1026 | 1089 | 1152 | 1216 | 1280 | 1344 | 1409 | 1474 | 1540 | 1606 | 1673 | 1741 | 1809 |
| 52 | 1027 | 1090 | 1153 | 1217 | 1281 | 1345 | 1410 | 1475 | 1541 | 1608 | 1675 | 1742 | 1810 |
| 53 | 1028 | 1091 | 1154 | 1218 | 1282 | 1346 | 1411 | 1476 | 1542 | 1609 | 1676 | 1743 | 1811 |
| 54 | 1029 | 1092 | 1155 | 1219 | 1283 | 1347 | 1412 | 1477 | 1543 | 1610 | 1677 | 1744 | 1813 |
| 55 | 1030 | 1093 | 1156 | 1220 | 1284 | 1348 | 1413 | 1479 | 1544 | 1611 | 1678 | 1746 | 1814 |
| 56 | 1031 | 1094 | 1157 | 1221 | 1285 | 1349 | 1414 | 1480 | 1546 | 1612 | 1679 | 1747 | 1815 |
| 57 | 1032 | 1095 | 1158 | 1222 | 1286 | 1350 | 1415 | 1481 | 1547 | 1613 | 1680 | 1748 | 1816 |
| 58 | 1033 | 1096 | 1159 | 1223 | 1287 | 1352 | 1416 | 1482 | 1548 | 1614 | 1681 | 1749 | 1817 |
| 59 | 1034 | 1097 | 1160 | 1224 | 1288 | 1353 | 1418 | 1483 | 1549 | 1615 | 1682 | 1750 | 1818 |

| $\lambda$ | 29°  | 30°  | 31°  | 32°  | 33°  | 34°  | 35°  | 36°  | 37°  | 38°  | 39°  | 40°  | 41°  |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0         | 1819 | 1888 | 1958 | 2028 | 2100 | 2171 | 2244 | 2318 | 2393 | 2468 | 2545 | 2623 | 2702 |
| 1         | 1821 | 1890 | 1959 | 2030 | 2101 | 2173 | 2246 | 2319 | 2394 | 2470 | 2546 | 2624 | 2703 |
| 2         | 1822 | 1891 | 1960 | 2031 | 2102 | 2174 | 2247 | 2320 | 2395 | 2471 | 2548 | 2625 | 2704 |
| 3         | 1823 | 1892 | 1962 | 2032 | 2103 | 2175 | 2248 | 2322 | 2396 | 2472 | 2549 | 2627 | 2706 |
| 4         | 1824 | 1893 | 1963 | 2033 | 2104 | 2176 | 2249 | 2323 | 2398 | 2473 | 2550 | 2628 | 2707 |
| 5         | 1825 | 1894 | 1964 | 2034 | 2105 | 2178 | 2250 | 2324 | 2399 | 2475 | 2551 | 2629 | 2708 |
| 6         | 1826 | 1895 | 1965 | 2035 | 2107 | 2179 | 2252 | 2325 | 2400 | 2479 | 2553 | 2631 | 2710 |
| 7         | 1827 | 1896 | 1966 | 2037 | 2108 | 2180 | 2253 | 2327 | 2401 | 2477 | 2554 | 2632 | 2711 |
| 8         | 1829 | 1898 | 1967 | 2038 | 2109 | 2181 | 2254 | 2328 | 2403 | 2478 | 2555 | 2633 | 2712 |
| 9         | 1830 | 1899 | 1969 | 2039 | 2110 | 2182 | 2255 | 2329 | 2404 | 2480 | 2557 | 2634 | 2714 |
| 10        | 1831 | 1900 | 1970 | 2040 | 2111 | 2184 | 2257 | 2330 | 2405 | 2481 | 2558 | 2636 | 2715 |
| 11        | 1832 | 1901 | 1971 | 2041 | 2113 | 2185 | 2258 | 2332 | 2406 | 2482 | 2559 | 2637 | 2716 |
| 12        | 1833 | 1902 | 1972 | 2043 | 2114 | 2186 | 2259 | 2333 | 2408 | 2484 | 2560 | 2638 | 2718 |
| 13        | 1834 | 1903 | 1973 | 2044 | 2115 | 2187 | 2260 | 2334 | 2409 | 2485 | 2562 | 2640 | 2719 |
| 14        | 1835 | 1905 | 1974 | 2045 | 2116 | 2188 | 2261 | 2335 | 2410 | 2486 | 2563 | 2641 | 2720 |
| 15        | 1837 | 1906 | 1976 | 2046 | 2117 | 2190 | 2263 | 2337 | 2411 | 2487 | 2564 | 2642 | 2722 |
| 16        | 1838 | 1907 | 1977 | 2047 | 2119 | 2191 | 2264 | 2338 | 2413 | 2489 | 2566 | 2644 | 2723 |
| 17        | 1839 | 1908 | 1978 | 2048 | 2120 | 2192 | 2265 | 2339 | 2414 | 2490 | 2567 | 2645 | 2724 |
| 18        | 1840 | 1909 | 1979 | 2050 | 2121 | 2193 | 2266 | 2340 | 2415 | 2491 | 2568 | 2646 | 2726 |
| 19        | 1841 | 1910 | 1980 | 2051 | 2122 | 2194 | 2268 | 2342 | 2416 | 2492 | 2569 | 2648 | 2727 |
| 20        | 1842 | 1912 | 1981 | 2052 | 2123 | 2196 | 2269 | 2343 | 2418 | 2494 | 2571 | 2649 | 2728 |
| 21        | 1843 | 1913 | 1983 | 2053 | 2125 | 2197 | 2270 | 2344 | 2419 | 2495 | 2572 | 2650 | 2729 |
| 22        | 1845 | 1914 | 1984 | 2054 | 2126 | 2198 | 2271 | 2345 | 2420 | 2496 | 2573 | 2651 | 2731 |
| 23        | 1846 | 1915 | 1985 | 2056 | 2127 | 2199 | 2272 | 2346 | 2422 | 2498 | 2575 | 2653 | 2732 |
| 24        | 1847 | 1916 | 1986 | 2057 | 2128 | 2200 | 2274 | 2348 | 2423 | 2499 | 2576 | 2654 | 2733 |
| 25        | 1848 | 1917 | 1987 | 2058 | 2129 | 2202 | 2275 | 2349 | 2424 | 2500 | 2577 | 2655 | 2735 |
| 29        | 1849 | 1918 | 1988 | 2059 | 2131 | 2203 | 2266 | 2350 | 2425 | 2501 | 2578 | 2657 | 2736 |
| 27        | 1850 | 1920 | 1990 | 2060 | 2132 | 2204 | 2267 | 2351 | 2427 | 2503 | 2580 | 2658 | 2737 |
| 28        | 1852 | 1921 | 1991 | 2061 | 2133 | 2205 | 2279 | 2353 | 2428 | 2504 | 2581 | 2659 | 2739 |
| 29        | 1853 | 1922 | 1992 | 2063 | 2134 | 2207 | 2280 | 2354 | 2429 | 2505 | 2582 | 2661 | 2740 |
| 30        | 1854 | 1923 | 1993 | 2064 | 2135 | 2208 | 2281 | 2355 | 2430 | 2506 | 2584 | 2662 | 2742 |
| 31        | 1855 | 1924 | 1994 | 2065 | 2137 | 2209 | 2282 | 2356 | 2432 | 2508 | 2585 | 2663 | 2743 |
| 32        | 1856 | 1925 | 1995 | 2066 | 2138 | 2210 | 2283 | 2358 | 2433 | 2509 | 2586 | 2665 | 2744 |
| 33        | 1857 | 1927 | 1997 | 2067 | 2139 | 2211 | 2285 | 2359 | 2434 | 2510 | 2588 | 2666 | 2746 |
| 34        | 1858 | 1928 | 1998 | 2069 | 2140 | 2213 | 2286 | 2360 | 2435 | 2512 | 2589 | 2667 | 2747 |
| 35        | 1860 | 1929 | 1999 | 2070 | 2141 | 2214 | 2287 | 2361 | 2437 | 2513 | 2590 | 2669 | 2748 |
| 36        | 1861 | 1930 | 2000 | 2071 | 2143 | 2215 | 2288 | 2363 | 2438 | 2514 | 2591 | 2670 | 2750 |
| 37        | 1892 | 1931 | 2001 | 2072 | 2144 | 2216 | 2290 | 2364 | 2439 | 2515 | 2593 | 2671 | 2751 |
| 38        | 1863 | 1932 | 2002 | 2073 | 2145 | 2217 | 2291 | 2365 | 2440 | 2517 | 2594 | 2673 | 2752 |
| 39        | 1864 | 1934 | 2004 | 2075 | 2146 | 2219 | 2292 | 2366 | 2442 | 2518 | 2595 | 2674 | 2754 |
| 40        | 1865 | 1935 | 2005 | 2076 | 2147 | 2220 | 2293 | 2368 | 2443 | 2519 | 2597 | 2675 | 2755 |
| 41        | 1866 | 1936 | 2006 | 2077 | 2149 | 2221 | 2295 | 2369 | 2444 | 2521 | 2598 | 2676 | 2756 |
| 42        | 1868 | 1937 | 2077 | 2078 | 2150 | 2222 | 2296 | 2370 | 2445 | 2522 | 2599 | 2678 | 2758 |
| 43        | 1869 | 1938 | 2008 | 2079 | 2151 | 2224 | 2297 | 2371 | 2447 | 2523 | 2601 | 2679 | 2759 |
| 44        | 1870 | 1939 | 2010 | 2080 | 2152 | 2225 | 2298 | 2373 | 2448 | 2524 | 2602 | 2680 | 2760 |
| 45        | 1871 | 1941 | 2011 | 2082 | 2153 | 2226 | 2299 | 2374 | 2449 | 2526 | 2603 | 2682 | 2762 |
| 46        | 1872 | 1942 | 2012 | 2083 | 2155 | 2227 | 2301 | 2375 | 2451 | 2527 | 2604 | 2683 | 2763 |
| 47        | 1873 | 1943 | 2013 | 2084 | 2156 | 2228 | 2302 | 2376 | 2452 | 2528 | 2606 | 2684 | 2764 |
| 48        | 1875 | 1944 | 2014 | 2085 | 2157 | 2230 | 2303 | 2378 | 2453 | 2530 | 2607 | 2686 | 2766 |
| 49        | 1876 | 1945 | 2015 | 2086 | 2158 | 2231 | 2304 | 2379 | 2454 | 2531 | 2608 | 2687 | 2767 |
| 50        | 1877 | 1946 | 2017 | 2088 | 2159 | 2232 | 2306 | 2380 | 2456 | 2532 | 2610 | 2688 | 2768 |
| 51        | 1878 | 1948 | 2018 | 2089 | 2161 | 2233 | 2307 | 2381 | 2457 | 2533 | 2611 | 2690 | 2770 |
| 52        | 1879 | 1949 | 2019 | 2090 | 2162 | 2235 | 2308 | 2383 | 2458 | 2535 | 2612 | 2691 | 2771 |
| 53        | 1880 | 1950 | 2020 | 2091 | 2163 | 2236 | 2309 | 2384 | 2459 | 2536 | 2614 | 2692 | 2772 |
| 54        | 1881 | 1951 | 2021 | 2092 | 2164 | 2237 | 2311 | 2385 | 2461 | 2537 | 2615 | 2694 | 2774 |
| 55        | 1883 | 1952 | 2022 | 2094 | 2165 | 2238 | 2312 | 2386 | 2462 | 2538 | 2616 | 2695 | 2775 |
| 56        | 1884 | 1953 | 2024 | 2095 | 2167 | 2239 | 2313 | 2388 | 2463 | 2540 | 2617 | 2696 | 2776 |
| 57        | 1885 | 1955 | 2025 | 2096 | 2168 | 2241 | 2314 | 2389 | 2464 | 2541 | 2619 | 2698 | 2778 |
| 58        | 1886 | 1956 | 2026 | 2097 | 2169 | 2242 | 2316 | 2390 | 2466 | 2542 | 2620 | 2699 | 2779 |
| 59        | 1887 | 1957 | 2027 | 2098 | 2170 | 2243 | 2317 | 2391 | 2467 | 2544 | 2621 | 2700 | 2780 |

TABLE IV.

Meridional Parts.

97

|    | 42°  | 43°  | 44°  | 45°  | 46°  | 47°  | 48°  | 49°  | 50°  | 51°  | 52°  | 53°  | 54°  |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0  | 2782 | 2863 | 2946 | 3030 | 3116 | 3203 | 3292 | 3382 | 3474 | 3569 | 3665 | 3764 | 3865 |
| 1  | 2783 | 2864 | 2947 | 3031 | 3117 | 3204 | 3293 | 3384 | 3476 | 3570 | 3667 | 3765 | 3866 |
| 2  | 2784 | 2866 | 2949 | 3033 | 3118 | 3206 | 3295 | 3385 | 3478 | 3572 | 3668 | 3767 | 3868 |
| 3  | 2786 | 2867 | 2950 | 3034 | 3120 | 3207 | 3296 | 3387 | 3479 | 3573 | 3670 | 3769 | 3870 |
| 4  | 2787 | 2869 | 2951 | 3036 | 3121 | 3209 | 3298 | 3388 | 3481 | 3575 | 3672 | 3770 | 3871 |
| 5  | 2788 | 2870 | 2953 | 3037 | 3123 | 3210 | 3299 | 3390 | 3482 | 3577 | 3673 | 3772 | 3873 |
| 6  | 2790 | 2871 | 2954 | 3038 | 3123 | 3212 | 3301 | 3391 | 3484 | 3578 | 3675 | 3774 | 3875 |
| 7  | 2791 | 2873 | 2956 | 3040 | 3126 | 3213 | 3302 | 3393 | 3485 | 3580 | 3677 | 3775 | 3877 |
| 8  | 2792 | 2874 | 2957 | 3041 | 3127 | 3214 | 3303 | 3394 | 3487 | 3582 | 3678 | 3777 | 3878 |
| 9  | 2794 | 2875 | 2958 | 3043 | 3129 | 3216 | 3305 | 3396 | 3488 | 3583 | 3680 | 3779 | 3880 |
| 10 | 2795 | 2877 | 2960 | 3044 | 3130 | 3217 | 3306 | 3397 | 3490 | 3585 | 3681 | 3780 | 3882 |
| 11 | 2797 | 2878 | 2961 | 3046 | 3131 | 3219 | 3308 | 3399 | 3492 | 3586 | 3683 | 3782 | 3883 |
| 12 | 2798 | 2880 | 2963 | 3047 | 3133 | 3220 | 3309 | 3400 | 3493 | 3588 | 3685 | 3784 | 3885 |
| 13 | 2799 | 2881 | 2964 | 3048 | 3134 | 3222 | 3311 | 3402 | 3495 | 3590 | 3686 | 3785 | 3887 |
| 14 | 2801 | 2882 | 2965 | 3050 | 3136 | 3224 | 3312 | 3403 | 3496 | 3591 | 3688 | 3787 | 3889 |
| 15 | 2802 | 2884 | 2967 | 3051 | 3137 | 3225 | 3314 | 3405 | 3498 | 3593 | 3690 | 3789 | 3890 |
| 16 | 2803 | 2885 | 2968 | 3053 | 3139 | 3226 | 3316 | 3407 | 3499 | 3594 | 3691 | 3790 | 3892 |
| 17 | 2805 | 2886 | 2970 | 3054 | 3140 | 3228 | 3317 | 3408 | 3501 | 3596 | 3693 | 3792 | 3894 |
| 18 | 2806 | 2888 | 2971 | 3055 | 3142 | 3229 | 3319 | 3410 | 3503 | 3598 | 3695 | 3794 | 3895 |
| 19 | 2807 | 2889 | 2972 | 3057 | 3143 | 3231 | 3320 | 3411 | 3504 | 3599 | 3696 | 3795 | 3897 |
| 20 | 2809 | 2891 | 2974 | 3058 | 3144 | 3232 | 3322 | 3413 | 3506 | 3601 | 3698 | 3797 | 3899 |
| 21 | 2810 | 2892 | 2975 | 3060 | 3146 | 3234 | 3323 | 3414 | 3507 | 3602 | 3699 | 3799 | 3901 |
| 22 | 2811 | 2893 | 2976 | 3061 | 3147 | 3235 | 3325 | 3416 | 3509 | 3604 | 3701 | 3800 | 3902 |
| 23 | 2813 | 2895 | 2978 | 3063 | 3149 | 3237 | 3326 | 3417 | 3510 | 3606 | 3703 | 3802 | 3904 |
| 24 | 2814 | 2896 | 2979 | 3064 | 3150 | 3238 | 3328 | 3419 | 3512 | 3607 | 3704 | 3804 | 3906 |
| 25 | 2815 | 2897 | 2981 | 3065 | 3152 | 3240 | 3329 | 3420 | 3514 | 3609 | 3706 | 3806 | 3907 |
| 26 | 2817 | 2899 | 2982 | 3067 | 3153 | 3241 | 3331 | 3422 | 3515 | 3610 | 3708 | 3807 | 3909 |
| 27 | 2818 | 2900 | 2983 | 3068 | 3155 | 3242 | 3332 | 3423 | 3517 | 3612 | 3709 | 3809 | 3911 |
| 28 | 2820 | 2902 | 2985 | 3070 | 3156 | 3244 | 3334 | 3425 | 3518 | 3614 | 3711 | 3811 | 3913 |
| 29 | 2821 | 2903 | 2986 | 3071 | 3157 | 3245 | 3335 | 3427 | 3520 | 3615 | 3713 | 3812 | 3914 |
| 30 | 2822 | 2904 | 2988 | 3073 | 3159 | 3247 | 3337 | 3428 | 3521 | 3617 | 3714 | 3814 | 3916 |
| 31 | 2824 | 2906 | 2989 | 3074 | 3160 | 3248 | 3338 | 3430 | 3523 | 3618 | 3716 | 3816 | 3918 |
| 32 | 2825 | 2907 | 2991 | 3075 | 3162 | 3250 | 3340 | 3431 | 3525 | 3620 | 3717 | 3817 | 3919 |
| 33 | 2826 | 2908 | 2992 | 3077 | 3163 | 3251 | 3341 | 3433 | 3526 | 3622 | 3719 | 3819 | 3921 |
| 34 | 2828 | 2910 | 2993 | 3078 | 3165 | 3253 | 3343 | 3434 | 3528 | 3623 | 3721 | 3821 | 3923 |
| 35 | 2829 | 2911 | 2995 | 3080 | 3166 | 3254 | 3344 | 3436 | 3529 | 3625 | 3722 | 3822 | 3925 |
| 36 | 2830 | 2913 | 2996 | 3081 | 3168 | 3256 | 3346 | 3437 | 3531 | 3626 | 3724 | 3824 | 3926 |
| 37 | 2832 | 2914 | 2998 | 3083 | 3169 | 3257 | 3347 | 3439 | 3532 | 3628 | 3726 | 3826 | 3928 |
| 38 | 2833 | 2915 | 2999 | 3084 | 3171 | 3259 | 3349 | 3440 | 3534 | 3630 | 3727 | 3827 | 3930 |
| 39 | 2834 | 2917 | 3000 | 3085 | 3172 | 3260 | 3350 | 3442 | 3536 | 3631 | 3729 | 3829 | 3932 |
| 40 | 2836 | 2918 | 3002 | 3087 | 3173 | 3262 | 3352 | 3543 | 3537 | 3633 | 3731 | 3831 | 3933 |
| 41 | 2837 | 2919 | 3003 | 3088 | 3175 | 3263 | 3353 | 3545 | 3539 | 3634 | 3732 | 3832 | 3935 |
| 42 | 2839 | 2921 | 3005 | 3090 | 3176 | 3265 | 3355 | 3547 | 3540 | 3636 | 3734 | 3834 | 3937 |
| 43 | 2840 | 2922 | 3006 | 3091 | 3178 | 3266 | 3356 | 3548 | 3542 | 3638 | 3736 | 3836 | 3938 |
| 44 | 2841 | 2924 | 3007 | 3093 | 3179 | 3268 | 3358 | 3550 | 3543 | 3639 | 3737 | 3838 | 3940 |
| 45 | 2843 | 2925 | 3009 | 3094 | 3181 | 3269 | 3359 | 3551 | 3545 | 3641 | 3739 | 3839 | 3942 |
| 46 | 2844 | 2926 | 3010 | 3095 | 3182 | 3271 | 3361 | 3553 | 3547 | 3643 | 3741 | 3841 | 3944 |
| 47 | 2845 | 2928 | 3012 | 3097 | 3184 | 3272 | 3362 | 3554 | 3548 | 3644 | 3742 | 3843 | 3945 |
| 48 | 2847 | 2929 | 3013 | 3098 | 3185 | 3274 | 3364 | 3556 | 3550 | 3646 | 3744 | 3844 | 3947 |
| 49 | 2848 | 2931 | 3014 | 3100 | 3187 | 3275 | 3365 | 3557 | 3551 | 3647 | 3746 | 3846 | 3949 |
| 50 | 2849 | 2932 | 3016 | 3101 | 3188 | 3277 | 3367 | 3559 | 3553 | 3649 | 3747 | 3848 | 3951 |
| 51 | 2851 | 2933 | 3017 | 3103 | 3190 | 3278 | 3368 | 3560 | 3555 | 3651 | 3749 | 3849 | 3952 |
| 52 | 2852 | 2935 | 3019 | 3104 | 3191 | 3280 | 3370 | 3562 | 3556 | 3652 | 3750 | 3851 | 3954 |
| 53 | 2854 | 2936 | 3020 | 3105 | 3192 | 3281 | 3371 | 3564 | 3558 | 3654 | 3752 | 3853 | 3956 |
| 54 | 2855 | 2937 | 3021 | 3107 | 3194 | 3283 | 3373 | 3565 | 3559 | 3655 | 3754 | 3854 | 3958 |
| 55 | 2856 | 2939 | 3023 | 3108 | 3195 | 3284 | 3374 | 3567 | 3561 | 3657 | 3755 | 3856 | 3959 |
| 56 | 2858 | 2940 | 3024 | 3110 | 3197 | 3286 | 3376 | 3568 | 3562 | 3659 | 3757 | 3858 | 3961 |
| 57 | 2859 | 2942 | 3026 | 3111 | 3198 | 3287 | 3378 | 3570 | 3564 | 3660 | 3759 | 3860 | 3963 |
| 58 | 2860 | 2943 | 3027 | 3113 | 3200 | 3289 | 3379 | 3571 | 3566 | 3662 | 3760 | 3861 | 3964 |
| 59 | 2862 | 2944 | 3029 | 3114 | 3201 | 3290 | 3381 | 3573 | 3567 | 3664 | 3762 | 3863 | 3966 |

|    | 55°  | 56°  | 57°  | 58°  | 59°  | 60°  | 61°  | 62°  | 63°  | 64°  | 65°  | 66°  | 67°  |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0  | 3968 | 4074 | 4183 | 4294 | 4409 | 4527 | 4649 | 4775 | 4905 | 5039 | 5179 | 5324 | 5474 |
| 1  | 3970 | 4076 | 4184 | 4296 | 4411 | 4529 | 4651 | 4777 | 4907 | 5042 | 5181 | 5326 | 5477 |
| 2  | 3971 | 4077 | 4186 | 4298 | 4413 | 4531 | 4653 | 4779 | 4909 | 5044 | 5184 | 5328 | 5479 |
| 3  | 3973 | 4079 | 4188 | 4300 | 4415 | 4533 | 4655 | 4781 | 4912 | 5046 | 5186 | 5331 | 5482 |
| 4  | 3975 | 4081 | 4190 | 4302 | 4417 | 4535 | 4657 | 4784 | 4914 | 5049 | 5188 | 5333 | 5484 |
| 5  | 3977 | 4083 | 4192 | 4304 | 4419 | 4537 | 4660 | 4786 | 4916 | 5051 | 5191 | 5336 | 5487 |
| 6  | 3978 | 4085 | 4194 | 4306 | 4421 | 4539 | 4662 | 4788 | 4918 | 5053 | 5193 | 5338 | 5489 |
| 7  | 3980 | 4086 | 4195 | 4308 | 4423 | 4541 | 4664 | 4790 | 4920 | 5055 | 5195 | 5341 | 5492 |
| 8  | 3982 | 4088 | 4197 | 4309 | 4425 | 4543 | 4666 | 4792 | 4923 | 5058 | 5198 | 5343 | 5495 |
| 9  | 3984 | 4080 | 4199 | 4311 | 4427 | 4545 | 4668 | 4794 | 4925 | 5060 | 5200 | 5346 | 5497 |
| 10 | 3985 | 4092 | 4202 | 4313 | 4429 | 4547 | 4670 | 4796 | 4927 | 5062 | 5203 | 5348 | 5500 |
| 11 | 3987 | 4094 | 4203 | 4315 | 4431 | 4549 | 4672 | 4798 | 4929 | 5065 | 5205 | 5351 | 5502 |
| 12 | 3989 | 4095 | 4205 | 4317 | 4433 | 4551 | 4674 | 4801 | 4931 | 5067 | 5207 | 5353 | 5505 |
| 13 | 3991 | 4097 | 4207 | 4319 | 4434 | 4553 | 4676 | 4805 | 4934 | 5069 | 5210 | 5356 | 5507 |
| 14 | 3992 | 4099 | 4208 | 4321 | 4436 | 4555 | 4678 | 4808 | 4936 | 5071 | 5212 | 5358 | 5510 |
| 15 | 3994 | 4101 | 4210 | 4323 | 4438 | 4557 | 4680 | 4807 | 4938 | 5074 | 5214 | 5361 | 5513 |
| 16 | 3996 | 4103 | 4212 | 4325 | 4440 | 4559 | 4682 | 4809 | 4940 | 5076 | 5217 | 5363 | 5515 |
| 17 | 3998 | 4104 | 4214 | 4327 | 4442 | 4562 | 4684 | 4811 | 4943 | 5078 | 5219 | 5366 | 5518 |
| 18 | 3999 | 4106 | 4216 | 4328 | 4444 | 4564 | 4687 | 4814 | 4945 | 5081 | 5222 | 5368 | 5520 |
| 19 | 4001 | 4108 | 4218 | 4330 | 4446 | 4566 | 4689 | 4816 | 4947 | 5083 | 5224 | 5371 | 5523 |
| 20 | 4003 | 4110 | 4220 | 4332 | 4448 | 4568 | 4691 | 4818 | 4949 | 5085 | 5226 | 5373 | 5526 |
| 21 | 4005 | 4112 | 4221 | 4334 | 4450 | 4570 | 4693 | 4820 | 4951 | 5088 | 5229 | 5376 | 5528 |
| 22 | 4006 | 4113 | 4223 | 4336 | 4452 | 4572 | 4695 | 4822 | 4954 | 5090 | 5231 | 5378 | 5531 |
| 23 | 4008 | 4115 | 4225 | 4338 | 4454 | 4574 | 4697 | 4824 | 4956 | 5092 | 5234 | 5380 | 5533 |
| 24 | 4010 | 4117 | 4227 | 4340 | 4456 | 4576 | 4699 | 4826 | 4958 | 5095 | 5236 | 5383 | 5536 |
| 25 | 4012 | 4119 | 4229 | 4342 | 4458 | 4578 | 4701 | 4829 | 4960 | 5097 | 5238 | 5385 | 5539 |
| 26 | 4014 | 4121 | 4231 | 4344 | 4460 | 4580 | 4703 | 4831 | 4963 | 5099 | 5241 | 5388 | 5541 |
| 27 | 4015 | 4122 | 4232 | 4346 | 4462 | 4582 | 4705 | 4833 | 4965 | 5102 | 5243 | 5390 | 5544 |
| 28 | 4017 | 4124 | 4234 | 4347 | 4464 | 4584 | 4707 | 4835 | 4967 | 5104 | 5246 | 5393 | 5546 |
| 29 | 4019 | 4126 | 4236 | 4349 | 4466 | 4586 | 4710 | 4837 | 4969 | 5106 | 5248 | 5395 | 5549 |
| 30 | 4021 | 4128 | 4238 | 4351 | 4468 | 4588 | 4712 | 4839 | 4972 | 5108 | 5250 | 5398 | 5552 |
| 31 | 4022 | 4130 | 4240 | 4353 | 4470 | 4590 | 4714 | 4842 | 4974 | 5111 | 5253 | 5401 | 5554 |
| 32 | 4024 | 4132 | 4242 | 4355 | 4472 | 4592 | 4716 | 4844 | 4976 | 5113 | 5255 | 5403 | 5557 |
| 33 | 4026 | 4133 | 4244 | 4357 | 4474 | 4594 | 4718 | 4846 | 4978 | 5115 | 5258 | 5406 | 5559 |
| 34 | 4028 | 4135 | 4246 | 4359 | 4476 | 4596 | 4720 | 4848 | 4981 | 5118 | 5260 | 5408 | 5562 |
| 35 | 4029 | 4137 | 4247 | 4361 | 4478 | 4598 | 4722 | 4850 | 4983 | 5120 | 5263 | 5411 | 5565 |
| 36 | 4031 | 4139 | 4249 | 4363 | 4480 | 4600 | 4724 | 4852 | 4985 | 5122 | 5265 | 5413 | 5567 |
| 37 | 4033 | 4141 | 4251 | 4365 | 4482 | 4602 | 4726 | 4855 | 4987 | 5125 | 5267 | 5416 | 5570 |
| 38 | 4035 | 4142 | 4253 | 4367 | 4484 | 4604 | 4728 | 4857 | 4990 | 5127 | 5270 | 5418 | 5573 |
| 39 | 4037 | 4144 | 4255 | 4369 | 4486 | 4606 | 4731 | 4859 | 4992 | 5129 | 5272 | 5421 | 5575 |
| 40 | 4038 | 4146 | 4257 | 4370 | 4488 | 4608 | 4733 | 4861 | 4994 | 5132 | 5275 | 5423 | 5578 |
| 41 | 4040 | 4148 | 4259 | 4372 | 4490 | 4610 | 4735 | 4863 | 4996 | 5134 | 5277 | 5426 | 5580 |
| 42 | 4042 | 4150 | 4260 | 4374 | 4492 | 4612 | 4736 | 4865 | 4999 | 5136 | 5280 | 5428 | 5583 |
| 43 | 4044 | 4152 | 4262 | 4376 | 4494 | 4614 | 4739 | 4868 | 5001 | 5139 | 5282 | 5431 | 5586 |
| 44 | 4045 | 4153 | 4264 | 4378 | 4495 | 4616 | 4741 | 4870 | 5003 | 5141 | 5284 | 5433 | 5588 |
| 45 | 4047 | 4155 | 4266 | 4380 | 4497 | 4618 | 4743 | 4872 | 5005 | 5143 | 5287 | 5436 | 5591 |
| 46 | 4049 | 4157 | 4268 | 4382 | 4499 | 4620 | 4745 | 4874 | 5008 | 5146 | 5289 | 5438 | 5594 |
| 47 | 4051 | 4159 | 4270 | 4384 | 4501 | 4623 | 4747 | 4876 | 5010 | 5148 | 5292 | 5441 | 5596 |
| 48 | 4052 | 4161 | 4272 | 4386 | 4503 | 4625 | 4750 | 4879 | 5012 | 5151 | 5294 | 5443 | 5599 |
| 49 | 4054 | 4162 | 4274 | 4388 | 4505 | 4627 | 4752 | 4881 | 5014 | 5153 | 5297 | 5446 | 5602 |
| 50 | 4056 | 4164 | 4275 | 4390 | 4507 | 4629 | 4754 | 4883 | 5017 | 5155 | 5299 | 5448 | 5604 |
| 51 | 4058 | 4166 | 4277 | 4392 | 4509 | 4631 | 4756 | 4885 | 5019 | 5158 | 5301 | 5451 | 5607 |
| 52 | 4060 | 4168 | 4279 | 4394 | 4511 | 4633 | 4758 | 4887 | 5021 | 5160 | 5304 | 5454 | 5610 |
| 53 | 4061 | 4170 | 4281 | 4396 | 4513 | 4635 | 4760 | 4890 | 5023 | 5162 | 5306 | 5456 | 5612 |
| 54 | 4063 | 4172 | 4283 | 4398 | 4515 | 4637 | 4762 | 4892 | 5026 | 5165 | 5309 | 5458 | 5615 |
| 55 | 4065 | 4173 | 4285 | 4399 | 4517 | 4639 | 4764 | 4894 | 5028 | 5167 | 5311 | 5461 | 5617 |
| 56 | 4067 | 4175 | 4287 | 4401 | 4519 | 4641 | 4766 | 4896 | 5030 | 5169 | 5314 | 5464 | 5620 |
| 57 | 4069 | 4177 | 4289 | 4403 | 4521 | 4643 | 4769 | 4898 | 5033 | 5172 | 5316 | 5466 | 5623 |
| 58 | 4070 | 4179 | 4291 | 4405 | 4523 | 4645 | 4771 | 4901 | 5035 | 5174 | 5319 | 5469 | 5625 |
| 59 | 4072 | 4181 | 4292 | 4407 | 4525 | 4647 | 4773 | 4903 | 5037 | 5176 | 5321 | 5471 | 5628 |

TABLE IV.

## Meridional Parts.

99

|    | 68°  | 69°  | 70°  | 71°  | 72°  | 73°  | 74°  | 75°  | 76°  | 77°  | 78°  | 79°  | 80°  |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0  | 5631 | 5795 | 5966 | 6146 | 6335 | 6534 | 6746 | 6970 | 7210 | 7467 | 7745 | 8046 | 8375 |
| 1  | 5633 | 5797 | 5969 | 6149 | 6338 | 6538 | 6749 | 6974 | 7214 | 7472 | 7749 | 8051 | 8381 |
| 2  | 5636 | 5800 | 5972 | 6152 | 6341 | 6541 | 6753 | 6978 | 7218 | 7476 | 7754 | 8056 | 8387 |
| 3  | 5639 | 5803 | 5975 | 6155 | 6345 | 6545 | 6757 | 6982 | 7222 | 7481 | 7759 | 8061 | 8393 |
| 4  | 5642 | 5806 | 5978 | 6158 | 6348 | 6548 | 6760 | 6986 | 7227 | 7485 | 7764 | 8067 | 8398 |
| 5  | 5644 | 5809 | 5981 | 6161 | 6351 | 6552 | 6764 | 6990 | 7231 | 7490 | 7769 | 8072 | 8404 |
| 6  | 5646 | 5811 | 5984 | 6164 | 6354 | 6555 | 6768 | 6994 | 7235 | 7494 | 7774 | 8077 | 8410 |
| 7  | 5650 | 5814 | 5986 | 6167 | 6358 | 6558 | 6771 | 6997 | 7239 | 7498 | 7778 | 8083 | 8416 |
| 8  | 5652 | 5817 | 5989 | 6170 | 6361 | 6562 | 6775 | 7001 | 7243 | 7503 | 7783 | 8088 | 8422 |
| 9  | 5655 | 5820 | 5992 | 6173 | 6364 | 6565 | 6779 | 7005 | 7247 | 7507 | 7788 | 8093 | 8427 |
| 10 | 5658 | 5823 | 5995 | 6177 | 6367 | 6569 | 6782 | 7009 | 7252 | 7512 | 7793 | 8099 | 8433 |
| 11 | 5660 | 5825 | 5998 | 6180 | 6371 | 6572 | 6786 | 7013 | 7256 | 7516 | 7798 | 8104 | 8439 |
| 12 | 5663 | 5828 | 6001 | 6183 | 6374 | 6576 | 6790 | 7017 | 7260 | 7521 | 7803 | 8109 | 8445 |
| 13 | 5666 | 5831 | 6004 | 6186 | 6377 | 6579 | 6793 | 7021 | 7264 | 7525 | 7808 | 8115 | 8451 |
| 14 | 5668 | 5834 | 6007 | 6189 | 6380 | 6583 | 6797 | 7025 | 7268 | 7530 | 7813 | 8120 | 8457 |
| 15 | 5671 | 5837 | 6010 | 6192 | 6384 | 6586 | 6801 | 7029 | 7273 | 7535 | 7817 | 8125 | 8463 |
| 16 | 5674 | 5839 | 6013 | 6195 | 6387 | 6590 | 6804 | 7033 | 7277 | 7539 | 7821 | 8131 | 8469 |
| 17 | 5676 | 5842 | 6016 | 6198 | 6390 | 6593 | 6808 | 7027 | 7281 | 7544 | 7827 | 8136 | 8474 |
| 18 | 5679 | 5845 | 6019 | 6201 | 6394 | 6597 | 6812 | 7041 | 7285 | 7548 | 7832 | 8141 | 8480 |
| 19 | 5682 | 5848 | 6022 | 6205 | 6397 | 6600 | 6815 | 7045 | 7289 | 7553 | 7837 | 8147 | 8486 |
| 20 | 5685 | 5851 | 6025 | 6208 | 6400 | 6603 | 6819 | 7048 | 7294 | 7557 | 7842 | 8152 | 8492 |
| 21 | 5687 | 5854 | 6028 | 6211 | 6403 | 6607 | 6823 | 7052 | 7298 | 7562 | 7847 | 8158 | 8498 |
| 22 | 5690 | 5856 | 6031 | 6214 | 6407 | 6610 | 6826 | 7056 | 7302 | 7566 | 7852 | 8163 | 8504 |
| 23 | 5693 | 5859 | 6034 | 6217 | 6410 | 6614 | 6830 | 7060 | 7306 | 7571 | 7857 | 8168 | 8510 |
| 24 | 5695 | 5862 | 6037 | 6220 | 6413 | 6617 | 6834 | 7064 | 7311 | 7576 | 7862 | 8174 | 8516 |
| 25 | 5698 | 5865 | 6040 | 6223 | 6417 | 6621 | 6838 | 7068 | 7315 | 7580 | 7867 | 8179 | 8522 |
| 26 | 5701 | 5868 | 6043 | 6226 | 6420 | 6624 | 6841 | 7072 | 7319 | 7585 | 7872 | 8185 | 8528 |
| 27 | 5704 | 5871 | 6046 | 6230 | 6423 | 6628 | 6845 | 7076 | 7323 | 7589 | 7877 | 8190 | 8534 |
| 28 | 5706 | 5874 | 6049 | 6233 | 6427 | 6631 | 6849 | 7080 | 7328 | 7594 | 7882 | 8196 | 8540 |
| 29 | 5709 | 5876 | 6052 | 6236 | 6430 | 6635 | 6853 | 7084 | 7332 | 7599 | 7887 | 8201 | 8546 |
| 30 | 5712 | 5879 | 6055 | 6239 | 6433 | 6639 | 6856 | 7088 | 7336 | 7603 | 7892 | 8207 | 8552 |
| 31 | 5715 | 5882 | 6058 | 6242 | 6437 | 6642 | 6860 | 7092 | 7341 | 7608 | 7897 | 8212 | 8558 |
| 32 | 5717 | 5885 | 6061 | 6245 | 6440 | 6646 | 6864 | 7096 | 7345 | 7612 | 7902 | 8218 | 8565 |
| 33 | 5720 | 5888 | 6064 | 6249 | 6443 | 6649 | 6868 | 7100 | 7349 | 7617 | 7907 | 8223 | 8571 |
| 34 | 5723 | 5891 | 6067 | 6252 | 6447 | 6653 | 6871 | 7104 | 7353 | 7622 | 7912 | 8229 | 8577 |
| 35 | 5725 | 5894 | 6070 | 6255 | 6450 | 6656 | 6875 | 7108 | 7358 | 7626 | 7917 | 8234 | 8583 |
| 36 | 5728 | 5896 | 6073 | 6258 | 6453 | 6660 | 6879 | 7112 | 7362 | 7631 | 7922 | 8240 | 8589 |
| 37 | 5731 | 5899 | 6076 | 6261 | 6457 | 6663 | 6883 | 7116 | 7366 | 7636 | 7927 | 8245 | 8595 |
| 38 | 5734 | 5902 | 6079 | 6264 | 6460 | 6667 | 6886 | 7120 | 7371 | 7640 | 7932 | 8251 | 8601 |
| 39 | 5736 | 5905 | 6082 | 6268 | 6463 | 6670 | 6890 | 7124 | 7375 | 7645 | 7937 | 8256 | 8607 |
| 40 | 5739 | 5908 | 6085 | 6271 | 6467 | 6674 | 6894 | 7128 | 7379 | 7650 | 7942 | 8262 | 8614 |
| 41 | 5742 | 5911 | 6088 | 6274 | 6470 | 6677 | 6898 | 7132 | 7384 | 7654 | 7948 | 8267 | 8620 |
| 42 | 5745 | 5914 | 6091 | 6277 | 6473 | 6681 | 6901 | 7136 | 7388 | 7659 | 7953 | 8273 | 8626 |
| 43 | 5747 | 5917 | 6094 | 6280 | 6477 | 6685 | 6905 | 7140 | 7392 | 7664 | 7958 | 8279 | 8632 |
| 44 | 5750 | 5919 | 6097 | 6283 | 6480 | 6688 | 6909 | 7145 | 7397 | 7668 | 7963 | 8284 | 8638 |
| 45 | 5753 | 5922 | 6100 | 6287 | 6483 | 6692 | 6913 | 7149 | 7401 | 7673 | 7968 | 8290 | 8644 |
| 46 | 5756 | 5925 | 6103 | 6290 | 6487 | 6695 | 6917 | 7153 | 7406 | 7678 | 7973 | 8295 | 8651 |
| 47 | 5758 | 5928 | 6106 | 6293 | 6490 | 6699 | 6920 | 7157 | 7410 | 7683 | 7978 | 8301 | 8657 |
| 48 | 5761 | 5931 | 6109 | 6296 | 6494 | 6702 | 6924 | 7161 | 7414 | 7687 | 7983 | 8307 | 8663 |
| 49 | 5764 | 5934 | 6112 | 6299 | 6497 | 6706 | 6928 | 7165 | 7419 | 7692 | 7989 | 8312 | 8669 |
| 50 | 5767 | 5937 | 6115 | 6303 | 6500 | 6710 | 6932 | 7169 | 7423 | 7697 | 7994 | 8318 | 8676 |
| 51 | 5770 | 5940 | 6118 | 6306 | 6504 | 6713 | 6936 | 7173 | 7427 | 7702 | 7999 | 8324 | 8682 |
| 52 | 5772 | 5943 | 6121 | 6309 | 6507 | 6717 | 6940 | 7177 | 7432 | 7706 | 8004 | 8329 | 8688 |
| 53 | 5775 | 5946 | 6124 | 6312 | 6511 | 6720 | 6943 | 7181 | 7436 | 7711 | 8009 | 8335 | 8695 |
| 54 | 5778 | 5948 | 6127 | 6315 | 6514 | 6724 | 6947 | 7185 | 7441 | 7716 | 8014 | 8341 | 8701 |
| 55 | 5781 | 5951 | 6130 | 6319 | 6517 | 6728 | 6951 | 7189 | 7445 | 7721 | 8020 | 8347 | 8707 |
| 56 | 5783 | 5954 | 6133 | 6322 | 6521 | 6731 | 6955 | 7194 | 7449 | 7725 | 8025 | 8352 | 8714 |
| 57 | 5786 | 5957 | 6136 | 6325 | 6524 | 6735 | 6959 | 7198 | 7454 | 7730 | 8030 | 8358 | 8720 |
| 58 | 5789 | 5960 | 6140 | 6328 | 6528 | 6738 | 6963 | 7202 | 7458 | 7735 | 8035 | 8364 | 8726 |
| 59 | 5792 | 5963 | 6143 | 6332 | 6531 | 6742 | 6966 | 7206 | 7463 | 7740 | 8040 | 8369 | 8733 |

